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A-infinity GL(N)-equivariant matrix integrals-II. The nc-BV differential, (stable) ribbon graphs and compacti fied moduli spaces.

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$A-$infinity $GL(N)-$equivariant matrix integrals-II. The nc-BV differential, (stable) ribbon graphs and compactified moduli spaces.

Serguei Barannikov

IMJ, CNRS

22/06/2010

The nc-Batalin-Vilkovisky formalism, stable ribbon graphs and compactified moduli spaces (B1,2006)

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- Theorem: stable ribbon graph complex = Feynman transform of the modular operad of "associative algebras with scalar product" (odd/even d)

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- Let I - an *odd derivation* acting on associative superalgebra V, $\dim_k V < \infty$, with scalar product, $tr([a,\cdot])$ on V_0, and let $\exists \widetilde{I}$, $[I,\widetilde{I}] = 1$, $\rightarrow$Cohomology classes in $H^*(\overline{\mathcal{M}}_{g,n}^{comb})$([B2])

The nc-Batalin-Vilkovisky operator and compactified moduli spaces (B1,2006)

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- U -$\mathbb{Z}/2\mathbb{Z}$ graded vector space/$\mathbb{C}$, l-scalar product on U of degree $d \in \mathbb{Z}/2\mathbb{Z}$,

$$C_\lambda = \oplus_{j=0}^{\infty}(U[1]^{\otimes j})^{\mathbb{Z}/j\mathbb{Z}}$$

-the symmetric (resp. exterior) powers for odd (resp even) d, of cyclic tensors

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- nc-BV equation

$$\begin{aligned} \hbar \Delta S + \frac{1}{2}\{S, S\} &= 0,\ S = \sum_{g \geq 0, i} \hbar^{2g-1+i} S_{g,i}, \quad S_{g,i} \in Symm^i(C_\lambda[1+d]), \\ &\Leftrightarrow \quad \Delta \exp(S/\hbar) = 0 \end{aligned}$$

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- Theorem: S satisfies nc-BV equation $\Longleftrightarrow$this is a closed chain in the stable ribbon graph complex ($\Rightarrow$in $H_*(\overline{\mathcal{M}}_{g,n}^{comb})$)

Solutions to nc BV equation

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- Theorem ([B6]). Summation over ribbon graphs$\longrightarrow$solution to the nc Batalin-Vilkovisky equation from dg-associative algebras (summation over trees$\longrightarrow$A-infinity algebra structure)

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- Conjecture ([B1]). Counting of holomorphic curves $(\Sigma, \partial\Sigma, p_i) \to (M, \coprod L_i, \oplus H_*(L_i \bigcap L_j))$, with $\mathbb{Z}/2\mathbb{Z}$-graded local systems, gives solution to the nc-BV equations.

Derivations and the cohomology classes

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- $\widetilde{I}$ is homotopy inverse to the derivation $\rightarrow$ this defines cohomology class

Strange associative superalgebra with odd trace and psi-classes.

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- $q(N) = \{[X, \pi] = 0 | X \in gl(N|N)\}$,where $\pi-$odd involution, $q(N)$ has *odd trace otr*, $I = [\Xi, \cdot]$, Ξ- odd element $\Xi = \begin{pmatrix} 0 & |diag(\lambda_1, \ldots, \lambda_n) \end{pmatrix}$, ($I^2 \neq 0$ (!))

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- Similarly, with even scalar product and an *odd* derivation, in particular for $gl(N|N)$ and $I = [\Xi, \cdot]$, $\Xi \in gl(N|N)_{odd}$.

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