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Francesco Dell'Isola, Stefano Vidoli. Damping of bending waves in truss beams by electrical transmission lines with PZT actuators. *Archive of Applied Mechanics*, 1998, pp.11. hal-00495549

HAL Id: hal-00495549

<https://hal.science/hal-00495549>

Submitted on 28 Jun 2010

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Damping of bending waves in truss beams by electrical transmission lines with PZT actuators

F. dell'Isola, S. Vidoli

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“Having thus obtained the data for a mathematical theory of electromagnetism, and having shown how this theory may be applied to the calculation of phenomena, I shall endeavour to place in as clear a light as I can the relations between the mathematical form of this theory and that of the fundamental science of Dynamics, in order that we may be in some degree prepared to determine the kind of dynamical phenomena among which we are to look for illustrations or explanations of the electromagnetic phenomena.”

J.C. Maxwell, A Treatise on Electricity and Magnetism

Summary A new device to damp mechanical waves in modular truss beams has been proposed in [1]. It is based on the electro-mechanical coupling of the truss beam with an electrical transmission line by a line distribution of PZT actuators. It has been proved in [1] that extensional and torsional waves can be damped using a standard second-order transmission line, and that such a line is not suitable to damp bending waves. In the present paper, we propose to couple the beam with a fourth-order transmission line, obtained from the standard one by adding a voltage-driven current generator, thus electrically paralleling the structure of the bending wave equation. As a detailed description of the system would require huge numerical programming, to test qualitatively the efficiency of the proposed electro-mechanical coupling we consider a coarse continuum model of PiezoElectro-Mechanical (PEM) beams, using an identification procedure based on the principle of virtual power [4]. We define the critical value for line impedance maximizing the electro-mechanical energy exchange for every wave frequency, thus proving that the electric damping of bending waves by distributed PZT control is technically feasible.

Key words generalized principle of virtual power, piezoelectro-mechanical coupling, vibration, damping

1

Introduction

The aim of this paper is to show that it is possible to damp bending waves in truss modular beams by coupling them with fourth-order electric transmission lines and adding PZT actuators in every module. The actuators behave as bars when regarded as mechanical components of the beam, and as capacitances when regarded as electrical components of the line. We show that the PZT actuators now available, see [7], allow for an energy exchange from its mechanical form to the electrical one. As a consequence, the system which we conceive in this paper, called PEM beam, seems to be suitable for space applications. Indeed, when adding dissipative elements in the electrical circuitry (eventually charging batteries) we show that it is possible to damp those mechanical waves which in a frictionless environment would damage the structure.

Received 24 September 1997; accepted for publication 17 June 1998

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One of the authors (S.V.) has been supported by the research contract of Agenzia Spaziale Italiana, issued under the scientific responsibility of Prof. A. Di Carlo. Both authors acknowledge his friendly and accurate criticism.

The electro-mechanical system which we study is characterized by the following features:

- the conceived placement of PZT actuators allows for a continuously distributed control of the mechanical structure;
- the transmission line connecting the PZT actuators allows for a global electro-mechanical response of the structure to external loading;
- the equations governing mechanical and electrical phenomena are of the same type and of the same differential order, thus showing the same modal behavior.

We think that a strong modal coupling represents the most efficient way to control mechanical structures by means of electrical devices.

In [1], the previous ideas were used to prove the feasibility of damping of extensional and torsional waves; we used there the standard second-order transmission line. Moreover, this second-order line was shown in [1] to be not so efficient for bending waves. Thus, we were led to conceive a non-standard electrical transmission line adding a capacitance and a voltage-driven current generator. The reason for this choice is intuitive: we need a line whose governing hyperbolic equation has exactly the same form as well-known Euler-Bernoulli dynamic bending equation.

We identify the constitutive equations for these fourth-order PEM beams in a way similar to that done in [2]. Both for the mechanical structure and for the proposed non-standard transmission line we obtain a coarse continuum model, easily deriving its governing equations by an identification procedure based on the principle of virtual power as generalized in [4]. Of course, the subsequent modal analysis we perform can exactly describe PEM beams only when the truss module lengths are negligible with respect to the considered wave lengths.

In particular, studying the dispersion formula for fourth-order PEM beams we find in the frequency-line inductance plane the locus of the “maximal electro-mechanical exchange” inductances.

For homogeneous boundary conditions and purely mechanical initial conditions, we study a time-evolution problem both for a dissipative and a non-dissipative transmission line, the appropriate choice of the line inductance value being suggested by the aforementioned locus. Contrary to what happens in PEM beams coupled with second-order transmission lines, efficient piezoelectric damping of bending waves is shown to be possible for fourth-order PEM beams by means of PZT actuators already available, [7].

Finally, we find the modal damping ratios as functions of the line resistance; we prove that there exists a critical value for which the electric dissipation of mechanical energy is maximum.

2

System description and modelling procedure

A modular truss beam is assumed to be coupled with a transmission electrical line by introducing piezoelectric actuators which act as bars in the beam module and as capacitances in the electrical line, Fig. 1.

Figure 2 shows the electric mesh constituting a line module: the dashed element represents a voltage-driven current generator that measures the voltage in the capacitance of the module k corresponding to a current i'_k , and generates in the module $(k + 1)$ the current $-i'_k$.

As done in the standard transmission-line theory, we introduce a coarse model for the electric line by defining the function $\gamma(\zeta, t)$ representing the electrical charge stored in the $[0, \zeta]$ -segment of the line at the instant t . The current field is given by its time derivative $i = \dot{\gamma}$.

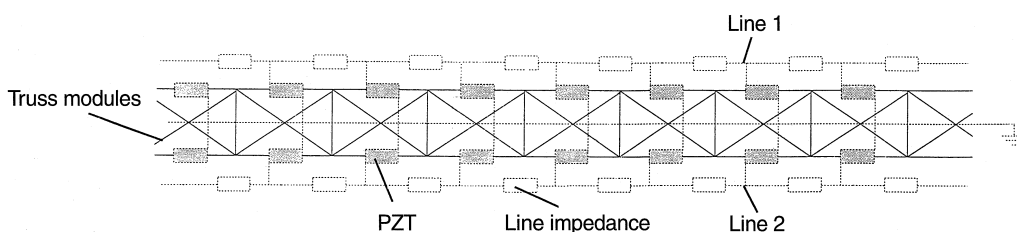


Fig. 1. A scheme of the piezo-electromechanical beam

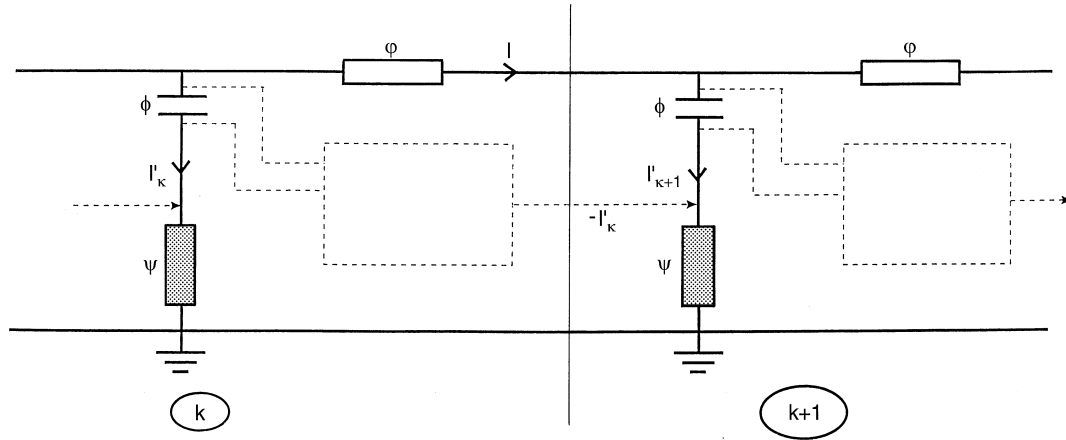


Fig. 2. Modular electric mesh; shaded boxes represent PZT actuators

2.1 Electrical balance equations

The power expended in the continuous electric line, [4], is given by

$$\mathcal{P} = \int_0^L (\varphi \cdot i + \phi \cdot i' + L\psi \cdot i'') d\zeta, \quad (1)$$

where L is the module length, φ is the potential drop per unit length due to the line impedance, ϕ is the potential drop faced at the capacitance plates and ψ is the potential drop at the end points of the piezoelectric bar, Fig. 2.

We remark that the last addend in Eq. (1) arises when the following homogenizing identity is considered:

$$i'_{k+1} - i'_k \simeq Li'' , \quad (2)$$

which is peculiar of the fourth-order transmission line we use.

Integrating by parts we get

$$\mathcal{P} = \int_0^L [(\varphi - \phi' + L \cdot \psi'') \cdot i] d\zeta + [\phi \cdot i - L\psi' \cdot i + L\psi \cdot i']_0^L, \quad (3)$$

that must vanish because of the weak form of Kirchhoff circuital laws, also known as the Tellegen theorem, see [10], for every regular test field $i(\zeta)$ verifying specified boundary conditions. This means that

$$\varphi - \phi' + L \cdot \psi'' = 0, \quad (4)$$

$$[(\phi - L \cdot \psi') \cdot i + L\psi \cdot i']_0^L = 0. \quad (5)$$

Equations (4) and (5) represent the localized electric balance equations.

2.2 Constitutive relations

For PEM truss beams we introduce a coarse continuum model in which the constitutive relations for the dynamical actions may be identified with an approach based on the principle of virtual power as proposed in [1] and in [2]. For the sake of completeness we recall here the most important lines of the procedure developed in [1]. We deal with two models of the system: in the refined one, the PEM structure is a set of elastic and piezoelectric bars connected by discrete circuits, while in the coarse one, it is an equivalent beam coupled with the some electric transmission lines.

A relation between the refined and the continuum models is established by assigning a map from the local state variables in the coarse model, say: the position p_o , attitude \mathbf{d} and charge

fields γ , to the state variables of a module in the refined model, say: the positions $x_{o\pm}$, the attitudes $\mathbf{g}_{k\pm}$ and the charges q_{\pm} of the two diaphragms; for example, we used the following map:

$$\begin{aligned} x_{o-} &= p_o - \frac{L}{2} p'_o, & x_{o+} &= p_o + \frac{L}{2} p'_o, \\ \mathbf{g}_{k-} &= \mathbf{d}_k - \frac{L}{2} \mathbf{d}'_k, & \mathbf{g}_{k+} &= \mathbf{d}_k + \frac{L}{2} \mathbf{d}'_k, \\ q_- &= \gamma - \frac{L}{2} \gamma', & q_+ &= \gamma + \frac{L}{2} \gamma', \end{aligned} \quad (6)$$

denoting by L the reference length of the module.

In order to identify the constitutive functions of the continuum we assume that for any pair of corresponding motions the power expended by the actions (mechanical and electrical) prescribed within the fine model equals the power expended by the corresponding actions within the coarse model

$$\begin{aligned} \mathcal{W}_m^{\text{coarse}} &= \mathcal{W}_m^{\text{refined}}, \\ \mathcal{W}_e^{\text{coarse}} &= \mathcal{W}_e^{\text{refined}}. \end{aligned}$$

In the coarse model, we quote here the expressions for the mechanical contact force and couple (\mathbf{s} , \mathbf{S}), and for the PZT potential drops (ψ_j) $j = 1, \dots, M$ in the case of a linear 2D PEM beam connected with M electrical lines

$$\begin{aligned} \mathbf{s} &= \sum_b (k_{mmb} \varepsilon_b + k_{meb} \rho_b) (A_b \mathbf{t} + \Delta_b \mathbf{n}), \\ \mathbf{S} &= \sum_b \bar{y}_b A_b (k_{mmb} \varepsilon_b + k_{meb} \rho_b) \mathbf{n} \wedge \mathbf{t}, \\ \psi_j &= k_{emj} \varepsilon_j + k_{eej} \rho_j, \end{aligned} \quad (7)$$

where ε_b is the bar elongation, L_b the bar length, $\rho_b = (q_{b+} - q_{b-})/L_b$ the difference of charge per unit length at the end points of the PZT bars and $A_b := L/L_b$, $\Delta_b := (y_{b+} - y_{b-})/L_b$, $\bar{y}_b := (y_{b+} + y_{b-})/2$; the subscript b refers to the generic bar in a module, L_b is the length and $y_{b\pm}$ the placements of the end points of the bar b , \mathbf{t} and \mathbf{n} are the tangent and normal vectors to the axis of the beam and, finally, the k 's are the piezoelectro-mechanical stiffnesses labeled by m (mechanical) and by e (electrical) indices.

Let u , v be the axial and transversal displacements and ϑ the fiber rotation in the coarse 2D continuum model; in [2], bar elongation ε_b is assumed to depend on them as follows:

$$\varepsilon_b = A_b [A_b u' + \Delta_b (v' - \vartheta) - A_b \bar{y}_b \vartheta'] , \quad (8)$$

which in the limit of a purely flexible beam becomes

$$\varepsilon_b = -A_b^2 \bar{y}_b v'' . \quad (9)$$

For the electrical mesh considered, the charge at the end points of the piezoelectric actuators - all initially discharged- is assumed to be

$$\rho_j = LA_j \gamma_j'' . \quad (10)$$

With these assumptions the constitutive relations in the coarse continuum model for the electric potentials in the transmission line j become

$$\begin{aligned} \varphi_j &= K_{Lj} \ddot{\gamma}_j + K_{Rj} \dot{\gamma}_j, \\ \phi_j &= K_{Cj} \gamma_j', \\ \psi_j &= -A_j^2 \bar{y}_j k_{emj} v'' + LA_j k_{eej} \gamma_j'' , \end{aligned} \quad (11)$$

where the K 's and k 's material constants.

2.3 Evolution equations for fourth-order PEM beams

From now on we shall consider the piezo-mechanical module shown in Fig. 3, in which the two PZT bars are connected to two different transmission lines characterized by the same parameters.

Introducing the constitutive relations into the mechanical and balance equations, we get the following evolution equations for v , $\delta := \gamma_1 - \gamma_2$ and $\sigma := \gamma_2 + \gamma_1$:

$$\begin{aligned} K_{mm}v^{IV} + K_{me}\delta^{IV} + \rho\ddot{v} &= 0 , \\ K_{em}v^{IV} + K_{ee}\delta^{IV} - K_C\delta'' + K_L\ddot{\delta} + K_R\dot{\delta} &= 0 , \\ K_{ee}\sigma^{IV} - K_C\sigma'' + K_L\ddot{\sigma} + K_R\dot{\sigma} &= 0 , \end{aligned} \quad (12)$$

generalizing the Euler-Bernoulli dynamic bending beam equation, [5] and [6]. In (12), the K 's are the material constants defined in Appendix and ρ is the beam mass density per unit length. We remark that according to (12)₃ the field σ is not coupled with v and δ ; this is due to the symmetry of the considered module. This means that, with vanishing electric initial data, the field σ is vanishing at every instant t . However, we remark that the field σ , as we have shown in [1], plays a relevant role in damping and controlling the extensional vibrations.

Equations (12)_{1,2} form a system of linear fourth-order hyperbolic equations. In the following section we show that it predicts, when choosing the maximal electro-mechanical exchange inductance K_L^* , a relevant electro-mechanical energy exchange. Similarly, because of the presence of K_R -terms, it predicts the possibility of electric damping of mechanical bending waves.

3 Numerical results

3.1 Waves in infinite PEM beams

We start evaluating the dispersion formula associated to (12)_{1,2} by assuming $K_R = 0$; it is found out, as usual, when searching for solutions in the form

$$\begin{pmatrix} v \\ \delta \end{pmatrix} = \begin{pmatrix} v_o \\ \delta_o \end{pmatrix} e^{\lambda s + j\omega t} , \quad (13)$$

where $\lambda \in \mathcal{C}$ and $\omega \in \mathcal{R}$. We get

$$(K_{mm}\lambda^4 - \rho\omega^2)(k_{ee}\lambda^4 - k_C\lambda^2 - K_L\omega^2) - K_{em}K_{me}\lambda^8 = 0 . \quad (14)$$

The initial energy associated to a given wave form is proportional to

$$E = \begin{pmatrix} v_o \\ \delta_o \end{pmatrix}^T \cdot \begin{pmatrix} K_{mm} & K_{me} \\ K_{em} & K_{ee} \end{pmatrix} \cdot \begin{pmatrix} v_o \\ \delta_o \end{pmatrix} . \quad (15)$$

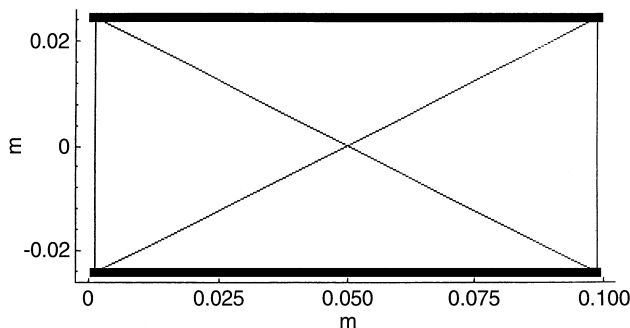


Fig. 3. Truss module; bold lines for PZT bars

The fraction of E related to the electro-mechanical exchange energy is given by

$$R_{ex} = \frac{(K_{me} + K_{em})v_o\delta_o}{E}, \quad (16)$$

which depends on K_L and ω by means of the eigenvector (v_o, δ_o) .

Figure 4 shows the graph of $R_{ex}(K_L, \omega)$. For every ω we define as “maximal exchange” inductances K_L^* those values which locally maximize R_{ex} ; in Fig. 5 we show their locus. We remark that for high frequencies the two branches of the locus merge. Equation (14) has eight solutions for λ at every K_L and ω . Let $\Sigma := \{\lambda_1, \dots, \lambda_8\}$ denote this set of solutions. We partition Σ into four pairs by associating to every root λ_i the closest (in the metric of the complex plane) root $\lambda_{j \neq i}$. Physically this partition means to group together closest wave forms.

Our numerical simulations show that a critical K_L^* minimizes the distance between some of these pairs. This mathematical fact substantiates our intuition which leads us to expect electro-mechanical coupling when the wave forms, the wave lengths and the frequencies are close.

In Fig. 6 we plot the distances of the introduced pairs as functions of K_L .

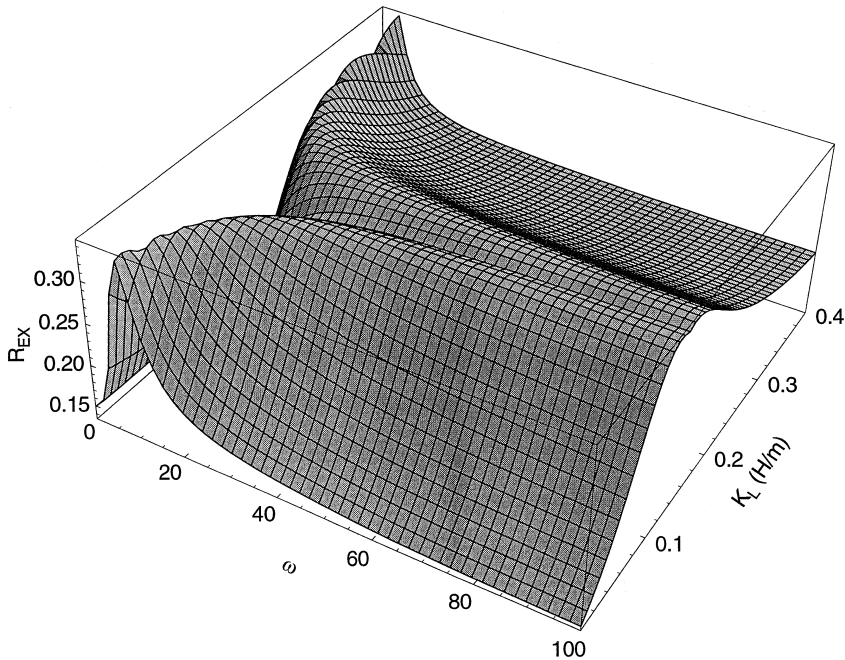


Fig. 4. Energy exchange R_{ex} in the frequency-inductance plane

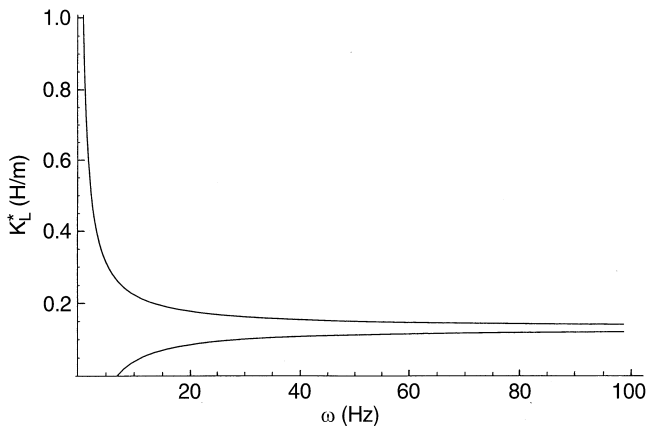


Fig. 5. Locus of maximal exchange value for inductance

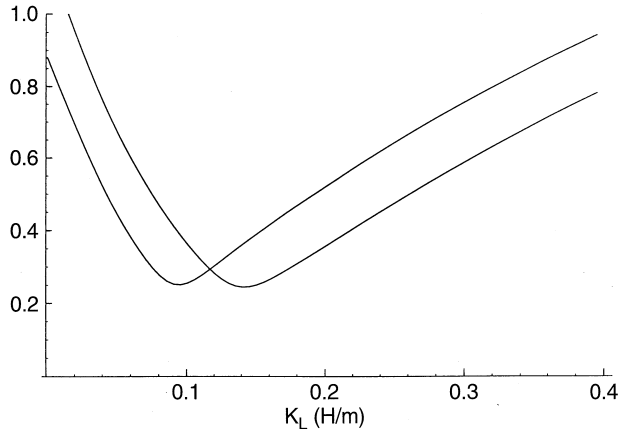


Fig. 6. Distance between different wave lengths vs. K_L ; $\omega = 2\pi \cdot 40 \text{ s}^{-1}$

3.2

Waves in clamped-clamped, electrically insulated PEM beams

We add to (12)_{1,2} the following boundary conditions for a 20-modules (total length $L_T = 2m$) truss beam:

$$\begin{aligned} v(0, t) = v'(0, t) = v(L_T, t) = v'(L_T, t) = 0, \\ \delta(0, t) = \delta'(0, t) = \delta(L_T, t) = \delta'(L_T, t) = 0. \end{aligned} \quad (17)$$

The initial value problem for (12)_{1,2} and (17) is reduced to a system of ordinary time-differential equations by a standard, component-wise, Fourier-Galerkin projection of the following differential operator:

$$D(\phi) = \phi^{IV}, \quad \phi(0, t) = \phi'(0, t) = \phi(L_T, t) = \phi'(L_T, t) = 0,$$

on the set of the eigenfunctions $\{\phi_n(\zeta)\}$.

We remark that when $K_C \rightarrow 0$ the set $\{\phi_n(\zeta)\}$ forms the set of eigenfunctions for the system (12)_{1,2}, (17). Let $\mathbf{x}(t)$ be the obtained set of Fourier coefficients. Then Eqs. (12)_{1,2} take the form

$$\mathbf{M}_{(K_L)} \ddot{\mathbf{x}} + \mathbf{R}_{(K_R)} \dot{\mathbf{x}} + \mathbf{K}_{(K_C)} \mathbf{x} = 0. \quad (18)$$

We look for the solutions of (18) having the form

$$\mathbf{x}(t) = \bar{\mathbf{x}} e^{(r+j\omega)t} = \bar{\mathbf{x}} e^{\left(\frac{r}{\omega} + j\right)\omega t}, \quad (19)$$

which physically represents the time-dependent coefficient vector of the spatial modal forms, where r/ω represents the dimensionless damping ratio and ω the pulsation.

In Fig. 7, the damping ratios r/ω and frequencies $|\omega|/2\pi$ are plotted as functions of K_R when $K_L = K_L^*$ for the first modal form. Figure 7 shows that:

1. both branches 1 and 2, related respectively to mainly mechanical and mainly electrical wave forms, are distorted by the electro-mechanical coupling;
2. related to branch 1, there exists a range \mathcal{S}_R for K_R in which a frequency veering and a relevant increase of the mechanical damping ratio occur;
3. because of the electro-mechanical coupling (maximal at $K_L = K_L^*$), the eigenvectors $\bar{\mathbf{x}}_{1,2}$ related to branch 1 and 2 have comparable mechanical and electrical parts when $K_R \in \mathcal{S}_R$; on the other hand, when $K_R \notin \mathcal{S}_R$, $\bar{\mathbf{x}}_1$ and $\bar{\mathbf{x}}_2$ have, respectively, purely mechanical and purely electrical nature.

Figure 8 shows the solution of a time-evolution problem starting from the initial data

$$v(\cdot, 0) = \phi_1(\cdot), \quad \dot{v}(\cdot, 0) = 0, \quad \delta(\cdot, 0) = 0, \quad \dot{\delta}(\cdot, 0) = 0. \quad (20)$$

A mutual exchange of mechanical and electrical energies is observed when the line is assumed to be non-dissipative, i.e. $K_R = 0$; this demonstrates that the critical value K_L^* efficiently couples the electrical and mechanical modes.

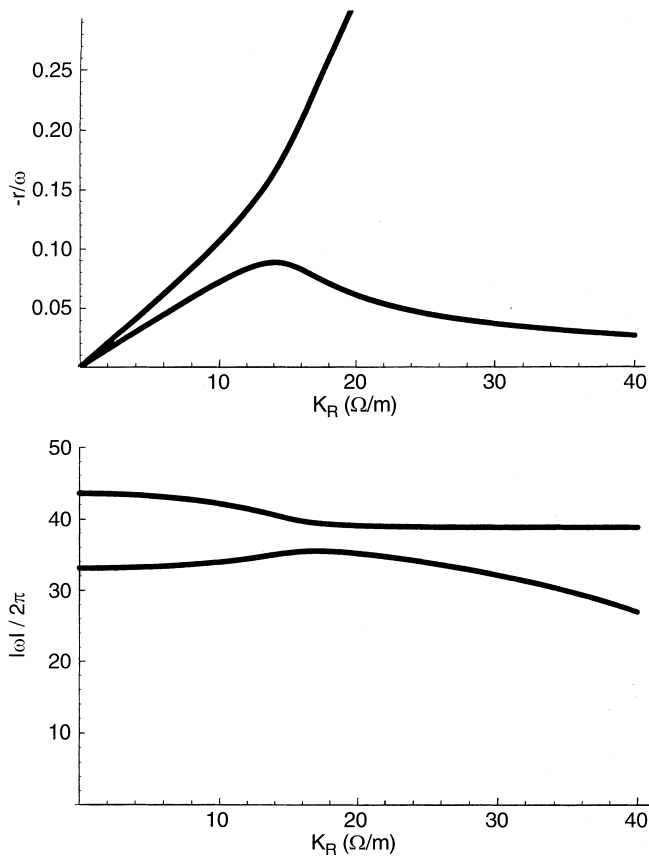


Fig. 7. Arguments of the time-exponential in (19) as functions of K_R

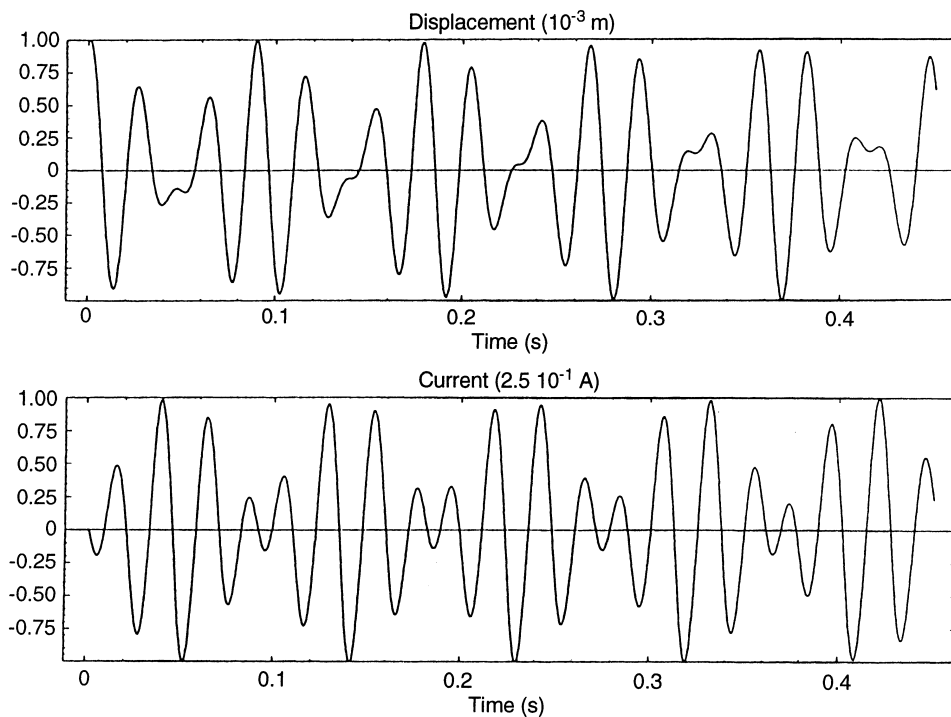


Fig. 8. Graph of $v(L_T/2, t)$ and $\dot{\delta}(L_T/2, t)$ non dissipative line, $K_L = K_L^*$

Figure 9 shows time evolution starting from the same initial data (20), but with a dissipative line: the energy once transformed in the electrical form is now dissipated by Joule effect.

Often we are interested to damp a mechanical vibration as efficiently as possible. A reasonable measure of damping efficiency can be defined as follows: let us consider the solution of

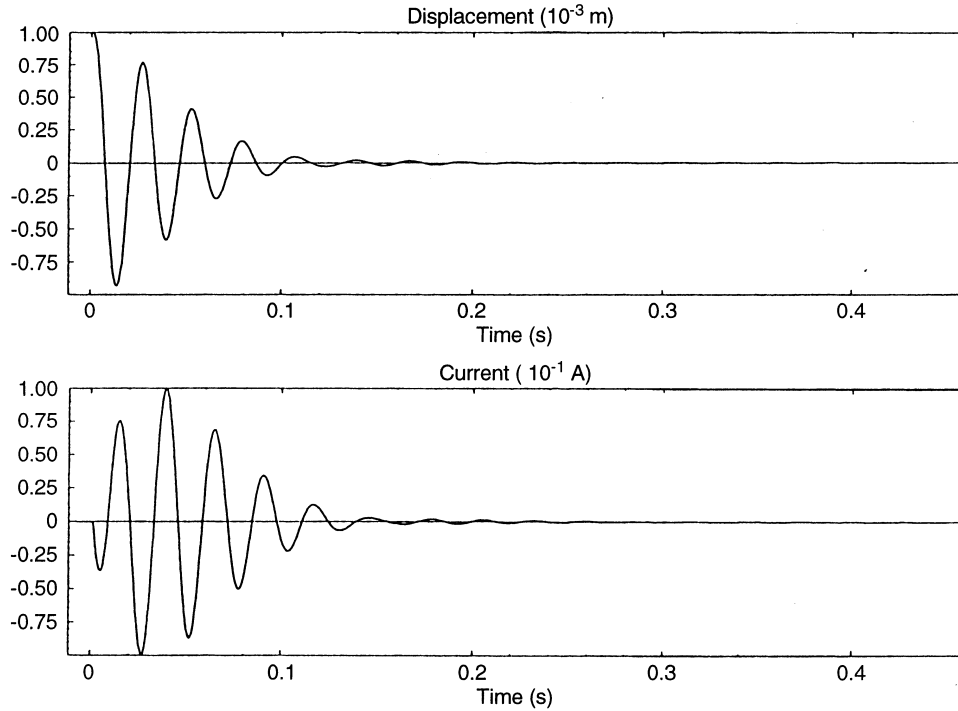


Fig. 9. Graph of $v(L_T/2, t)$ and $\dot{\delta}(L_T/2, t)$, dissipative line, $K_R = 12 \Omega/\text{m}$, $K_L = K_L^*$

(12)_{1,2} with (17) and some purely mechanical initial data; then, calculate the energy dissipated by the line in a period of the main harmonic. We call E_d the ratio between this dissipated energy and the initial energy. Figure 10 shows the contour plots of E_d as function of the electrical parameters K_L and K_R for two choices of initial data ($k = 1$ in Fig. 10a, $k = 2$ in Fig. 10b)

$$v(\cdot, 0) = \phi_k(\cdot), \quad \dot{v}(\cdot, 0) = 0, \quad \delta(\cdot, 0) = 0, \quad \dot{\delta}(\cdot, 0) = 0. \quad (21)$$

We can easily locate an optimal bounded range \mathcal{S}_{LR} for the electrical parameters K_L and K_R ; they behave as control parameters, respectively, for the electro-mechanical coupling and for the dissipated energy. We remark that:

1. the value for inductance maximizing E_d is constant as a function of K_R ; this constant value coincides with the “maximal exchange” value as defined by Fig. 5;
2. however, the “maximal dissipation” value for the line resistance K_R^* depends on the mode number;
3. this last value belongs to the veering range \mathcal{S}_R in which the mechanical damping ratio attains its maximum.

A physical interpretation of the existence of a critical value K_R^* may be the following: the line resistance has to be large enough to assure an electrical dissipation, and small enough to allow for the flow of the electro-mechanically induced current.

4

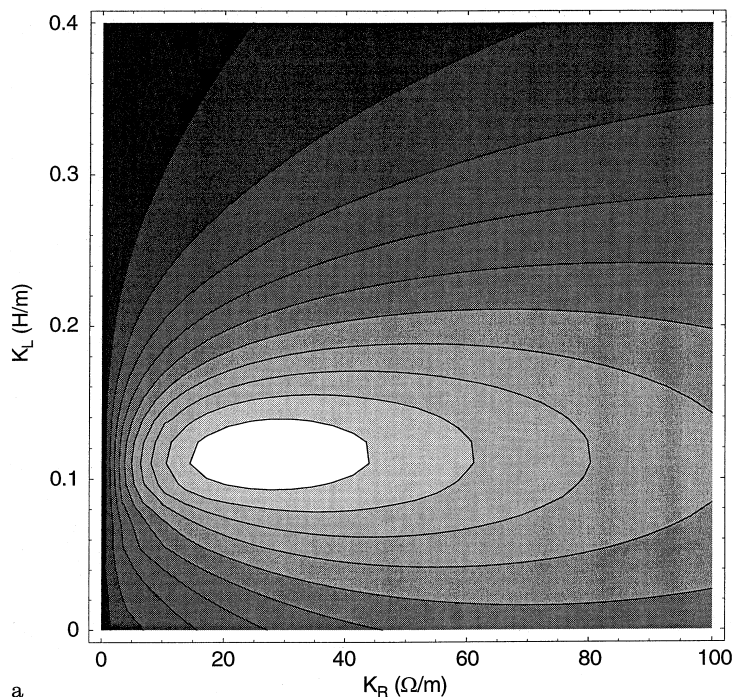
Concluding remarks

In this paper, as announced in [1], we prove that a fourth-order transmission line coupled with a beam by piezoelectric extensional actuators allows for an efficient modal coupling and, therefore, for an efficient vibration damping.

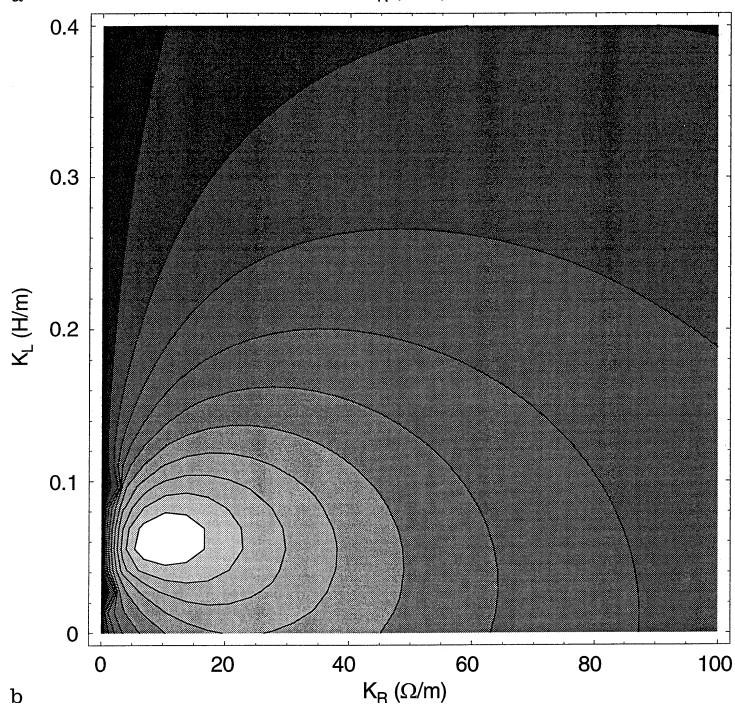
The conceived control device features very high damping ratios (≈ 0.1) and a bandwidth coinciding with intrinsic one of the PZT actuators in use; indeed one can choose optimal and efficient control parameters for every vibration frequency. We recall that a damping ratio ≈ 0.1 means that the time decrease of the wave amplitude is governed by

$$e^{\frac{-2\pi t}{10T}} \simeq e^{-\frac{t}{1.6T}},$$

where T is the wave period: after the time interval of two periods the amplitude is less than 30% of the initial value.



a



b

Fig. 10. Dissipated energies as functions of K_R and K_L

Further investigations will be devoted to active control, by simply introducing in Eqs. (12) displacement-current driven voltages, and to the study of shearable PEM beams. All the numerical simulations have been performed considering the piezoelectric constants of available PZT actuators; we remark that, due to their improved performances, see [7], the maximal exchange impedance we find in the present paper is rather small when compared with those found in [1].

Appendix

Equations (12) are obtained introducing the constitutive relations (7) through (11) into the electric balance equation (4) and into the mechanical balance equations, see [2],

$$\mathbf{s}' - \rho \ddot{\mathbf{v}} = 0 \quad , \quad (22)$$

$L_T = 2 \text{ m}$	$\rho = 1.6 \text{ kg/m}$
$K_C = 10^2 (\text{m} \cdot \text{F})^{-1}$	
$K_{mm} = 1.75 \cdot 10^6 \text{ N}$	$K_{me} = -4 \cdot 10^5 \text{ N} \cdot \text{m/C}$
$K_{em} = -4 \cdot 10^5 \text{ V}$	$K_{ee} = 2.5 \cdot 10^6 \text{ m/F}$

$$\mathbf{S}' + \mathbf{t} \wedge \mathbf{s} = \mathbf{0} . \quad (23)$$

An easy symbolic calculation leads us to write the current balance equation for the line j and the moment equation as

$$K_{mn} v^{IV} + \sum_j (K_{mej} \dot{\gamma}_j^{IV}) + \rho \ddot{v} = 0 , \quad (24)$$

$$K_{eej} \dot{\gamma}_j^{IV} + K_{emj} v^{IV} - K_{Cj} \dot{\gamma}_j'' + K_{Lj} \dot{\gamma}_j + K_{Rj} \dot{\gamma}_j = 0 , \quad (25)$$

where

$$K_{mm} = \sum_b (\bar{y}_b^2 A_b^3 K_{mmb}) , \quad (26)$$

$$K_{mej} = -\bar{y}_j A_j^2 L K_{mej} , \quad (27)$$

$$K_{eej} = L^2 A_j K_{eej} , \quad (28)$$

$$K_{emj} = -\bar{y}_j A_j^2 L K_{emj} . \quad (29)$$

Finally, we remark that, due to the module symmetry, we get for the two lines

$$\begin{aligned} K_{L1} &= K_{L2}, & K_{C1} &= K_{C2}, & K_{R1} &= K_{R2} , \\ K_{ee1} &= K_{ee2}, & K_{em1} &= -K_{em2}, & K_{me1} &= -K_{me2} , \end{aligned}$$

so that the index j can be skipped, and we obtain Eqs. (12). The Table shows the numerical parameters used in simulations (the module geometry may be evaluated by Fig. 3).

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