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Corrigendum to “Testing for Unit Roots with Flow Data
and Varying Sampling Frequency”

[Journal of Econometrics 119(1) (2004) 1–18]

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Chambers (2004) considers the properties of tests for unit roots in flow data satisfying

$$dy(t) = \gamma y(t)dt + \sigma dw(t), \quad t > 0, \quad (1)$$

where $w(t)$ denotes a standard Wiener process and $y(0)$ is assumed to be fixed. Observations on $y(t)$ are made at intervals of length $h > 0$ over the interval $0 < t \leq N$ and take the form

$$y_{th} = \int_{th-h}^{th} y(r)dr, \quad t = 1, \dots, T, \quad (2)$$

where the sample size $T = N/h$. Theorem 1 shows that, under the maintained/alternative hypothesis $\gamma \neq 0$, the observations satisfy $y_{th} = \alpha_h y_{th-h} + u_{th}$, where $\alpha_h = e^{\gamma h}$, while under the null hypothesis $\gamma = 0$ they satisfy $y_{th} = y_{th-h} + u_{th}$. It is claimed that these relations hold for $t = 1, \dots, T$, implying that the first observation satisfies $y_h = \alpha_h y(0) + u_h$ under the maintained hypothesis and $y_h = y(0) + u_h$ under the null. In fact, the stated solutions only hold for $t = 2, \dots, T$, because integrating the solution to (1), given by

$$y(t) = e^{\gamma t} y(0) + \sigma \int_0^t e^{\gamma(t-s)} dw(s), \quad t > 0,$$

over the interval $(0, 1]$ shows that under the alternative hypothesis the first observation satisfies $y_h = \gamma^{-1}(e^{\gamma h} - 1)y(0) + u_h$, while under the null $y_h = hy(0) + u_h$. In each case the coefficient on $y(0)$ is $O(h)$ rather than $O(1)$, and here this correct treatment of the initial condition is incorporated to amend the relevant results presented in Chambers (2004).

The asymptotic analysis considers three scenarios in which the sampling frequency increases ($h \downarrow 0$) while the sample size $T \uparrow \infty$ in each case; these are further classified into two cases concerning $y(0)$, namely: (1) $y(0) = 0$; and (2) $y(0) \neq 0$. It is also convenient to index each of h , N and T by a generic index $n \uparrow \infty$, in which case $T_n = N_n/h_n$. The three scenarios are: (a) $N_n = N$ (fixed) and $h_n \downarrow 0$; (b) $N_n \uparrow \infty$ and $h_n \downarrow 0$ such that $h_n N_n^{1/2} \uparrow \infty$; and (c) $N_n \downarrow 0$ and $h_n \downarrow 0$. The motivation for considering these is discussed in the original article.

Two least squares regressions are considered in Chambers (2004) based on data indexed by n of the form $y_{nt} = y_{th_n}$. The first is given by $y_{nt} = \hat{\alpha}_n y_{nt-1} + \hat{u}_{nt}$ while the second includes an intercept and is of the form $y_{nt} = \tilde{\mu} + \tilde{\alpha}_n y_{nt-1} + \tilde{u}_{nt}$. All results in Chambers (2004) concerning $\hat{\alpha}_n$ when $y(0) = 0$ are correct as are all the results for $\tilde{\alpha}_n$ because the second regression eradicates the effects of $y(0)$ regardless of its value. It is therefore **only the results for the regression without an intercept when $y(0) \neq 0$ that need amending i.e. those for $\hat{\alpha}_n$ in Case (2)**.

Under the null hypothesis $\gamma = 0$ the data satisfy

$$y_{nt} = y_{nt-1} + u_{nt}, \quad t = 2, \dots, T_n; \quad y_{n1} = h_n y(0) + u_{n1}, \quad (3)$$

implying that $y_{nt} = \sum_{j=1}^t u_{nj} + h_n y(0)$; the original article used $y_{nt} = \sum_{j=1}^t u_{nj} + y(0)$. It is

convenient to define the following functionals (where $W(r)$ is a Wiener process on $r \in [0, 1]$):

$$\begin{aligned} P(N, y(0), W) &= \frac{N\sigma^2}{2} \left(W(1)^2 - \frac{2}{3} \right) + N^{1/2}\sigma y(0)W(1), \\ Q(N, y(0), W) &= N^2\sigma^2 \int_0^1 W(r)^2 dr + 2N^{3/2}\sigma y(0) \int_0^1 W(r) dr + Ny(0)^2, \\ Z(W) &= \frac{\frac{1}{2} \left(W(1)^2 - \frac{2}{3} \right)}{\int_0^1 W(r)^2 dr}. \end{aligned}$$

The amended parts of Theorem 2, and Lemma A.1 upon which it is based, are:

Theorem 2. Let $\{\{y_{nt}\}_{t=1}^{T_n}\}_{n=1}^\infty$ denote a triangular array of random variables generated by (3). Then, as $n \uparrow \infty$ and $h_n \downarrow 0$, when $y(0) \neq 0$:

- (a) If $N_n = N$, $\frac{1}{h_n}(\hat{\alpha}_n - 1) \Rightarrow \frac{P(N, y(0), W)}{Q(N, y(0), W)}$;
- (b) If $N_n \uparrow \infty$ and $h_n N_n^{1/2} \uparrow \infty$, $T_n(\hat{\alpha}_n - 1) \Rightarrow Z(W)$;
- (c) If $N_n \downarrow 0$, $\frac{N_n^{1/2}}{h_n}(\hat{\alpha}_n - 1) \Rightarrow \frac{\sigma W(1)}{y(0)}$.

Lemma A.1. Let $\{\{y_{nt}\}_{t=1}^{T_n}\}_{n=1}^\infty$ denote a triangular array of random variables generated by (3). Then, as $n \uparrow \infty$ and $h_n \downarrow 0$, when $y(0) \neq 0$:

- (a) If $N_n = N$, $\frac{1}{h_n^2} \sum_{t=1}^{T_n} y_{nt-1} u_{nt} \Rightarrow P(N, y(0), W)$, $\frac{1}{h_n} \sum_{t=1}^{T_n} y_{nt-1}^2 \Rightarrow Q(N, y(0), W)$;
- (b) If $N_n \uparrow \infty$ and $h_n N_n^{1/2} \uparrow \infty$, the limits in Case (1) apply;
- (c) If $N_n \downarrow 0$, $\frac{1}{h_n^2 N_n^{1/2}} \sum_{t=1}^{T_n} y_{nt-1} u_{nt} \Rightarrow \sigma y(0)W(1)$, $\frac{1}{h_n N_n} \sum_{t=1}^{T_n} y_{nt-1}^2 \xrightarrow{p} y(0)^2$.

Proof of Lemma A.1. The objective is to express the sample moments in terms of the random function $X_n(r) = T_n^{-1/2} h_n^{-3/2} \sum_{j=1}^{[T_n r]} \Rightarrow \sigma W(r)$ as $n \uparrow \infty$. It can be shown that

$$\sum_{t=1}^{T_n} y_{nt-1} u_{nt} = \frac{N_n h_n^2}{2} X_n(1)^2 - \frac{h_n^2 N_n}{2} \frac{1}{T_n} \sum_{t=1}^{T_n} (u_{nt}^h)^2 + N_n^{1/2} h_n^2 y(0) X_n(1), \quad (4)$$

$$\sum_{t=1}^{T_n} y_{nt-1}^2 = N_n^2 h_n \int_0^1 X_n(r)^2 dr + 2h_n N_n^{3/2} y(0) \int_0^1 X_n(r) dr + N_n h_n y(0)^2, \quad (5)$$

where $u_{nt}^h = h_n^{-3/2} u_{nt}$. The results follow straightforwardly from these expressions. \square

Under the alternative hypothesis $\gamma \neq 0$ the data satisfy

$$y_{nt} = \alpha_n y_{nt-1} + u_{nt}, \quad t = 2, \dots, T_n; \quad y_{n1} = \gamma^{-1}(\alpha_n - 1)y(0) + u_{n1}, \quad (6)$$

implying that $y_{nt} = \sum_{j=1}^t e^{(t-j)\gamma h_n} u_{nj} + \gamma^{-1} e^{t\gamma h_n} (1 - e^{-\gamma h_n}) y(0)$; the original article used

$y_{nt} = \sum_{j=1}^t e^{(t-j)\gamma h_n} u_{nj} + e^{t\gamma h_n} y(0)$. It is convenient to define the functionals

$$\begin{aligned}\Theta(N, y(0), J) &= \sigma^2 N \left(J(1)^2 - \frac{2}{3} \right) + 2\sigma N^{1/2} e^{\gamma N} y(0) J(1) + \left(e^{2\gamma N} - 1 \right) y(0)^2, \\ \Phi(N, y(0), J) &= \sigma^2 N^2 \int_0^1 J(r)^2 dr + 2\sigma y(0) N^{3/2} \int_0^1 e^{r\gamma N} J(r) dr + \frac{(e^{2\gamma N} - 1)}{2\gamma} y(0)^2,\end{aligned}$$

where $J(r) = \int_0^r e^{(r-s)\gamma N} dW(s)$. The amended parts of Theorem 3 and Lemma A.2 are:

Theorem 3. Let $\{\{y_{nt}\}_{t=1}^{T_n}\}_{n=1}^\infty$ denote a triangular array of random variables generated by (6). Then, as $n \uparrow \infty$ and $h_n \downarrow 0$, when $y(0) \neq 0$:

- (a) If $N_n = N$, $\frac{1}{h_n}(\hat{\alpha}_n - \alpha_n) \Rightarrow \frac{\Theta(N, y(0), J)}{2\Phi(N, y(0), J)} - \gamma$ and $\frac{1}{h_n}(\hat{\alpha}_n - 1) \Rightarrow \frac{\Theta(N, y(0), J)}{2\Phi(N, y(0), J)}$;
- (b)(i) If $\gamma > 0$, $N_n \uparrow \infty$ and $h_n N_n^{1/2} \uparrow \infty$, $\frac{1}{h_n}(\hat{\alpha}_n - \alpha_n) \xrightarrow{p} 0$ and $\frac{1}{h_n}(\hat{\alpha}_n - 1) \xrightarrow{p} \gamma$;
- (b)(ii) If $\gamma < 0$, $N_n \uparrow \infty$ and $h_n N_n^{1/2} \uparrow \infty$, $\frac{1}{h_n}(\hat{\alpha}_n - \alpha_n) \xrightarrow{p} \gamma \left(1 + \frac{2}{3}\sigma^2\right)$ and $\frac{1}{h_n}(\hat{\alpha}_n - 1) \xrightarrow{p} 2\gamma \left(1 + \frac{1}{3}\sigma^2\right)$;
- (c) If $N_n \downarrow 0$, $\frac{N_n^{1/2}}{h_n}(\hat{\alpha}_n - \alpha_n) \Rightarrow \frac{\sigma W(1)}{y(0)}$ and $\frac{N_n^{1/2}}{h_n}(\hat{\alpha}_n - 1) \Rightarrow \frac{\sigma W(1)}{y(0)}$.

Lemma A.2. Let $\{\{y_{nt}\}_{t=1}^{T_n}\}_{n=1}^\infty$ denote a triangular array of random variables generated by (6). Then, as $n \uparrow \infty$ and $h_n \downarrow 0$, when $y(0) \neq 0$:

- (a) If $N_n = N$, $\frac{1}{h_n^2} \sum_{t=1}^{T_n} y_{nt-1} u_{nt} \Rightarrow \frac{1}{2} [\Theta(N, y(0), J) - 2\gamma \Phi(N, y(0), J)]$,
 $\frac{1}{h_n} \sum_{t=1}^{T_n} y_{nt-1}^2 \Rightarrow \Phi(N, y(0), J)$;
- (b)(i) If $\gamma > 0$, $N_n \uparrow \infty$ and $h_n N_n^{1/2} \uparrow \infty$, $\frac{1}{h_n^2 e^{2\gamma N_n}} \sum_{t=1}^{T_n} y_{nt-1} u_{nt} \xrightarrow{p} 0$,
 $\frac{1}{h_n e^{2\gamma N_n}} \sum_{t=1}^{T_n} y_{nt-1}^2 \xrightarrow{p} \frac{1}{4\gamma^2} \left(\eta + \sqrt{2\gamma} y(0) \right)^2$;
- (b)(ii) If $\gamma < 0$, $N_n \uparrow \infty$ and $h_n N_n^{1/2} \uparrow \infty$, $\frac{1}{h_n^2 N_n} \sum_{t=1}^{T_n} y_{nt-1} u_{nt} \xrightarrow{p} -\frac{1}{2} \left(1 + \frac{2}{3}\sigma^2\right)$,
 $\frac{1}{h_n N_n} \sum_{t=1}^{T_n} y_{nt-1}^2 \xrightarrow{p} -\frac{1}{2\gamma}$;
- (c) If $N_n \downarrow 0$, $\frac{1}{h_n N_n^{1/2}} \sum_{t=1}^{T_n} y_{nt-1} u_{nt} \Rightarrow \sigma y(0) W(1)$, $\frac{1}{h_n N_n} \sum_{t=1}^{T_n} y_{nt-1}^2 \xrightarrow{p} y(0)^2$.

Proof of Lemma A.2. Defining $y_{n0} = \gamma^{-1}(e^{\gamma h_n} - 1)y(0)$, some algebra yields

$$\begin{aligned} \sum_{t=1}^{T_n} y_{nt-1}^2 &= e^{-2\gamma h_n} N_n^2 h_n \int_0^1 \left(\int_0^{r-1/T_n} e^{(r-s)\gamma N_n} dX_n(s) \right)^2 dr \\ &+ 2e^{-2\gamma h_n} N_n^{3/2} y(0) \frac{(1 - e^{-\gamma h_n})}{\gamma} \int_0^1 e^{r\gamma N_n} \left(\int_0^{r-1/T_n} e^{(r-s)\gamma N_n} dX_n(s) \right) dr \\ &+ \frac{(1 - e^{-\gamma h_n})^2 (e^{2\gamma N_n} - e^{2\gamma h_n})}{\gamma^2 (e^{2\gamma h_n} - 1)} y(0)^2 + y_{n0}^2, \\ \sum_{t=1}^{T_n} y_{nt-1} u_{nt} &= \frac{1}{2e^{\gamma h_n}} \left[y_{nT_n}^2 - y_{n0}^2 - (e^{2\gamma h_n} - 1) \sum_{t=1}^{T_n} y_{nt-1}^2 - \sum_{t=1}^{T_n} u_{nt}^2 \right], \end{aligned}$$

where $y_{nT_n} = h_n N_n^{1/2} \int_0^1 e^{(1-s)\gamma N_n} dX_n(s) + \gamma^{-1}(1 - e^{-\gamma h_n})e^{\gamma N_n} y(0)$. The results then follow by examining each case in turn and also utilising Lemma A.3 (taking $c = \gamma N_n$). \square

The results in Theorems 2 and 3 enable the consistency properties of tests for unit roots based on the normalised estimators to be established. The amended part of Theorem 4 is:

Theorem 4. Let $\{\{y_{nt}\}_{t=1}^{T_n}\}_{n=1}^{\infty}$ denote a triangular array of random variables generated by (6), and consider testing the null hypothesis $\gamma = 0$ against a fixed alternative $\gamma \neq 0$. Then, as $n \uparrow \infty$ and $h_n \downarrow 0$, when $y(0) \neq 0$:

- (a) If $N_n = N$, $h_n^{-1}(\hat{\alpha}_n - 1)$ is not consistent.
- (b) If $N_n \uparrow \infty$ and $h_n N_n^{1/2} \uparrow \infty$, $T_n(\hat{\alpha}_n - 1)$ is consistent.
- (c) If $N_n \downarrow 0$, $N_n^{1/2} h_n^{-1}(\hat{\alpha}_n - 1)$ is not consistent.

From a qualitative point of view it is now clear that it is not possible to consistently test for a unit root in cases (a) and (c). As shown by Perron (1991) for the case of a stock variable, an increasing span of data is also required with flow variables for consistent unit root testing based on the statistics considered here.

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