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Small-scale anisotropy in Lagrangian turbulence

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Abstract. We report measurements of the second-order Lagrangian structure function and the Lagrangian velocity spectrum in an intensely turbulent laboratory flow. We find that the asymmetries of the large-scale flow are reflected in the small-scale statistics. In addition, we present new measurements of the Lagrangian structure function scaling constant C_0 , which is of central importance to stochastic turbulence models as well as to the understanding of turbulent pair dispersion and scalar mixing. The scaling of C_0 with the turbulence level is also investigated, and found to be in agreement with an existing model.

Turbulence governs the vast majority of fluid flows in nature and in industrial applications, including the dynamics of weather systems and clouds, the spread of odour plumes and pollutants, and mixing in chemical reactors. Despite the importance of turbulence, however, our fundamental understanding of the subject remains poor. Indeed, Feynman called turbulence one of the last great unsolved problems of classical physics [1].

Due to the complexity of the fluid equations of motion, we are forced to turn to phenomenological modelling to gain insight into the behaviour of turbulent flows. Statistical turbulence modelling has been dominated by the ideas of Kolmogorov [2], whose 1941 hypotheses have so influenced the field that they are simply known as the ‘K41’ model. Taken together, the K41 hypotheses assume that, in intense turbulence and well away from any boundaries or singularities, the statistics of turbulent flow should be universal at length

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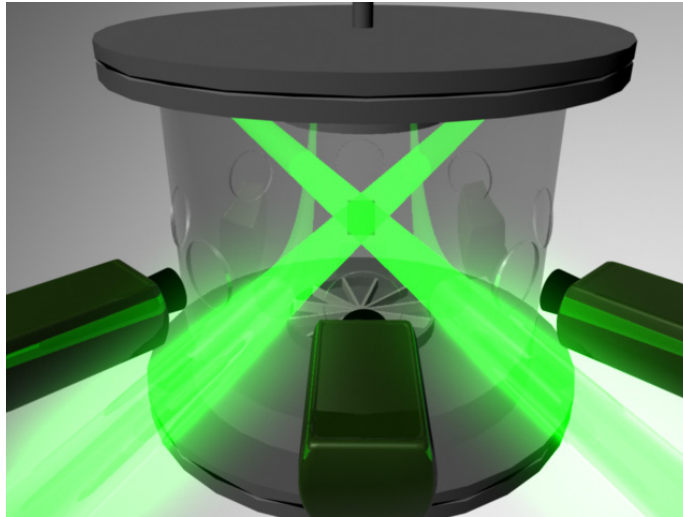


Figure 1. Sketch of the experimental apparatus. The trajectories of tracer particles were recorded by three high-speed cameras in a $5 \times 5 \times 5 \text{ cm}^3$ subvolume in the centre of the tank. The tracers were illuminated by two pulsed Nd:YAG lasers with a combined power of roughly 150 W. The cameras were arranged in a single plane in the forward scattering direction from the lasers with an angular separation of roughly 45° . The discs rotated about the z -axis.

and timescales that are small compared with the injection of energy into the flow. If these small-scale statistics are to be universal, they must be independent of the large-scale flow structure. In particular, K41 predicts that at small scales the turbulence should ‘forget’ any preferred directions of the large-scale flow and that the small-scale fluctuations should be statistically homogeneous and isotropic. Models and simulations of turbulence therefore commonly assume isotropic flow. Real flows, however, are never homogeneous and isotropic at large scales. Careful study of the effects of large-scale anisotropy on the small-scale turbulent fluctuations is therefore very important for understanding the behaviour of turbulent flows in nature. In addition, such study is necessary in order to relate current turbulence theory, modelling, and simulation to practical applications.

We have investigated the K41 hypothesis of local isotropy in an optical three-dimensional (3D) particle tracking experiment. Our experimental facility consists of a closed cylindrical chamber where turbulence is generated between counter-rotating discs, as sketched in figure 1. The tank has a diameter of 48.3 cm, and the discs are separated by 43.9 cm. The flow is seeded with polystyrene tracer particles with a diameter of $25 \mu\text{m}$ and a density 1.06 times that of water, which have been shown to act as passive tracers in this flow [3]. The particles are illuminated with two pulsed Nd:YAG lasers with a combined power of roughly 150 W, and their motion is followed using three Phantom v7.1 CMOS cameras from Vision Research. These cameras are capable of recording 27 000 images per second at a resolution of 256×256 pixels. Tracer particle tracks are found from the image sequences using particle tracking algorithms [4], and their velocities are calculated by convolving the particle tracks with a Gaussian smoothing and differentiating kernel [5].

Because of the cylindrical symmetry of our apparatus, the large-scale flow is axisymmetric. We investigate the effects of this large-scale anisotropy in the context of the variance of the temporal increments of the turbulent velocity $\delta u_i(\tau) = u_i(t + \tau) - u_i(t)$, known as the second-order Lagrangian structure function $D_{ij}^L(\tau)$. Axisymmetric turbulence has been the subject of prior theoretical work [6]–[8], but has not yielded any experimentally verifiable predictions similar to those made by the K41 model. K41 theory predicts that the structure function should scale as $D_{ij}^L(\tau) = \langle \delta u_i(\tau) \delta u_j(\tau) \rangle = C_0 \epsilon \tau \delta_{ij}$ in the so-called inertial range where the only relevant flow parameter is the rate of energy dissipation per unit mass ϵ . According to K41, the structure function should be isotropic and C_0 should have a universal value for all turbulent flows. It is an important parameter in stochastic models of turbulent transport and dispersion [9]–[11] and is, remarkably, also connected both to the Richardson constant governing the separation of fluid element pairs, assuming that the covariance of the relative acceleration of the pair is stationary, and to the structure functions of the fluctuations of a scalar field passively advected by the turbulence [12]. Previously measured values of C_0 range from 2.1 to 7.0 [13], in part because Lagrangian experiments, where the trajectories of individual fluid particles are followed, have historically been very difficult. Here, we report new, better-resolved measurements of C_0 .

Despite recent experimental and numerical studies of Lagrangian turbulence [3], [14]–[18], most of our understanding of turbulence still comes from Eulerian measurements, where probes are fixed with respect to some laboratory reference frame. For example, while the value of the Lagrangian constant C_0 is very uncertain, the corresponding Eulerian constant C_2 has a well-measured value of 2.13 ± 0.22 [19]. Lagrangian statistics also seem to require higher turbulence levels to observe K41 scaling [18]. The turbulence level is quantified by the Reynolds number, which measures the relative importance of the nonlinear inertial terms and the linear viscous terms in the Navier–Stokes equations. In the present work, we report the Reynolds number based on the Taylor microscale, $R_\lambda \equiv u' \lambda / \nu$, where u' is the root mean square velocity of the turbulent fluctuations and ν is the kinematic viscosity. λ , the Taylor microscale, is defined to be $\sqrt{15 u'^2 \nu / \epsilon}$. To define our Reynolds numbers, we measure ϵ from the second- and third-order Eulerian structure functions, which give us the spherically averaged dissipation rate. Since, as mentioned above, the large-scale velocity is different in the axial direction and the radial directions, we define the Reynolds number based on the radial root mean square velocity. Yeung [18] has suggested that a Reynolds number of at least $R_\lambda = 600$ – 700 is required to observe K41 scaling of Lagrangian quantities, a range difficult to achieve both in experiments and simulations. In this work, we report measurements at Reynolds numbers up to $R_\lambda = 815$.

Figure 2 shows a single component of our measured Lagrangian structure functions compensated by $\tau \epsilon$ at $R_\lambda = 200, 350$, and 815 . Plotting the structure function in this fashion should display a plateau in the inertial range with value C_0 . A well-developed inertial range requires a large scale separation between the Kolmogorov timescale τ_η and the Lagrangian integral time T_L . T_L is usually measured from the Lagrangian velocity autocorrelation function, which decays approximately exponentially. We show the three diagonal components of the autocorrelation tensor $R(\tau)$ measured at $R_\lambda = 815$ in figure 3 along with fits of the function

$$R(\tau) = \frac{T_L e^{-\tau/T_L} - T_2 e^{-\tau/T_2}}{T_L - T_2} \quad (1)$$

proposed by Sawford [10] to account for the finite slope of the autocorrelation function at the origin. T_2 here is related to the Kolmogorov time τ_η . We fit (1) between 0 and $40\tau_\eta$, and find an

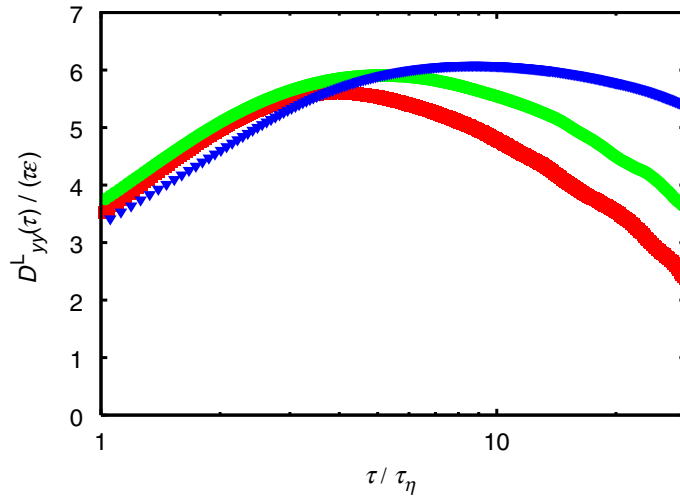


Figure 2. Compensated Lagrangian structure functions at $R_\lambda = 200$ (■), 350 (●) and 815 (▼). The structure functions have been scaled by $\tau\epsilon$ so that they should show a plateau in the inertial range. The scaling range is short at all Reynolds numbers shown, but grows larger as the Reynolds number increases.

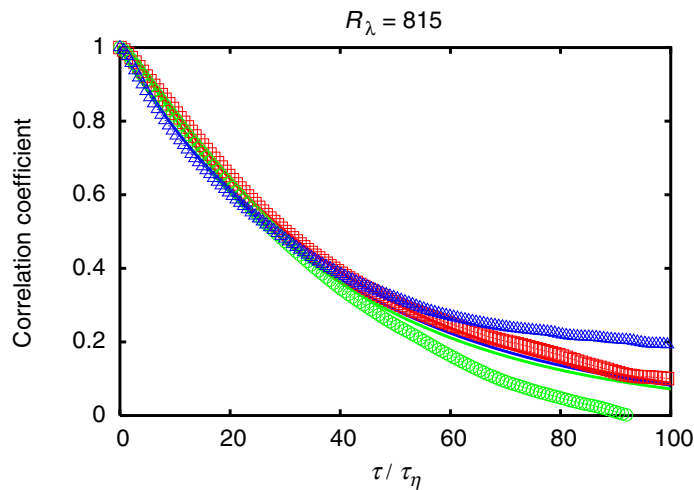


Figure 3. Lagrangian velocity autocorrelation function measured at $R_\lambda = 815$. The red and green symbols show the two radial velocity components, and the blue symbols show the axial velocity component. The corresponding solid lines are fits of (1). In a finite measurement volume like ours, the velocity autocorrelation function is heavily biased.

integral time of approximately $40\tau_\eta$ for each component. The autocorrelation function, however, is known to be heavily biased in a finite measurement volume like ours since it mixes effects at all scales [20, 21], and will give an integral time that is too short. We therefore also report the eddy turnover time T_E , estimated as $T_E = L/u'$ where L is the integral length scale. At $R_\lambda = 815$ in our experiment, $T_E = 208 \tau_\eta$, so that $T_L/T_E \approx 0.2$. This ratio is roughly independent of Reynolds

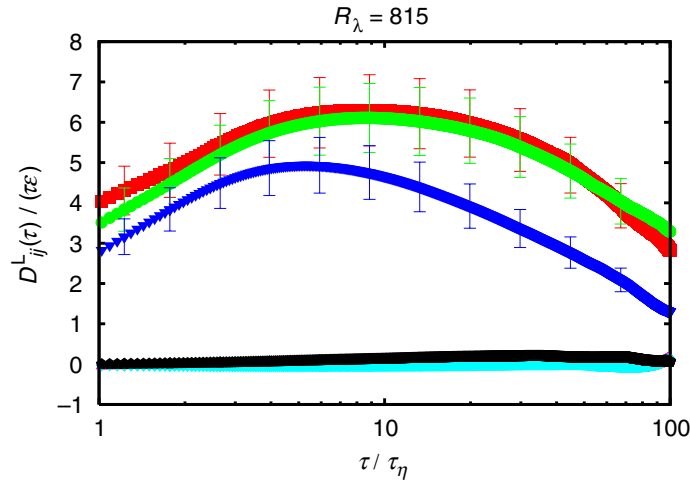


Figure 4. The xx (\blacksquare), yy (\bullet) and zz (\blacktriangledown) components of the compensated Lagrangian structure function at $R_\lambda = 815$. The other symbols show the off-diagonal components. The time axis has been normalized by the Kolmogorov time. The relative magnitude of the radial and axial components reflects the anisotropy of our large-scale flow.

number in our experiment. Our scaling ranges are therefore short, but do grow with Reynolds number.

In figure 4, we focus on the full structure function tensor at $R_\lambda = 815$. Two features of this tensor are particularly noteworthy. We see very short plateau regions for all three diagonal components of the structure function tensor, consistent with the K41 scaling prediction, though without a fully developed Lagrangian inertial range. It is clear, however, that this tensor is not isotropic, contradicting the K41 hypothesis of local isotropy. The zz component, measured in the axial direction of our cylindrical flow chamber, shows a peak value roughly 25% lower than that of the xx and yy components, measured in the radial direction. The xx and yy components are identical within experimental precision, reflecting the axisymmetry of the large-scale flow. Measuring ϵ from the Eulerian structure functions conditioned to lie in the axial or radial directions cannot account for the observed anisotropy in the Lagrangian structure functions. We note that the peak values of the compensated structure functions occur at very short times, less than a factor of 10 larger than the Kolmogorov time $\tau_\eta = \sqrt{\nu/\epsilon}$, the characteristic timescale of the fastest turbulent motion.

We have also measured the anisotropy present in the Lagrangian velocity spectrum, defined as the Fourier transform of the Lagrangian velocity autocorrelation function. Though the spectrum and the structure function are related, they can contain different information [22]. Again using K41 theory, the spectrum should scale as $E_{ij}^L(\omega) = B_0\epsilon\omega^{-2}\delta_{ij}$, and like the structure function should be isotropic. The constant B_0 is related to C_0 simply by a factor of π : $C_0 = \pi B_0$ [23].

The anisotropy we see in the structure function is also present in the spectrum. Figure 5 shows the diagonal components of the spectrum for $R_\lambda = 690$ compensated by $\epsilon\omega^{-2}$, where again a plateau corresponds to K41 scaling. The anisotropy between the axial and radial components of the spectrum is clear. While our spectral data are not as well resolved as our structure function data due to the scarcity of very long particle tracks observed in our experiment, we can still

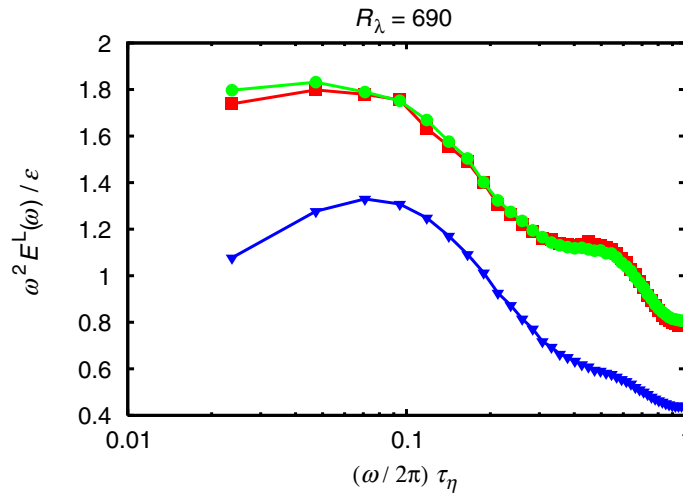


Figure 5. Compensated Lagrangian velocity spectra at $R_\lambda = 690$ in the x -direction (\blacksquare), y -direction (\bullet) and z -direction (\blacktriangledown). By scaling the spectra by $\epsilon\omega^{-2}$, we expect to see a plateau in the inertial range with value B_0 . The frequency axis has been scaled by the Kolmogorov frequency. As above, we note that the difference in magnitude between the radial spectra and the axial spectrum reflects the large-scale structure of our flow. The bump in the spectrum at high frequencies is due to noise in the measurements, but the inertial range behaviour is unaffected.

measure both the anisotropy and the scaling constant B_0 . The bump in the spectrum at high frequencies is due to noise in the measurements; by adjusting the width of our smoothing and differentiating filter, however, we have found that the inertial range values of the spectrum are not affected by the noise.

Persistent anisotropy has been noted previously in Eulerian studies of homogeneous shear flows [24, 25] and in the context of the SO(3) symmetry group [26, 27], as well as in Lagrangian studies of fluid particle acceleration [3, 15] but has not been investigated in the context of the statistics of the Lagrangian velocity. Indeed, only a small number of studies of the Lagrangian structure function and spectrum have been conducted. These experiments, however, suffered both from large experimental uncertainties and from large Lagrangian tracers that may have averaged out the smallest scales of the flow. Hanna [28] measured the Lagrangian spectra in the atmospheric boundary layer using neutrally buoyant balloons, but acknowledged significant (as much as 50%) uncertainty in the measurements, reporting a value of 4 ± 2 for C_0 . Lien *et al* [29] measured 1D spectra using large floaters (roughly 1 m in scale) in the oceanic boundary layer. Due to the considerable noise in their measurements, they were only able to estimate that the value of C_0 lies somewhere between 3.1 and 6.2. Mordant *et al* [16] measured the radial Lagrangian structure function in a 1D laboratory acoustic particle tracking experiment in a counter-rotating disc device similar to ours. While their tracer particles were significantly smaller than those of Hanna or Lien *et al*, they were still at least a factor of 10 larger than the Kolmogorov length scale $\eta = (\nu^3/\epsilon)^{1/4}$, the scale of the smallest turbulent motion in the flow. In contrast, our particles are smaller than η for all Reynolds numbers investigated. Mordant *et al* [16] obtained a maximum value of 4 for C_0 , which may be depressed due to the filtering effect of their tracer particles. Lien and D'Asaro [13] have estimated a value of 5.5 for

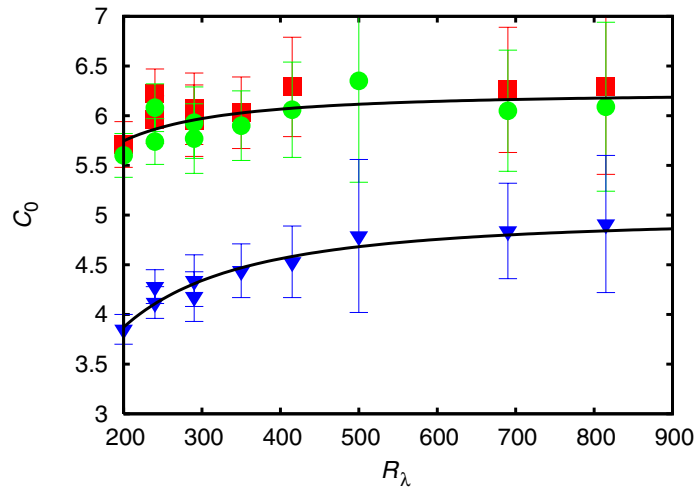


Figure 6. Measurements of C_0 from the Lagrangian structure function tensor for the xx component (\blacksquare), yy component (\bullet) and zz component (\blacktriangledown) as a function of Reynolds number. The zz component C_0 values are smaller than those measured for the two radial components, presumably due to the large-scale axisymmetry of our flow. C_0 is observed to increase weakly with Reynolds number. The solid lines are fits of Sawford’s model (2) for the Reynolds number dependence of C_0 [10]. We note that due to the time resolution in the $R_\lambda = 500$ data run, we encountered large uncertainties and were not able to measure a C_0 value from the xx component. We have therefore not included the $R_\lambda = 500$ data points in the fits of (2).

πB_0 from the spectral data published by Mordant *et al* [16]. None of these three experiments addressed the anisotropy of C_0 .

Lagrangian structure functions and spectra have also been investigated in direct numerical simulations (DNS) of the Navier–Stokes equations [17, 30]. These studies have provided a great deal of insight into Lagrangian turbulence and into the low Reynolds number scaling behaviour of these quantities. For instance, by fitting his stochastic model to low Reynolds number DNS, Sawford [10] has estimated that $C_0 = 7$ at high Reynolds numbers. DNS cannot, however, be seen as a replacement for experimental results. In addition, DNS is usually performed assuming homogeneous, isotropic flow, and so does not generally address issues of anisotropy. A review of published values for C_0 , including those from DNS, experiments, and theoretical results, is given by Lien and D’Asaro [13].

The anisotropy we find between the radial and axial components of both the structure function and the spectrum persists at all Reynolds numbers measured. In figure 6, we show values of C_0 determined from the plateaux of the compensated structure functions as a function of R_λ . For both the axial and radial structure functions, we also observe that C_0 increases weakly with Reynolds number. It is encouraging to note that figure 6 shows that our C_0 estimates seem to saturate as the Reynolds number increases; this result suggests that we can measure true inertial range behaviour at high Reynolds number despite the very short scaling range of the structure function, as can also be inferred from figure 2. To model this Reynolds number dependence of

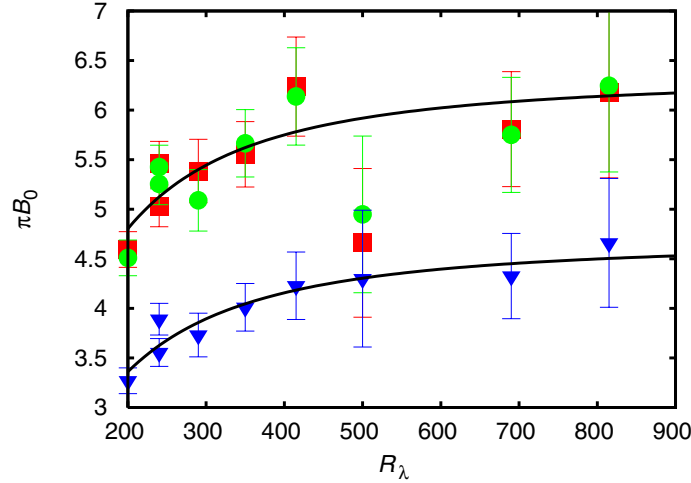


Figure 7. Measurements from the Lagrangian spectra for the x -direction (\blacksquare), y -direction (\bullet) and z -direction (\blacktriangledown). The solid lines are again fits of (2). Despite the larger degree of scatter in the data, we find good agreement with C_0 as calculated directly from the structure functions. Again, we have not included the $R_\lambda = 500$ data point in the fits.

C_0 , Sawford [10] has proposed that

$$C_0 = \frac{C_0^\infty}{1 + AR_\lambda^{-1.64}}, \quad (2)$$

where C_0^∞ is the asymptotic value of C_0 at infinite Reynolds number. Sawford suggested that $A \approx 365$, and we find values of A of the same order. Fits of this function to our C_0 data are shown in figure 6. We find that $C_0^\infty = 6.2 \pm 0.3$ for the radial structure functions and $C_0^\infty = 5.0 \pm 0.4$ for the axial structure function.

Figure 7 shows our measurements of $\pi B_0 = C_0$ as a function of Reynolds number. It is clear that our measurements of B_0 are significantly more uncertain than those of C_0 from the structure functions. Nevertheless, when we fit (2) to the πB_0 data, we find agreement with the values of C_0^∞ found above. For the radial spectral components, we find $\pi B_0^\infty = 6.3 \pm 0.4$, while for the axial component we find $\pi B_0^\infty = 4.7 \pm 0.4$.

We observe that the Reynolds number dependence of B_0 appears to be stronger than that of C_0 from our fits of (2). This is contrary to the prediction of Lien and D'Asaro [13], who suggested that B_0 has the weaker Reynolds number dependence of the two constants. The scatter seen in figure 7 is most probably not due to Reynolds number effects, but merely to the scarcity of long tracks in our experiment and the corresponding uncertainty in the low frequencies of the spectrum. The number of long tracks observed is independent of Reynolds number.

Our measurements of C_0 and πB_0 remain anisotropic even at the highest Reynolds number investigated. In figure 8, we plot the ratio of the radial measurements to the axial measurements. The anisotropy drops weakly with Reynolds number, but the decrease is very slow and the anisotropy remains strong even at the highest Reynolds number measured.

Taken together, our results suggest that any symmetries (or lack thereof) present at the large scales of the flow will also be reflected in the small-scale turbulent fluctuations. Clearly, therefore,

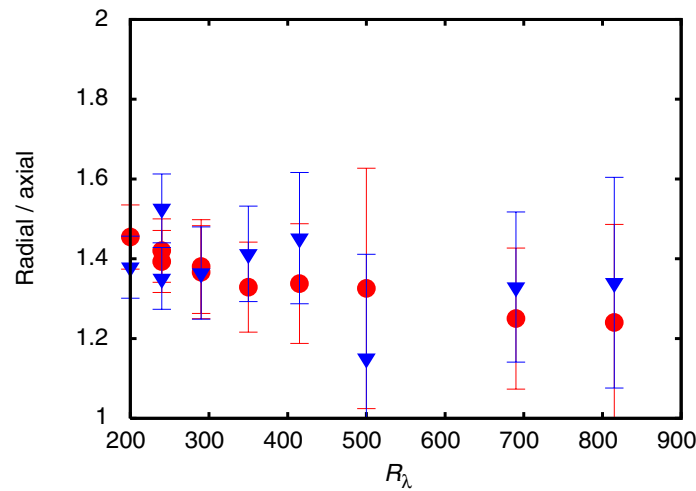


Figure 8. The ratio of the radial to the axial measurements of C_0 as a function of Reynolds number from both the structure function (●) and the spectrum (▼). While the anisotropy decreases weakly with increasing Reynolds number, the measurements remain far from isotropic even at the highest Reynolds numbers measured.

great care must be exercised when applying the results of isotropic turbulence theory to real experimental, industrial and natural flows. For instance, any climate or pollutant transport models must take the significant anisotropies present in the atmosphere into account. The significant difference between the scaling constants measured in the radial and axial directions reflects the large-scale axisymmetry in our flow. We do, however, see K41 scaling ranges for both the Lagrangian structure function and spectrum, suggesting that while our results contradict the K41 hypothesis of local isotropy, the K41 scaling hypotheses are fulfilled.

In summary, we have investigated the effects of large-scale anisotropy on the Lagrangian characteristics of small-scale turbulence. We find that the axisymmetry of our large-scale flow is also present in the small-scale fluctuations as measured by the Lagrangian second-order structure function and velocity spectrum, in contrast with Kolmogorov's hypothesis of local isotropy [2]. We have also measured the scaling constants for both the structure function and the spectrum. Using Sawford's model [10], we have extrapolated our results to the limit of infinite Reynolds number and found that $C_0^\infty = 6.2 \pm 0.3$ and $\pi B_0^\infty = 6.3 \pm 0.4$ for the radial components and that $C_0^\infty = 5.0 \pm 0.4$ and $\pi B_0^\infty = 4.7 \pm 0.4$ for the axial component in our swirling flow. It is our hope that these new measurements of scaling constants in an anisotropic flow will shed light on the nature of Lagrangian turbulence and will lead to improved models with more applicability to real flows.

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