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Florentin Smarandache

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# GENERALIZATION OF THE THEOREM OF MENELAUS USING A SELF-RECURRENT METHOD

Florentin Smarandache  
ENSIETA, Brest, France

## Abstract.

This generalization of the Theorem of Menelaus from a triangle to a polygon with  $n$  sides is proven by a self-recurrent method which uses the induction procedure and the Theorem of Menelaus itself.

The **Theorem of Menelaus for a Triangle** is the following:

If a line ( $d$ ) intersects the triangle  $\Delta A_1A_2A_3$  sides  $A_1A_2$ ,  $A_2A_3$ , and  $A_3A_1$  respectively in the points  $M_1$ ,  $M_2$ ,  $M_3$ , then we have the following equality:

$$\frac{M_1A_1}{M_1A_2} \cdot \frac{M_2A_2}{M_2A_3} \cdot \frac{M_3A_3}{M_3A_1} = 1$$

where by  $M_1A_1$  we understand the (positive) length of the segment of line or the distance between  $M_1$  and  $A_1$ ; similarly for all other segments of lines.

Let's generalize the Theorem of Menelaus for any  $n$ -gon (a polygon with  $n$  sides), where  $n \geq 3$ , using our Recurrence Method for Generalizations, which consists in doing an induction and in using the Theorem of Menelaus itself.

For  $n = 3$  the theorem is true, already proven by Menelaus.

## The **Theorem of Menelaus for a Quadrilateral**.

Let's prove it for  $n = 4$ , which will inspire us to do the proof for any  $n$ .

Suppose a line ( $d$ ) intersects the quadrilateral  $A_1A_2A_3A_4$  sides  $A_1A_2$ ,  $A_2A_3$ ,  $A_3A_4$ , and  $A_4A_1$  respectively in the points  $M_1$ ,  $M_2$ ,  $M_3$ , and  $M_4$ , while its diagonal  $A_2A_4$  into the point  $M$  [see Fig. 1 below].

We split the quadrilateral  $A_1A_2A_3A_4$  into two disjoint triangles (3-gons)  $\Delta A_1A_2A_4$  and  $\Delta A_4A_2A_3$ , and we apply the Theorem of Menelaus in each of them, respectively getting the following two equalities:

$$\frac{M_1A_1}{M_1A_2} \cdot \frac{MA_2}{MA_4} \cdot \frac{M_4A_4}{M_4A_1} = 1$$

and

$$\frac{MA_4}{MA_2} \cdot \frac{M_2A_2}{M_2A_3} \cdot \frac{M_3A_3}{M_3A_4} = 1.$$

Now, we multiply these last two relationships and we obtain the Theorem of Menelaus for  $n = 4$  (a quadrilateral):

$$\frac{M_1A_1}{M_1A_2} \cdot \frac{M_2A_2}{M_2A_3} \cdot \frac{M_3A_3}{M_3A_4} \cdot \frac{M_4A_4}{M_4A_1} = 1.$$

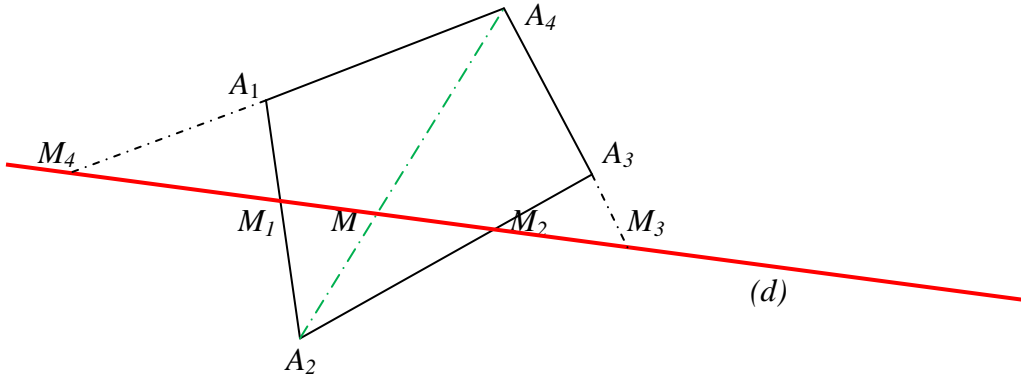


Fig. 1

Let's suppose by induction upon  $k \geq 3$  that the Theorem of Menelaus is true for any  $k$ -gon with  $3 \leq k \leq n-1$ , and we need to prove it is also true for  $k = n$ .

Suppose a line  $(d)$  intersects the  $n$ -gon  $A_1A_2 \dots A_n$  sides  $A_iA_{i+1}$  in the points  $M_i$ , while its diagonal  $A_2A_n$  into the point  $M$  {of course by  $A_nA_{n+1}$  one understands  $A_nA_1$ }.

We consider the  $n$ -gon  $A_1A_2 \dots A_{n-1}A_n$  and we split it similarly as in the case of quadrilaterals in a 3-gon  $\Delta A_1A_2A_n$  and an  $(n-1)$ -gon  $A_nA_2A_3 \dots A_{n-1}$  and we can respectively apply the Theorem of Menelaus according to our previously hypothesis of induction in each of them, and we respectively get:

$$\frac{M_1A_1}{M_1A_2} \cdot \frac{MA_2}{MA_n} \cdot \frac{MnAn}{MnA_1} = 1$$

and

$$\frac{MA_n}{MA_2} \cdot \frac{M_2A_2}{M_2A_3} \cdot \dots \cdot \frac{M_{n-2}A_{n-2}}{M_{n-2}A_{n-1}} \cdot \frac{M_{n-1}A_{n-1}}{M_{n-1}A_n} = 1$$

whence, by multiplying the last two equalities, we get

the **Theorem of Menelaus for any  $n$ -gon**:

$$\prod_{i=1}^n \frac{M_iA_i}{M_iA_{i+1}} = 1.$$

### Conclusion.

We hope the reader will find useful this self-recurrence method in order to generalize known scientific results by means of themselves!

*{Translated from French by the Author.}*

### References:

1. Alain Bouvier et Michel George, sous la direction de François Le Lionnais, *Dictionnaire des Mathématiques*, Presses Universitaires de France, Paris, p. 466 (*Ménélaüs d'Alexandrie*), 1979.
2. Florentin Smarandache, *Généralisation du Théorème de Ménélaüs*, Séminaire de Mathématiques, Lycée Sidi El Hassan Lyoussi, Sefrou, Morocco, 1984.
3. Florentin Smarandache, *Généralisations et Généralités*, Ed. Nouvelle, Fès, Morocco, 1984.
4. F. Smarandache, *Généralisation du Théorème de Ménélaüs*, Rabat, Morocco, Seminar for the selection and preparation of the Moroccan students for the International Olympiad of Mathematics in Paris - France, 1983.