# Tabu Split and Merge for the Simplification of Polygonal Curves 

Gildas Ménier, Pierre-François Marteau

## - To cite this version:

Gildas Ménier, Pierre-François Marteau. Tabu Split and Merge for the Simplification of Polygonal Curves. IEEE international conference on Systems, Man and Cybernetics, Oct 2009, San Antonio, Texas, United States. pp.1322-1327, 10.1109/ICSMC.2009.5346240 . hal-00492900

HAL Id: hal-00492900

## https://hal.science/hal-00492900

Submitted on 19 Jul 2010

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire HAL, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

# Tabu Split and Merge for the Simplification of Polygonal Curves 

Gildas Ménier<br>VALORIA, Université de Bretagne Sud<br>Université Européenne de Bretagne<br>France<br>gildas.menier@univ-ubs.fr

Pierre-François Marteau<br>VALORIA, Université de Bretagne Sud<br>Université Européenne de Bretagne<br>France<br>pierre-francois.marteau@univ-ubs.fr


#### Abstract

A Tabu Move Merge Split (TMMS) algorithm is proposed for the polygonal approximation problem. TMMS incorporates a tabu principle to avoid premature convergence into local minima. TMMS is compared to optimal, near to optimal top down Multi-Resolution (TDMR) and classical split and merge heuristics solutions. Experiments show that potential improvements for crudest approximations can be obtained. The evaluation is carried out on 2D geographic maps according to effectiveness and efficiency measures.


Keywords- Polygonal approximation, Tabu search, Split-and-Merge, top down multi resolution, dynamic programming.

## I. Introduction

Polygonal curve approximation has been widely studied in the past to scale up time consuming applications such as graphic display, contour detection or time series data mining. If we apprehend a discrete curve as a multidimensional vector, polygonal approximation can be seen as a dimension reduction technique that relates also to multidimensional scaling.

Following polygonal approximation is an optimization problem that can be tackled along two angles:

- Min- $\varepsilon$ problem: Given a polygonal curve $S_{c}$ having $N$ segments, find an approximation $A_{c}$ having $K$ segments such that the maximal approximation error $\varepsilon$ is minimized.
- Min-\# problem: Given a polygonal curve $S_{c}$ having $N$ segments, find an approximation $A_{c}$ with the minimum number of segments $K$ so that the maximal approximation error does not exceed a given tolerance $\varepsilon$.

Most of the proposed algorithms developed to solve these problems belongs either to graph-theoretic approaches [1,6,8,10, 11,12], dynamic programming [5,7] or to heuristic approaches $[2,4,12,13,15]$. Optimal solutions based on dynamic programming principle exist, nevertheless, there complexities are proved to be $O\left(K . N^{2}\right)$ leaving space for much faster but suboptimal solutions.

Finding fast algorithms as near-to-optimality as possible for long input curves is still an open challenge leading to fruitful applications. In [10] for instance, author proposed a fast and
dirty filtering approach dedicated to time series retrieval whose efficiency is highly correlated with the quality of polygonal approximations.

Recently, we have proposed a top town multiresolution algorithm (TDMR) [9] that solves the problem of polygonal curve approximation in linear time complexity $O(N)$. This algorithm ensures near to optimal solutions between each two successive levels of resolution, but, as we descend the resolution levels, the approximations depart further from optimality. The only other known algorithm showing a linear complexity is the Douglas-Peucker algorithm [2,4]. It is faster than TDMR but provides approximations that are much farther to optimality than the ones provided by TDMR. In this paper, we explore heuristic strategies based on three elementary operations (merge, split and move) associated to a Tabu search principle and try to evaluate how these strategies could boost suboptimal solutions such as TDMR, Merge-L2 [12] or Douglas Peucker [2] heuristics.
The second section of the paper states the problem definitions and introduces the three elementary operations at the basis of the heuristics we will detail into the third section of the paper. The fourth section presents and comments the experiments we have carried out on a set of 2D geographic maps; we conclude the paper and suggest some perspectives in the final section of the paper.

## II. PROBLEM DEFINITIONS AND THE THREE ELEMENTARY OPERATIONS

We attempt to correct an approximation curve $A_{c}$ of $k$ points approaching a (source) curve $S_{c}$ of $n$ points $(k \ll n)$ by the mean of elementary transformation operators: move, merge and split. To simplify the search space, and following the common definitions of the Min- $\mathcal{E}$ and Min-\# problems, we take approximation points among the points of the target curve: $A_{c}$ is therefore a sorted list of references (index) of points in $S_{c}$.

Let $S_{c}[i]$ be the $\mathrm{i}^{\text {th }}$ point of the curve $\mathrm{S}_{\mathrm{c}}$.
Let $\mathrm{A}_{\mathrm{c}}[]$ be the reference to a point in $\mathrm{Sc}-$ the $\mathrm{i}^{\text {th }}$ element of $\mathrm{A}_{c}$ is the $A_{c}[i]^{\text {th }}$ point of $S_{c}$, i.e. the point $S_{c}\left[A_{c}[i]\right]$ (Fig. 1).


Figure 1. The reference curve $\mathrm{S}_{\mathrm{c}}$ and an approximation $\mathrm{A}_{\mathrm{c}}$
Let $S_{c}[a, b]$ be the sub curve between the $a^{\text {th }}$ point and the $b^{\text {th }}$ point of the curve $S_{c}$.
Let $A_{c}[a, b]$ be the sub curve between the $a^{\text {th }}$ point and the $b^{\text {th }}$ point of the curve $\mathrm{A}_{\mathrm{c}}$.

## A. move operator

With the aim of minimizing the Min- $\varepsilon$ criterion, the move operator slides a potentially misplaced point thru the set of curve's indexes according to least surface error gradient direction. The error is computed as follows:

For the approximation point $i_{a}$ (coding the $i_{a}{ }^{\text {th }}$ element of the curve $A_{c}$ ), an evaluation of the surface between the sub curve $S_{c}\left[A_{c}\left[i_{a}-1\right], A_{c}\left[i_{a}\right]\right]$ and the sub curve of $A_{c}\left[i_{a}-1, i_{a}\right]$ is computed as $\mathrm{E}_{\text {left }}\left(\mathrm{S}_{\mathrm{c}}, \mathrm{i}_{\mathrm{a}}\right)$ summing the Euclidian distances between the points of $S_{c}$ and the approximation line provided by $\operatorname{Ac}\left[i_{a}-1, i_{a}\right]$ (Figure 2).

$$
E_{l e f t}\left(i_{a}\right)=\sum_{j=A c\left[i_{a}-1\right]}^{j<A c\left[i_{a}\right]} d_{\text {eucl }}\left(A_{c}\left[i_{a}-1, i_{a}\right], S_{c}[j]\right)
$$

$\mathrm{d}_{\text {eucl }}(D, P)$ stands for the Euclidian distance between a point $P$ and a line $D$.


Figure 2. Gradient of the errors for the move operation

Likewise,

$$
E_{r i g h t}\left(i_{a}\right)=\sum_{j=A c\left[i_{a}\right]}^{j<A c\left[i_{a}+1\right]} d_{e u c l}\left(A_{c}\left[i_{a}, i_{a}+1\right], S_{c}[j]\right)
$$

Let $\mathrm{E}_{\text {leffright }}(\mathrm{i})$ be the surface error balance supported by the $\mathrm{i}^{\text {th }}$ point:

$$
\mathrm{E}_{\text {lefflright }}\left(\mathrm{i}_{\mathrm{a}}\right)=\left|\mathrm{E}_{\text {left }}\left(\mathrm{i}_{\mathrm{a}}\right)-\mathrm{E}_{\text {right }}\left(\mathrm{i}_{\mathrm{a}}\right)\right| .
$$

The main idea here is to slide the $\mathrm{i}_{\mathrm{a}}^{\text {th }}$ point (in the $\mathrm{S}_{\mathrm{c}}$ indexes space) to balance the left and right errors. If $\mathrm{E}_{\text {right }}\left(\mathrm{i}_{\mathrm{a}}\right)>\mathrm{E}_{\text {left }}\left(\mathrm{i}_{\mathrm{a}}\right)$ then the $i_{a}^{\text {th }}$ point is moved by one position right in the $S_{c}$ indexes space (respectively left if $\mathrm{E}_{\text {right }}\left(\mathrm{i}_{\mathrm{a}}\right)<\mathrm{E}_{\text {left }}\left(\mathrm{i}_{\mathrm{a}}\right)$ ). The displacement is constrained by the segment boundary points $i_{a}-$ 1 and $i_{a}+1$ (Alg. 1).

## OPERATOR move <br> BEGIN

foreach point i in $\mathrm{A}_{\mathrm{c}}$ compute $\mathrm{E}_{\text {leffright }}(\mathrm{i})$
end foreach
select the point $\mathrm{i}_{\text {max }}$ having the maximum $\mathrm{E}_{\text {leftright }}(\mathrm{i})$ if $\mathrm{E}_{\text {left }}\left(\mathrm{i}_{\text {max }}\right)<\mathrm{E}_{\text {right }}\left(\mathrm{i}_{\text {max }}\right)$
then

$$
\text { if }\left(\mathrm{A}_{\mathrm{c}}\left[\mathrm{i}_{\max }-1\right]<\mathrm{A}_{\mathrm{c}}\left[\mathrm{i}_{\max }\right]-1\right)
$$

then
$\mathrm{A}_{\mathrm{c}}\left[\mathrm{i}_{\text {max }}\right]:=\mathrm{A}_{\mathrm{c}}\left[\mathrm{i}_{\text {max }}\right]-1$
// slides i to the left
end if
else
if $\mathrm{E}_{\text {left }}\left(\mathrm{i}_{\text {max }}\right)>\mathrm{E}_{\text {right }}\left(\mathrm{i}_{\text {max }}\right)$
then
if $\left(\mathrm{A}_{\mathrm{c}}\left[\mathrm{i}_{\text {max }}+1\right]>\mathrm{A}_{\mathrm{c}}\left[\mathrm{i}_{\text {max }}\right]+1\right)$
then
$\mathrm{A}_{\mathrm{c}}\left[\mathrm{i}_{\text {max }}\right]=\mathrm{A}_{\mathrm{c}}\left[\mathrm{i}_{\text {max }}\right]+1$
// slides it to the right end if
end if
end if
END

Algorithm 1

One execution of the operator move slides only one point - the point of $A_{c}$ having the worse $E_{\text {leffright }}(i)$ (thus being the most unbalanced point on the $\mathrm{A}_{\mathrm{c}}$ curve regarding the local surface error).

## B. split operator

The split operator (Alg. 2) finds one of the point $\mathrm{i}_{\mathrm{c}}$ in $\mathrm{S}_{\mathrm{c}}$ that maximises $d_{\text {eucl }}\left(\mathrm{A}_{\mathrm{c}}, \mathrm{S}_{\mathrm{c}}[\mathrm{i}]\right)$ and inserts a new point in $\mathrm{A}_{\mathrm{c}}$ to correct this error - thus decreasing (eventually not strictly) the Min- $\varepsilon$ value.

```
OPERATOR split
BEGIN
    foreach point ic in Sc
    compute d}\mp@subsup{\textrm{d}}{\mathrm{ eucl }}{}(\mp@subsup{\textrm{A}}{\textrm{c}}{},\mp@subsup{\textrm{S}}{\textrm{c}}{}[\textrm{i}]
    end foreach
    select the point ic max with the maximum distance
    insert a new point }\mp@subsup{\textrm{i}}{\mathrm{ new }}{}\mathrm{ in }\mp@subsup{\textrm{A}}{c}{
    that minimizes
END
```

Algorithm 2

## C. merge operator

The merge operator removes an element from $\mathrm{A}_{\mathrm{c}}$, thus merging the two adjacent approximating segments. Since the ratio k/error is important, the point to remove should be the point contributing the least to the global error $-i e$, it is important to discard first the points which removal increases the least the global error.


Figure 3. Application of the merge operation on element $i_{a}$.

The removal priority $R_{p}$ for an element $i_{a}$ is computed as follows : The error criterion Error (Min- $\mathcal{E}$ for instance) is computed on the segments $\left[\mathrm{A}_{c}\left[\mathrm{i}_{\mathrm{a}}-1\right], \mathrm{Ac}\left[\mathrm{i}_{\mathrm{a}}\right]\right],\left[\mathrm{A}_{c}\left[\mathrm{i}_{\mathrm{a}}\right], \mathrm{Ac}\left[\mathrm{i}_{\mathrm{a}}+1\right]\right]$ (Figure 3) and if the element $\mathrm{A}_{c}\left[\mathrm{i}_{\mathrm{a}}\right]$ were to be discarded, on the segment $\left[A_{c}\left[i_{a}-1\right], A_{c}\left[i_{a}+1\right]\right]$, the two segments $\left[A_{c}\left[i_{a}-1\right]\right.$, $\left.\mathrm{Ac}\left[\mathrm{i}_{\mathrm{a}}\right]\right]$ and $\left[\mathrm{A}_{\mathrm{c}}\left[\mathrm{i}_{\mathrm{a}}\right], \mathrm{Ac}\left[\mathrm{i}_{\mathrm{a}}+1\right]\right]$ are merged.

For instance, if the Min- $\varepsilon$ error is used:

$$
\begin{array}{r}
\mathrm{R}_{\mathrm{p}}\left(\mathrm{i}_{\mathrm{a}}\right)=\operatorname{Max}\left(\text { Error }_{\min -\mathrm{e}}\left[\mathrm{i}_{\mathrm{a}}-1, \mathrm{i}_{\mathrm{a}}\right],\right. \\
\text { Error min-e } \left.\left[\mathrm{i}_{\mathrm{a}}, \mathrm{i}_{\mathrm{a}}+1\right]\right) \\
\quad / \text { Error }_{\text {min- }-}\left[\mathrm{i}_{\mathrm{a}}-1, \mathrm{i}_{\mathrm{a}}+1\right]
\end{array}
$$

Alg. 3 describes the process :

```
OPERATOR merge
BEGIN
    foreach element \(i_{a}\) in \(A_{c}\)
                compute \(\mathrm{R}_{\mathrm{p}}\left(\mathrm{i}_{\mathrm{a}}\right)\)
    end foreach
    select the element \(i_{\text {a }}\) having the higher removal
        priority \(\mathrm{R}_{\mathrm{p}}\)
    remove it
END
```

Algorithm 3

## D. Refining the curve

As introduced above, we attempt to correct an existing approximation curve $\mathrm{A}_{\mathrm{c}}$ of $k$ points approaching a (source) curve $S_{c}$ of $n$ points by the mean of elementary transformation operators move, merge and split:

FUNCTION Refine ( $\mathrm{A}_{\mathrm{c}}, \mathrm{S}_{\mathrm{c}}$ )
// Refines the approximation $\mathrm{A}_{\mathrm{c}}$ (of k elements) of the curve $\mathrm{S}_{\mathrm{c}}$ (n points)
BEGIN
repeat $2 * \mathrm{k}$ loops of move(); merge(); split();
end repeat

## Algorithm 4

Unfortunately, a sequence of move, split and merge does not ensure that the error criterion will strictly decrease. In fact, the experiments (see Fig. 4) show a sequence of global error changes similar to those existing in incremental learning (such as Back Propagation Networks, or to a least amount in Genetic Algorithm) - ie in stabilizations and step breaks: this suggests that the optimization of the locations of the points is not a gradient process but rather a complex organizing mechanism necessary to explore the parameters space.

It is then necessary to keep aside the best solution found and to expect the next iterations to eventually improve this solution. In the simple algorithm described above (Alg. 4), we decided to perform a number of loops proportional to the $k$ elements of the approximation since we think that, in the ideal configuration, each element of the approximation should undergo - a least once - a move/merge/split operator.

Experimentally, we observe that $k$ loops is a minimum limit to achieve the best expectable error: experimentally, more than $2 * k$ loops doesn't increases the performances so far.

During its exploration of the parameter's space (as a vector of $k$ indexes in the $\mathrm{S}_{\mathrm{c}}$ curves), this algorithm seems to fall in gradient wells: Fig. 4 exhibits oscillating behaviours that prevents the search to go on: it seems that the sequence of movelmerge/split can lead to configurations that occur sequentially after $t$ loops.


Figure 4. Curve of $\mathrm{n}=49344$ points with $\mathrm{k}=50$ (best solution min-e $=$ 0.716 ) - No Tabu involved.

This suggests that the same configuration of elements in $\mathrm{A}_{\mathrm{c}}$ may be scanned more than once. Keeping the $n^{\text {th }}$ last configurations of $\mathrm{A}_{\mathrm{c}}{ }^{\mathrm{n}}$ and preventing (temporary) the algorithm to re-scan these previous solutions is somehow related to a search space strategy called Tabu Search [8].

In this example, we assumed that the cost to store and compare a new solution of $\mathrm{A}_{\mathrm{c}}$ (of $k$ points) to the $t$ previous solutions would be important. Mostly, it is the recurring choice of the points to move, merge and split that leads to an endless evaluation of sequences of already-seen configurations.

Therefore, we propose to introduce some Tabu search principles through the management a list of 'recently points chosen to perform movelmerge/split' and prevent the operator from picking (again) one of the tagged element - at least as long as these elements are enqueued in this FIFO list.

Restricting the search by the choice of one index point segments the search space in classes of approximation curves containing (or not) a specific point: this could be very restrictive at a first glance, but this strategy speeds up the search by selecting the curve containing the best / worse point in regard to the Min- $\varepsilon$ criterion.


Figure 5. Tabu with 5 elements (best solution min-e $=0.412$ )

For a Tabu list of size $t_{b}$ elements, not only this strategy seems to prevent the rapid oscillating behaviour under $t_{b}$ loops, but also seems to increase the search speed.


Increasing the size of the Tabu list above 10-15 elements decreases the performance of the algorithm without significant better results (mostly under $10^{-4}$ for the maximum error).

## III. APPROACHES

## A. RSDP: Reduced Search Dynamic Programming

The reference algorithms for curve approximations for the Min- $\mathcal{E}$ criterion are mostly based on dynamic programming [12]: they usually provide the minimal error at a computational cost of $O\left(n^{2}\right)$. It is possible to reduce this complexity, constraining the search when near-optimal solutions are acceptable - lowering the computational cost to $O\left(n^{2} / k\right)$ : in the following experiments, RSDP stands for Reduced Search Dynamic Programming [7].

## B. MR : Multi Resolution

In [9], we introduced a top-down multi resolution algorithm TDMR designed to compute iteratively nested approximations with a complexity (at the best case) of $O(n)$ : it features sequential processing of RSDP-like processing and outputs a multiresolution solution to the approximation problem.

## C. I-TMMS : Refine with equidistant initialization

This processing involves the refine function (described above) starting with a first curve of $k$ points to correct: each $k_{i}$ point is initially set at equidistant position on the $\mathrm{S}_{\mathrm{c}}$ curve (rounded to the nearest index). $2 * k$ loops are performed.

## D. SPLT : Split

Starting with an initial curve of $k_{0}=2$ points (the first and last point of $\mathrm{S}_{\mathrm{c}}$ ), this algorithm performs $k$ - 2 split operations to reach the final $k$ elements for $\mathrm{A}_{\mathrm{c}}$.

## E. MRG:Merge

Likewise, this algorithm starts with the complete curve $\mathrm{S}_{\mathrm{c}}$ - as the full collection of indexes for $\mathrm{A}_{\mathrm{c}}$ - and decimates iteratively the points (by the mean of merge operators) until the number of remaining elements reaches $k$ elements in $\mathrm{A}_{\mathrm{c}}$.

## F. $M R / T M M S$

A multiresolution process (MR) is performed [9], refined by the tabu movelmerge/split (TMMS) sequence of operators.

## G. SPLT/ TMMS

The SPLT (split) is performed, followed by the refine (tabu movelmergelsplit) TMMS process.

## H. MRG/TMMS

The MRG (merge) is performed, followed by the refine (tabu movelmergelsplit) TMMS process.

## IV. EXPERIMENTS

The experiments have been performed on 10 curves $S_{c}$ depicting the costal maps of Western Europe. The 10 curves $S_{c}$ have at least $\mathrm{n}=8192$ points.
We measure the fidelity of an approximation using the following formula:

$$
\mathrm{F}_{\text {method }}=\mathrm{E}_{\mathrm{RDSP}} / \mathrm{E}_{\text {method }}
$$

where E is the Min- $\varepsilon$ error.


Figure 7. $\mathrm{F}_{\text {method }}$ for different values of $k$ (number of points in $\mathrm{A}_{\mathrm{c}}$ )

RSDP, that is near-optimal, is used as reference solution. Fig. 7 shows the Fidelity for all the experimented methods. Basically, the TMMS procedure boosts the experimented methods for low $k$ values. ISM performs quite well for values of $k \ll n$ : it is only outperformed by MR and MR/TMMS : for $k>64$, ISM gives the worst results. MR/TMMS seems to improve marginally the error of MR. The TMMS procedure introduces a time cost that is measurable for all experimented methods in Fig.8.


Figure 8. Evaluation for all experimented methods of computation time (in sec.$)$ for different values of k .

## V. Conclusion

We have introduced the use of movelmerge/split operators using a Tabu-like selection to refine an existing approximation curve. We compared this approach with other sub optimal algorithms, namely top down multiresolution, split algorithm and merge algorithm. The TMMS procedure offers some boosting capability for the crudest approximations and could probably be used directly inside a multiresolution approach to improve the overall fidelity of the provided approximations a low level of resolution.

## References

[1] Chen D.Z., Daescu O.. "Space-efficient algorithms for approximating polygonal curves in two-dimensional space". In Int. Conf. on Computing and Combinatorics, Lecture Notes in Computer Science, volume 1449, pp 4554, 1998.

Douglas D., Peucker T., "Algorithms for the reduction of the number of points required to represent a digitized line or its caricature", The Canadian Cartographer 10(2), pp112-122, 1973.

Glover F., Laguna M. "Tabu Search". Kluwer, Norwell, MA, 1997.
[4] Hershberger J., Snoeyink J., "Speeding Up the Douglas-Peucker Line-Simplification Algorithm", Proceedings 5th Symp on Data Handling, 134-143. UBC Tech Report TR-92-07, 1992.
[5] Horng J.-H. "Improving fitting quality of polygonal approximation by using the dynamic programming technique". Pattern Recognition Letters, 23:pp1657-1673, 2002.
[6] Imai H., Iri. M. "Polygonal approximations of a curve formulations and algorithms". Computational Morphology, pp 71-86, 1988.
[7] Kolesnikov A., Fränti P., "Reduced-search Dynamic Programming for Approximation of Polygonal Curves" Pattern Recognition Letters 24:pp2243-2254, 2003.
[8] Kolesnikov A., Fränti P., Wu X., "Multiresolution Polygonal Approximation of Digital Curves" Proceedings of the 17th International Conference on Pattern Recognition ICPR'04 pp855-858 2004.
[9] Marteau P.F., Ménier G. "Adaptive multiresolution and dedicated elastic matching in linear time complexity for time series data mining". In Sixth International Coference on Intelligent Systems Design and Applications, pp 700-706, 2006.
[10] Marteau, P.F., Time Warp Edit Distances with Stiffness Adjustment for Time Series Matching, IEEE Transactions on Pattern Analysis and Machine Intelligence, Volume: 31, Issue: 2, pp. 306-318, Feb. 2009.
[11] Melkman A., O'Rourke J. "On Polygonal chain approximation" Computationnal Morphology. Elsevier Science Publishers B.V. North Holland pp 87-95, 1988.
[12] Perez JC., Vidal E., "Optimum Polygonal Approximation of Digitized Curves" Pattern Recognition Letters 15:pp743-750, 1994.
[13] Pikaz A., Dinstein I.. "An algorithm for polygonal approximation based on iterative point elimination". Pat- tern Recognition Letters, 16:pp557-563, 1995.
[14] Rosin R.L. "Techniques for assessing polygonal approximations of curves". IEEE Trans. Pattern Analysis and Machine Intelligence, 14:pp659-666, 1997.
[15] Salotti M.. "An efficient algorithm for the optimal polygonal approximation of digitized curves". Pattern Recognition Letters, 22:pp215-221, 2001.
[16] Visvalingam M., Whyatt J. "Line generalization by repeated elimination of points". Cartographic Journal, 30:pp46-51, 1993
[17] Y. Zhu and L.D. Seneviratne. "Optimal polygonal approximation of digitized curves". In IEEE Proc.-Vis. Image Signal Process, volume 144, pp814-1997.

