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► **To cite this version:**

Serguei Barannikov. A-infinity $gl(N)$ -equivariant matrix integrals. Workshop on Geometry and Physics of the Landau-Ginzburg model, May 2010, Grenoble, France. hal-00490107

HAL Id: hal-00490107

<https://hal.science/hal-00490107>

Submitted on 7 Jun 2010

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CNRS

02/06/2010

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$$\hbar \Delta S + \frac{1}{2} \{S, S\} = 0,$$

$S = \sum_{g \geq 0, i} \hbar^{2g-1+i} S_{g,i}$ where

$$S_{g,i} \in \text{Sym}^i(C_\lambda[1-d]), \quad C_\lambda = (\oplus_{j=0}^{\infty} ((V[1]^{\otimes j})^\vee)^{\mathbb{Z}/j\mathbb{Z}})$$

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- In the case of the algebra $e \cdot e = e$, the answer is the matrix Airy integral

$$\int \exp\left(\frac{1}{6} \text{Tr}(Y^3) - \frac{1}{2} \text{Tr}(\Lambda Y^2)\right) dY$$

The A-infinity equivariant matrix integrals ([B2],2006)

- The asymptotic expansion as $\Lambda \rightarrow \infty \Rightarrow$ sum over stable ribbon graphs \Rightarrow cohomology classes in $H^*(\overline{\mathcal{M}}_{g,n}^K)$ (in $H^*(\overline{\mathcal{M}}_{g,n}^K, \mathcal{L})$ for odd d)

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- This is the higher genus counterpart of the (nc)Hodge theory integration on CY projective manifolds, $(\hbar\Delta\gamma + \bar{\partial}\gamma + \frac{1}{2}[\gamma, \gamma] = 0, \gamma \in \Omega^{0,*}(M, \Lambda T))$

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- By setting $\widetilde{A} = A \oplus A^\vee[d]$ I extend the formalism to nonCY A_∞ -algebras, including weak CY algebras.
- My A_∞ equivariant matrix integrals give an integration framework in the noncommutative (derived algebraic) geometry, particularly adopted to the equation $\{m_{A_\infty}, m_{A_\infty}\} = 0$

$$\int \exp \widehat{S}(X, \Lambda) \widehat{\varphi} dX$$

$$\varphi \in \text{Ker}(\hbar\Delta + \{S, \cdot\}), \quad \varphi_{g,i} \in \text{Symm}^i(C_\lambda[1-d])$$

CY complex projective variety ($g=0$ calculations)

- M - projective manifold/ \mathbb{C} , $c_1(T_M) = 0$

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$$\Omega(t, \hbar) = \int_M \exp(\gamma)\omega$$

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- for γ^W , normalized via a filtration W opposite to F^{Hodge} ([B5]):

$$\frac{\partial^2}{\partial t^i \partial t^j} \Omega = \hbar^{-1} C_{ij}^k(t) \frac{\partial}{\partial t^k} \Omega$$

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$$C_{kij}(t) = \partial^3(\text{genus} = 0 \text{ GW-potential of } M^{\text{mirror}})$$

Noncommutative Hodge structures ([B5])

- The class $[\exp(\gamma^W)\omega] = \Omega(t, \hbar)$ is obtained as intersection

$$\Omega(t, \hbar) = \mathcal{L}(t) \cap (\text{Affine space}(W))$$

$$\mathcal{L}(t) \subset H_{DR}^*(M)((\hbar)) \hat{\otimes} \mathcal{O}_{\mathcal{M}_{\Lambda T}}$$

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(implies tt^* -equations, remarkably $\mathcal{D}_{\frac{\partial}{\partial \hbar}}$ -modules over \mathbb{A}^1 with similar properties appeared many years ago in works of Birkhoff, Malgrange, K.Saito and M.Saito)

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- Over moduli space of complex structures

$$\mathcal{L}(t) = \sum_r (F^{\text{Hodge}})^r \hbar^{-r} [[\hbar]]$$

Noncommutative Hodge structures ([B5]), cont'd

- $\mathcal{L}(t)$ corresponds via HKR and formality isomorphisms for $C^*(A, A) + k[\zeta, \frac{\partial}{\partial \zeta}]$ -module $C_*(A)$ to

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$$\frac{\partial}{\partial t} HC^-(A_t) \subset \hbar^{-1} HC^-(A_t), \quad \frac{\partial}{\partial t} - \text{Getzler flat connection on } HP(A_t)$$

where

$$rk_{C[[\hbar]]} HC^-(A) = rk_{C((\hbar))} HP$$

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- Real structure on HP in the case of arbitrary A_∞ -algebra?

Noncommutative Batalin-Vilkovisky operator ([B1])

- V $-\mathbb{Z}/2\mathbb{Z}$ graded vector space, l - scalar product on V^\vee of degree d ,

$$F = \text{Sym}(C_\lambda(V)[1-d])$$

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- Define the noncommutative BV differential on F via

$$\begin{aligned} \Delta(x_{\rho_1} \dots x_{\rho_r})_\lambda (x_{\tau_1} \dots x_{\tau_t})_\lambda &= \\ &= \sum_{p,q} (-1)^\varepsilon l_{\rho_p \tau_q} (x_{\rho_1} \dots x_{\rho_{p-1}} x_{\tau_{q+1}} \dots x_{\tau_{q-1}} x_{\rho_{p+1}} \dots x_{\rho_r})_\lambda + \\ &\sum_{p \pm 1 \neq q} (-1)^{\tilde{\varepsilon}} l_{\rho_p \rho_q} (x_{\rho_1} \dots x_{\rho_{p-1}} x_{\rho_{q+1}} \dots x_{\rho_r})_\lambda (x_{\rho_{p+1}} \dots x_{\rho_{q-1}})_\lambda (x_{\tau_1} \dots x_{\tau_t})_\lambda \\ &\sum_{p \pm 1 \neq q} (-1)^{\tilde{\varepsilon}} l_{\tau_p \tau_q} (x_{\rho_1} \dots x_{\rho_r})_\lambda (x_{\tau_1} \dots x_{\tau_{p-1}} x_{\tau_{q+1}} \dots x_{\tau_t})_\lambda (x_{\tau_{p+1}} \dots x_{\tau_{q-1}})_\lambda \end{aligned}$$

- signs are the standard Koszul signs taking into account that $(x_{\rho_1} \cdots x_{\rho_r})_\lambda = (1 - d) + \sum \overline{x_{\rho_i}}$, $x_i \in V[1]$.

Noncommutative Batalin-Vilkovisky differential cont'd

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- $\Delta_1 + \hbar\Delta_2 \rightarrow$ non-commutative Batalin–Vilkovisky geometry, stable ribbon graphs, compactified moduli spaces $H_*(\overline{\mathcal{M}}_{g,n}^K)$ ([B1])

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- $\text{Ker} \Delta_1 + \Delta_2 = \text{Im } \Delta_1 + \Delta_2$

Solutions to nc BV equation

- Conjecture ([B1]). Counting of holomorphic curves $(\Sigma, \partial\Sigma, p_i) \rightarrow (M, \coprod L_i, \oplus H_*(L_i \cap L_j))$, with $\mathbb{Z}/2\mathbb{Z}$ -graded local systems, gives solution to the nc-BV equations.

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- Theorem ([B6]). Summation over ribbon graphs \rightarrow solution to the nc Batalin-Vilkovisky equation from dg-associative algebras (summation over trees \rightarrow A-infinity algebra structure)

Strange associative superalgebra with odd trace and psi-classes.

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- Assume: I - an *odd derivation* acting on V , preserving the scalar product: ,
in general $I^2 \neq 0$ (!), $\exists \tilde{I}, [I, \tilde{I}] = 1, str([a, \cdot]) = 0$ for any $a \in A$.

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- Theorem ([B2],[B3]) This data \rightarrow Cohomology classes in $H^*(\overline{\mathcal{M}}_{g,n}^K)$
- Example $q(N)$, $q(N) = \{[X, \pi] = 0 | X \in gl(N|N)\}$,where π -odd involution, $q(N)$ has *odd trace* $otr, I = [\Xi, \cdot], \Xi$ - odd element
 $\Xi = (0 \quad | \quad diag(\lambda_1, \dots, \lambda_n)), (I^2 \neq 0 (!))$







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in general $I^2 \neq 0$ (!), $\exists \tilde{I}, [I, \tilde{I}] = 1, str([a, \cdot]) = 0$ for any $a \in A$.
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- Example $q(N)$, $q(N) = \{[X, \pi] = 0 | X \in gl(N|N)\}$,where π -odd involution, $q(N)$ has *odd trace* otr , $I = [\Xi, \cdot]$, Ξ - odd element $\Xi = (0 \mid diag(\lambda_1, \dots, \lambda_n))$, ($I^2 \neq 0$ (!))
- Theorem ([B2],[B3]) This is the generating function for products of classes $c_1(T_i)$.

Strange associative superalgebra with odd trace and psi-classes.

- V - associative algebra, odd/even scalar product
- Assume: I - an *odd derivation* acting on V , preserving the scalar product: , in general $I^2 \neq 0$ (!), $\exists \tilde{I}, [I, \tilde{I}] = 1, str([a, \cdot]) = 0$ for any $a \in A$.
- Theorem ([B2],[B3]) This data \rightarrow Cohomology classes in $H^*(\overline{\mathcal{M}}_{g,n}^K)$
- Example $q(N)$, $q(N) = \{[X, \pi] = 0 | X \in gl(N|N)\}$,where π -odd involution, $q(N)$ has *odd trace* $otr, I = [\Xi, \cdot], \Xi$ - odd element $\Xi = (0 \quad | \text{diag}(\lambda_1, \dots, \lambda_n)), (I^2 \neq 0$ (!))
- Theorem ([B2],[B3]) This is the generating function for products of classes $c_1(T_j)$.
- Similarly, with even scalar product and an *odd derivation*, in particular for $gl(N|N)$ and $I = [\Xi, \cdot], \Xi \in gl(N|N)_{odd}$.

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