



A-infinity $GL(N)$ -equivariant matrix integrals-III. Tree-level calculations and variations of nc-Hodge structure on complex projective manifolds.

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A –infinity $GL(N)$ –equivariant matrix integrals-III.
Tree-level calculations and variations of nc-Hodge
structure on complex projective manifolds.

Serguei Barannikov

IMJ, CNRS

23/06/2010

Tree level BV on complex projective manifold ($g=0$ calculations)

- nc-BV on $Symm(C_\lambda[1+d])$, $C_\lambda = \bigoplus_{j=0}^{\infty} (U[1]^{\otimes j})^{\mathbb{Z}/j\mathbb{Z}}$

$$\hbar \Delta S + \frac{1}{2} \{S, S\} = 0, \quad S = \sum_{g \geq 0, i} \hbar^{2g-1+i} S_{g,i}, \quad S_{g,i} \in Symm^i(C_\lambda[1+d]),$$

$$\Leftrightarrow \Delta \exp(S/\hbar) = 0$$

$$\{S_{0,1}, S_{0,1}\} = 0, \text{ - } A_\infty \text{ - CY algebra}$$

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- M - smooth projective/ $\mathbb{C}$, $c_1(T_M) = 0$, $\omega_0 \in \Gamma(M, K_M)$, $\rightarrow$ BV operator on $\Omega^{0,*}(M, \Lambda^* T)$,

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- (tree level) BV equation on $\Omega^{0,*}(M, \Lambda^* T)$

$$u \Delta \gamma + \bar{\partial} \gamma + \frac{1}{2} [\gamma, \gamma] = 0,$$

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γ_0 — A_∞ -deformations of $D^b Coh(M)$

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γ_0 – A_∞ -deformations of $D^b Coh(M)$

- The component

$$\gamma_0^{cl} \in \Omega^{0,1}(M, T)$$

$\bar{\partial}\gamma_0^{cl} + \frac{1}{2}[\gamma_0^{cl}, \gamma_0^{cl}] = 0$ - deformations of complex structure on M

Noncommutative periods map ($B[5]$)



$$u\Delta\gamma + \bar{\partial}\gamma + \frac{1}{2}[\gamma, \gamma] = 0, \Leftrightarrow (u\Delta + \bar{\partial}) \exp\left(\frac{1}{u}\gamma\right) = 0$$

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$$(u\Delta + \bar{\partial}) \rightarrow \Delta + \bar{\partial}$$

-local system of $(u\Delta + \bar{\partial})$ -cohomology over $\mathbb{A}_u^1 \setminus \{0\}$

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$$\Omega(t, u) = \int_M r_u \exp(\frac{1}{u}\gamma) \vdash \varpi_0$$

$$\Omega(t, u) \in H_{DR}^*(M)((u)) \hat{\otimes} \mathcal{O}_{\mathcal{M}_{\Lambda T}}$$

$$t \in \mathcal{M}_{\Lambda T}, \quad T_0 \mathcal{M}_{\Lambda T} = H^*(M, \Lambda^* T)$$

Noncommutative periods map and $g=0$ Gromov-Witten of the mirror



$$\Omega(t, u) = \int_M r_u \exp\left(\frac{1}{u}\gamma\right) \vdash \omega_0$$

For γ^W , normalized using a filtration W opposite to F^{Hodge} ([B5], 1999):

$$\frac{\partial^2}{\partial t^i \partial t^j} \Omega^W = u^{-1} C_{ij}^k(t) \frac{\partial}{\partial t^k} \Omega^W$$

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$$C_{kij}(t) = \partial_{kij}^3 (\text{genus} = 0 \text{ GW-potential of } M^{\text{mirror}})$$

Semi-infinite (Noncommutative) Hodge structures ([B5]).

- The class $[\exp(\gamma^W)\omega] = \Omega(t, u)$ is obtained as intersection of moving subspace

$$\Omega(t, u) = \mathcal{L}(t) \cap (\text{Affine space}(W))$$

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- The semi-infinite subspace $\mathcal{L}(t)$, $t \in \mathcal{M}_{\Lambda T}$, is defined for *arbitrary* projective manifold/ $\mathbb{C}$

$$\mathcal{L}(\gamma_0) : [r_u(\exp \frac{1}{u} i_{\gamma_0})(\varphi_0 + u\varphi_1 + \dots)], \varphi_i \in \Omega_{DR}$$

where $\gamma_0 \in \Omega^{0,*}(M, \Lambda^* T)$, $\bar{\partial}\gamma_0 + \frac{1}{2}[\gamma_0, \gamma_0] = 0$

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$$u\mathcal{L}(t) \subset \mathcal{L}(t), \quad \frac{\partial}{\partial t}\mathcal{L}(t) \subset u^{-1}\mathcal{L}(t)$$

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- Over mod space of complex structures it reduces to VHS

Noncommutative Hodge structures ([B5]), cont'd

- $\mathcal{L}(t)$ corresponds via HKR and formality isomorphisms for $C^*(A, A) + k[\tilde{\zeta}, \frac{\partial}{\partial \tilde{\zeta}}]$ -module $C_*(A)$ to

$$\mathcal{L}(t) = HC^-(A_t) \subset HP(A_t)$$

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- Let A be an arbitrary A_∞ -algebra, the $\frac{\infty}{2}$ -subspace $HC^-(A) \rightarrow HP(A)$,

$$uHC^-(A) \subset HC^-(A)$$

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$$\frac{\partial}{\partial t} HC^-(A_t) \subset u^{-1} HC^-(A_t), \quad \frac{\partial}{\partial t} - \text{Getzler flat connection on } HP(A_t)$$

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$$rk_{C[[u]]} HC^-(A) = rk_{C((u))} HP$$

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- Real structure on HP in the case of arbitrary A_∞ -algebra?

Summation over trees, BCOV lagrangian and BV cyclic operad.

- For $W = \overline{F^{Hodge}}$, $\gamma(t) = \Sigma \gamma_a t^a + \Delta \alpha(t)$, $\Delta(\gamma) = 0$

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- vertices $\rightarrow$ product tensor on $\Omega^{0,*}(M, \Lambda^* T)$
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- Meaning of this : the tree-level Feynman transform of the operad of BV algebras / $\Delta = 0 =$ operad of $H_*(\mathcal{M}_{0,n})$

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