

A level set approach for brittle damage.

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P.E. Bernard.**

Outline.

Level set
Approach to
damage.

I- Introduction

II -SLS
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- Motivations.
- A first, “sharp” level set damage model (SLS).
- Advanced model : the “thick” level set damage model (TLS).
- Conclusions.

The eXtended Finite Element Method for cracks : Achievement.

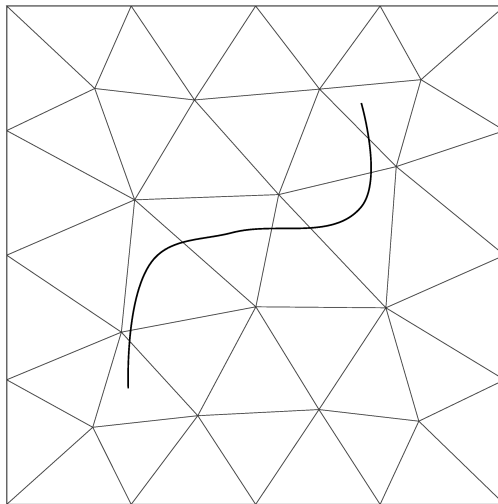
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- How to model the crack propagation in complex geometry, avoiding complex re-meshing ?
- Two level set functions are used to describe the crack, independently of the mesh.
- Discontinuities are represented using enrichment functions, based on the level set.



- Ready for industrial applications.

The eXtended Finite Element Method for cracks : Limits

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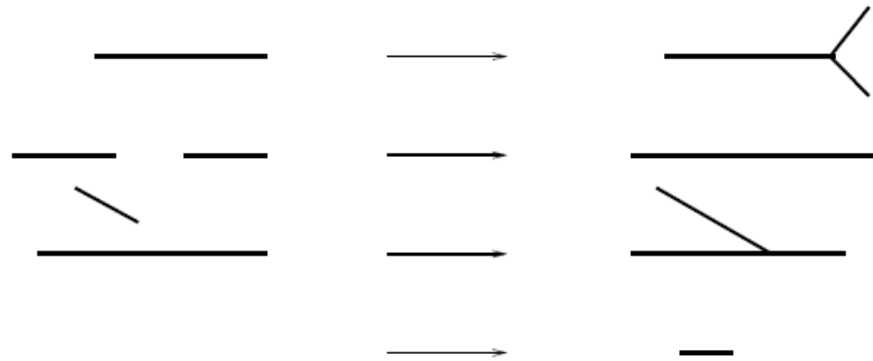
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Open questions remains:

- A crack that branch in two ?
- Two cracks that merge ?
- Crack Initiation ?



For this problems, the cracks pattern undergo a change of topology. The two levels set representation of each crack is not convenient.

For the geometrical representation, one level set only, on the other end, could represent this phenomenon correctly. To model the mechanical part, damage mechanics is probably the way to go.

Bibliography

In this cases, fracture mechanics is at it's limit. Damage model are necessary.

- One difficulty : were to place the crack ? ($d=1$). Progress in this direction (Perego et Al. 2007)
- And of course localization ...
 - Non local damage model (Bazant, Belytschko, Chang, 1984), (Pijaudier-Cabot 1987).
 - Higher order gradient models (Aifantlis 1984), (Triantafyllis and Aifantis 1986), (Shreyer and Chen 1986).
 - Micro-polar continuum model (Mühlhaus and Vardoulakis 1987).
 - Phase-field Approach (Hakim and Karma 2005), (Karma, Kessler and Levine 2001)
 - Variational approach (Francfort and Marigo 1998), (Bourdin, Francfort and Marigo, 2000, 2008).

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Damage model using level-set : Sharp Level Set Approach.

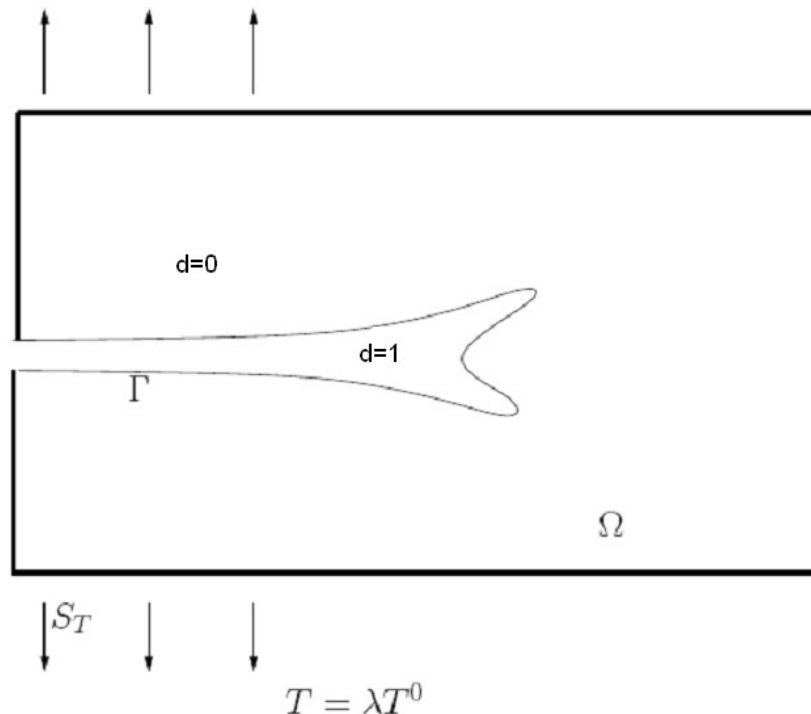
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- The damaged zone boundary is represented by a moving front, modeled by the iso-zero value of a level set. The front is the boundary between an undamaged and fully damaged zone (representing for example micro cracks.).



The first model (SLS Approach).

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Potential energy of the systeme

$$E(\Gamma, \mathbf{u}, \lambda) = \int_{\Omega} e d\Omega - \int_{S_T} \lambda \mathbf{T}^0 \cdot \mathbf{u} dS$$

$$\mathbf{u} = \lambda \mathbf{u}^0 \text{ on } S_u$$

$$e = \frac{1}{2} \boldsymbol{\epsilon}(\mathbf{u}) : \mathbf{E} : \boldsymbol{\epsilon}(\mathbf{u})$$

Equilibrium gives :

$$\mathbf{u} = \mathbf{u}(\Gamma, \lambda)$$

For given \mathbf{u} , energy variation with respect to the front position :

$$W(\Gamma, \lambda) = E(\Gamma, \mathbf{u}(\Gamma, \lambda), \lambda) \quad DW[v_n] = - \int_{\Gamma} e v_n dS$$

A propagation law (Nguyen , Pradeilles and Stolz 1989)

$$v_n \geq 0 \quad f = e - \left(Y_c + \frac{\gamma_c}{\rho} \right) \leq 0 \quad f v_n = 0$$

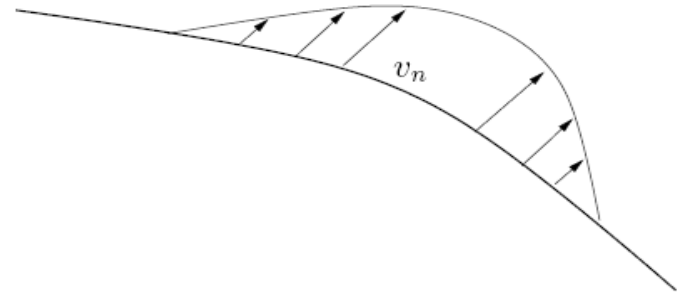
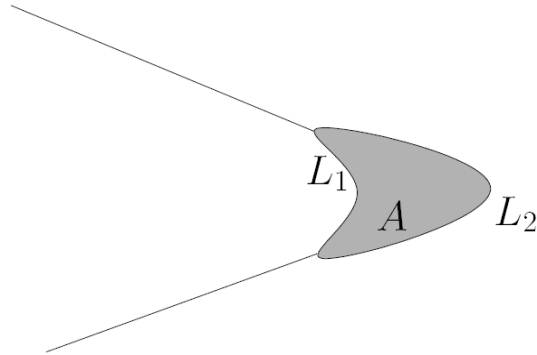
Geometrical interpretation of the dissipation

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Dissipation from 1 to 2

$$D_{12} = Y_c A + \gamma_c (L_2 - L_1)$$

The surface. Change is :

$$\dot{A} = \int_{\Gamma} v_n dS$$

The length change is.

$$\dot{L} = \int_{\Gamma} \frac{v_n}{\rho} dS$$

Dissipation is:

$$\dot{D} = \int_{\Gamma} Y_c v_n + \gamma_c \frac{v_n}{\rho} dS$$

Numerical experiments:

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- Extended finite element techniques : Mesh is not conforming with the front.
- Quad-tree type mesh adaptation.
- Level set advanced solving Hamilton Jacoby equation at each time step.
- Simple power law model for the front velocity :

$$v_n = \left\langle \frac{Y}{Y_c} - 1 - \frac{l_c}{\rho} \right\rangle_+^n$$

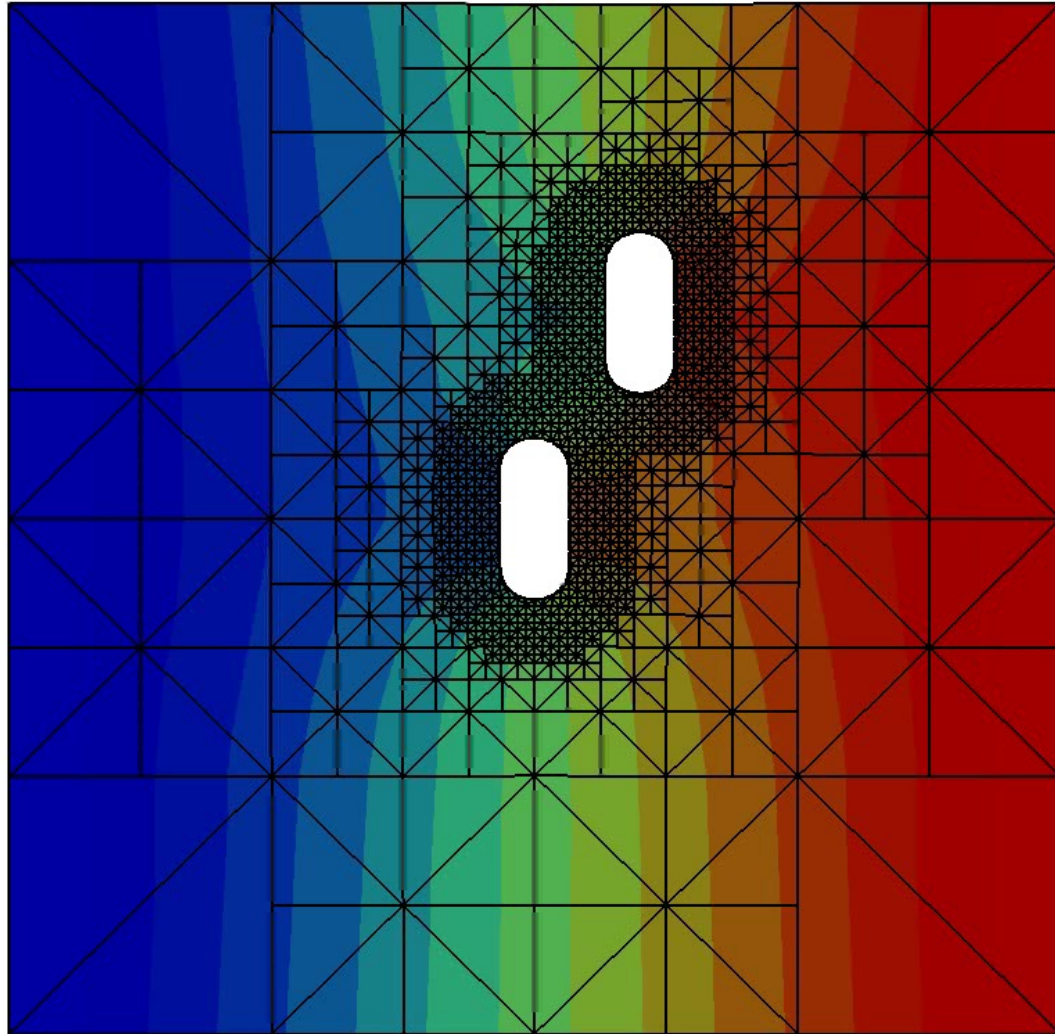
Damage coalescence.

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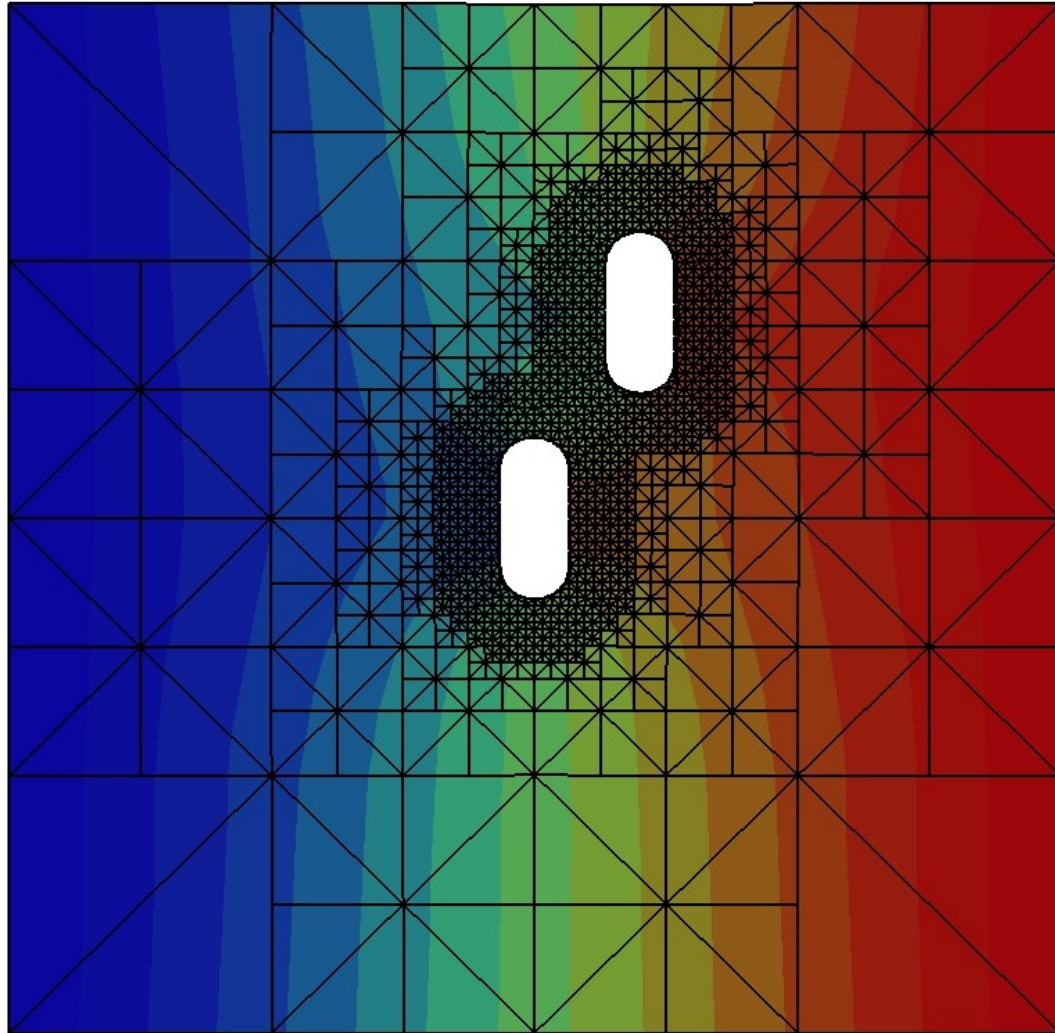
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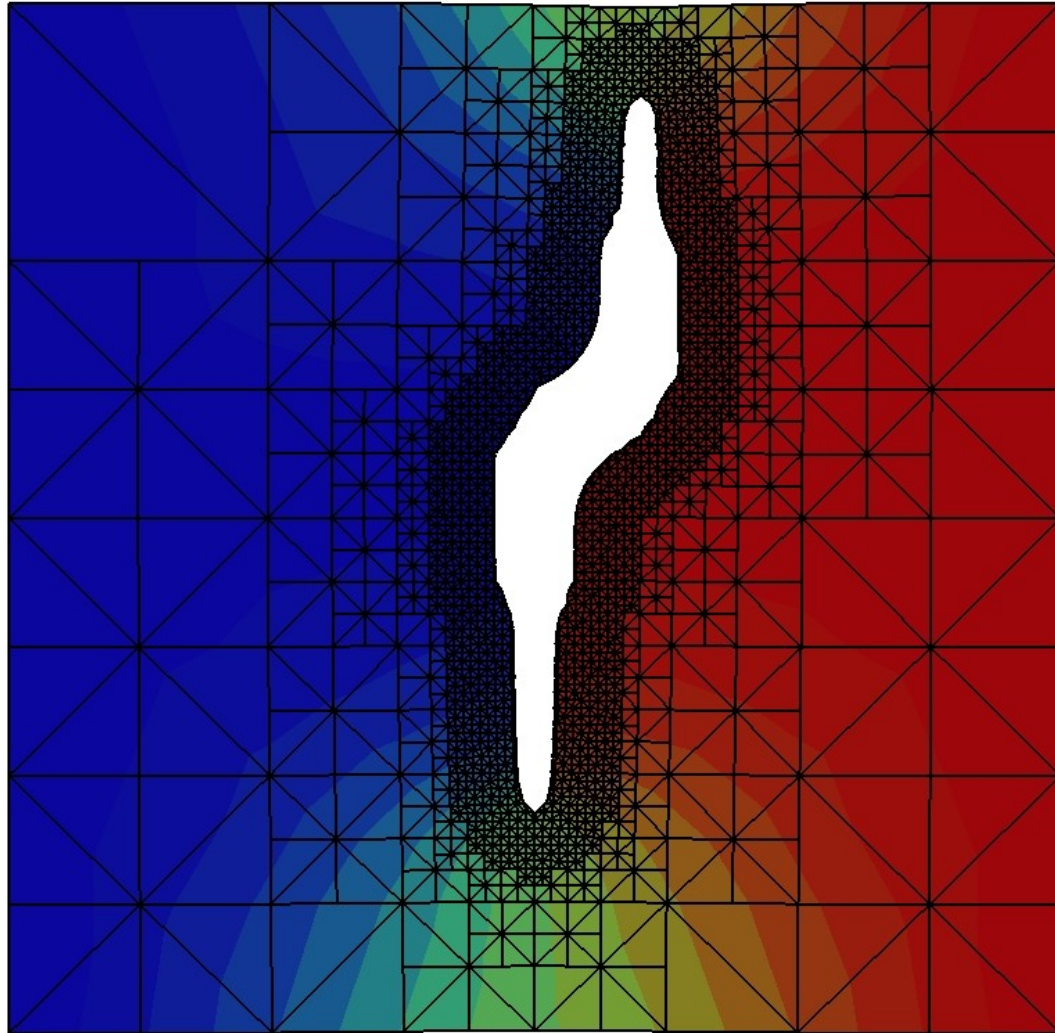
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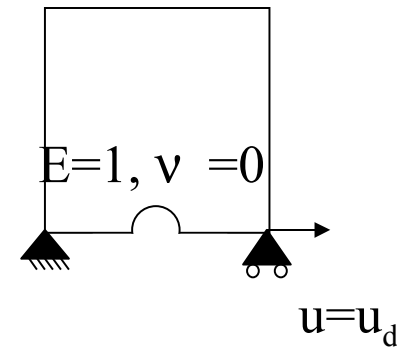
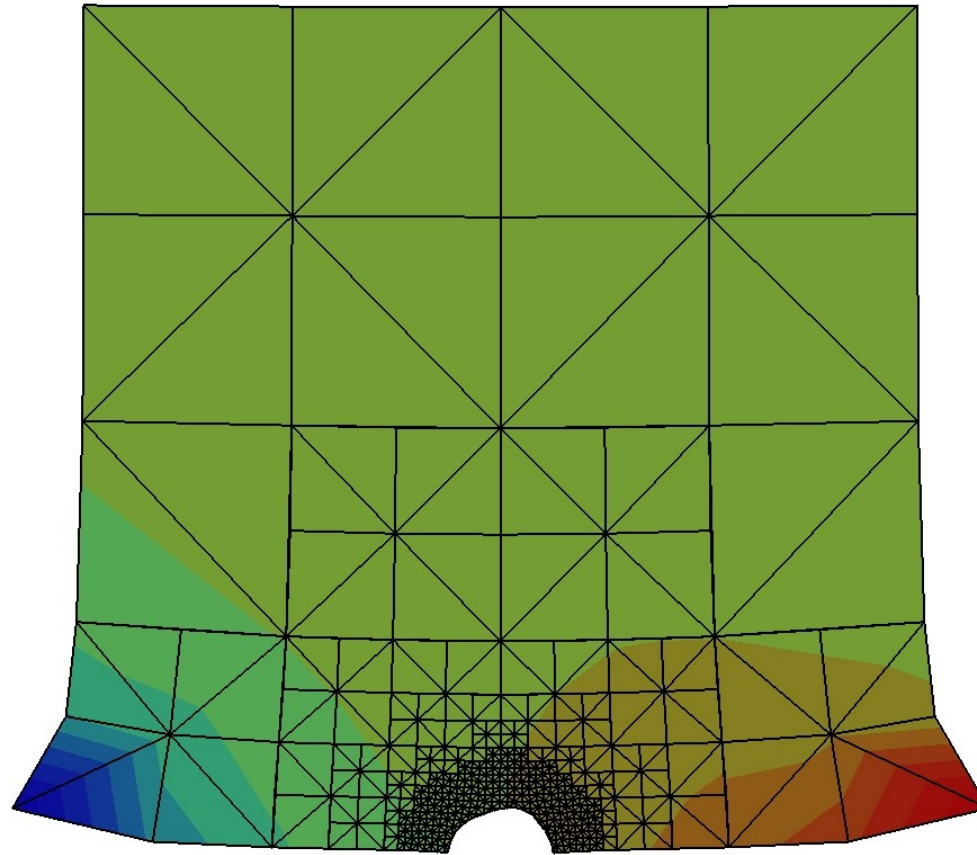
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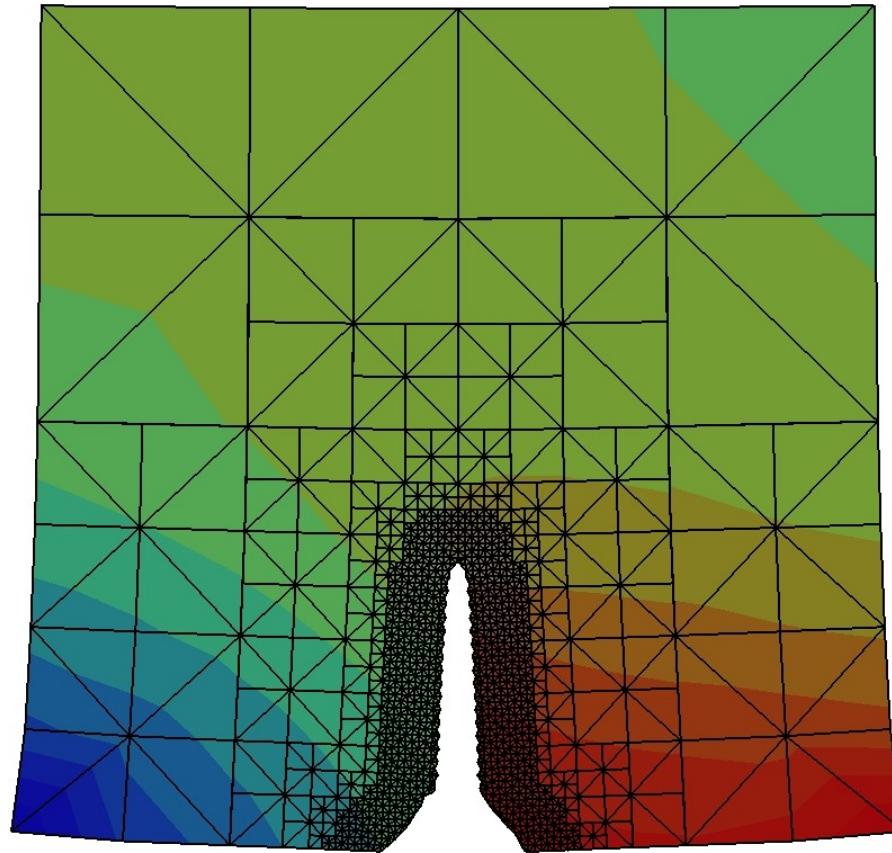
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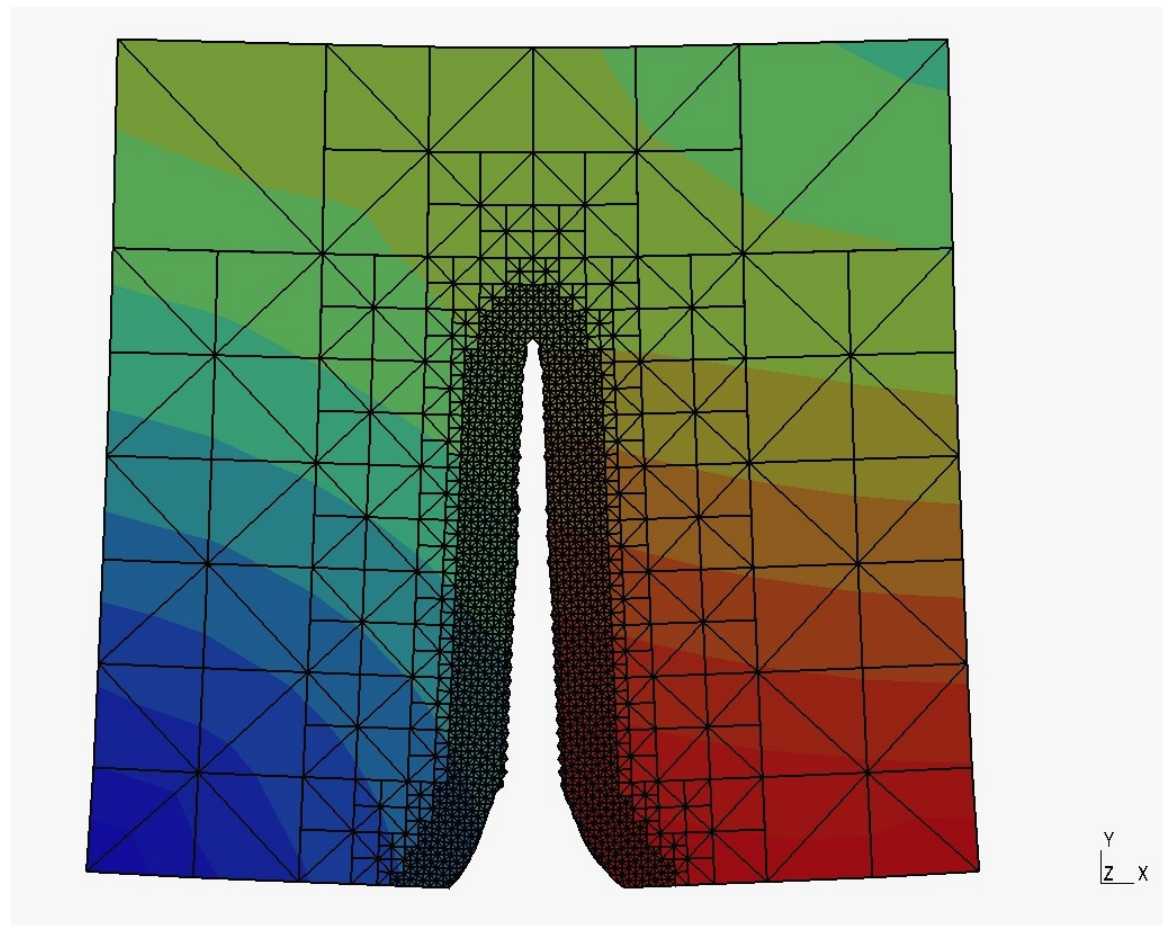
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Improving the model.

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Encouraging results, But :

- Whatever l_c , the damaged zone tend to crack of 0 thickness. The curvature term, seems at least in our numerical setting, enough to prevent localization
- Alternative model :

$$v_n = k_v \left\langle \frac{Y}{Y_c} - 1 - \frac{l_c}{\rho} \right\rangle_+^n H\left(\rho, \frac{d\rho}{dt}\right)$$

$$\begin{array}{ll} \text{if } \left(\frac{l_c}{\rho} > 1 \quad \text{and} \quad \frac{d\rho}{dt} < 0\right) & H = 0 \\ \text{else} & H = 1 \end{array}$$

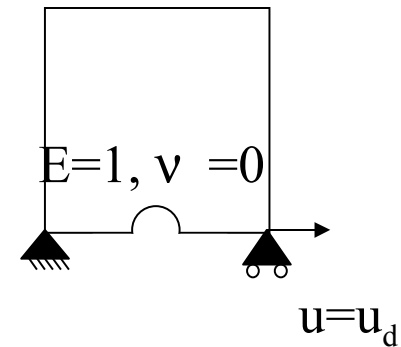
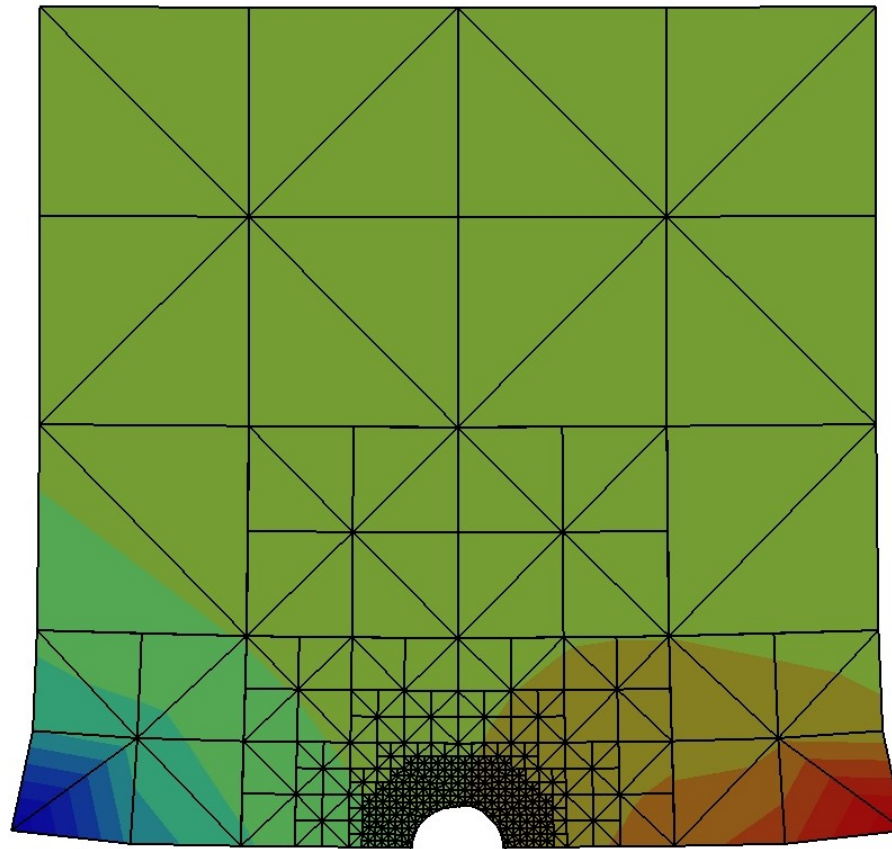
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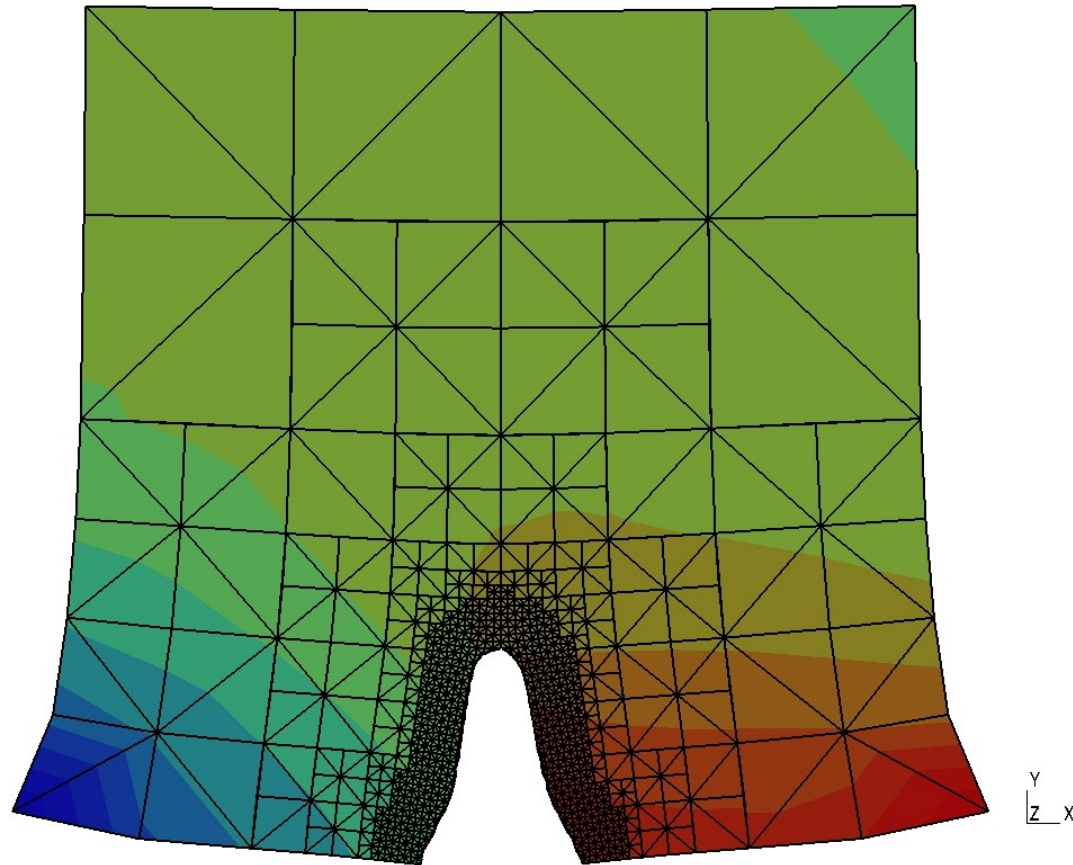
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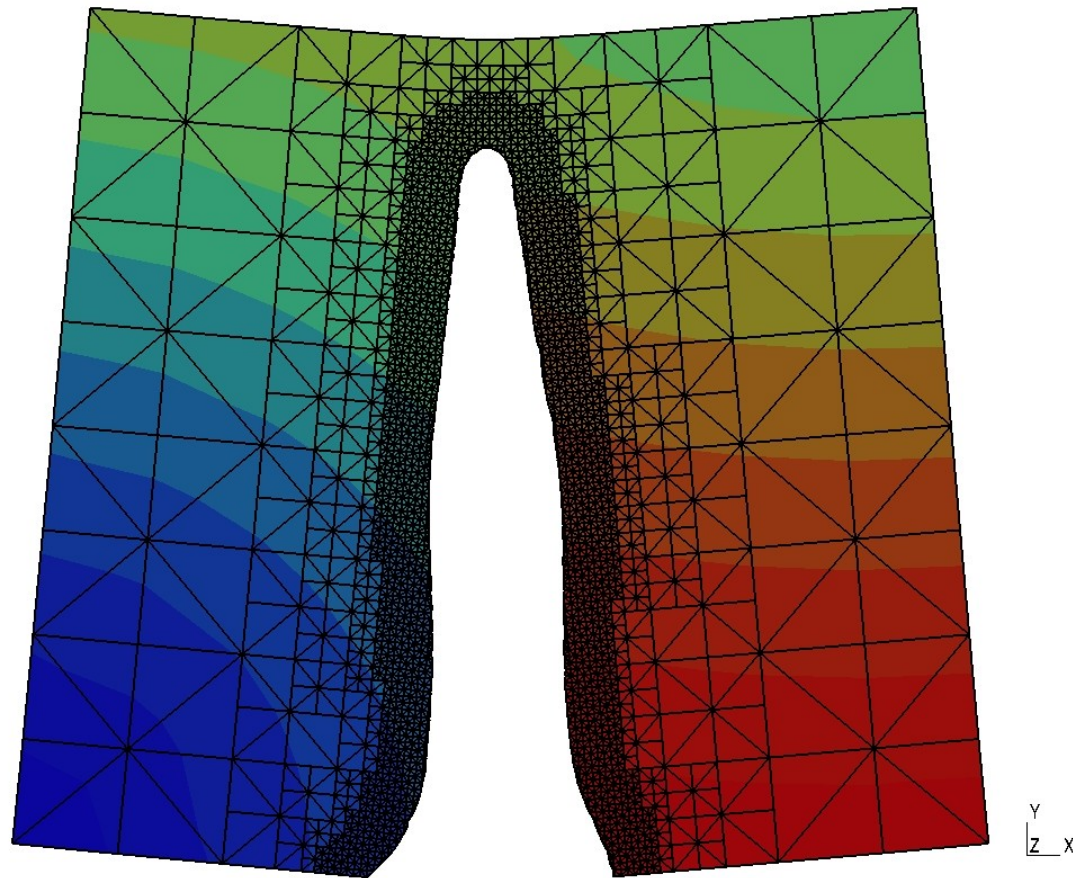
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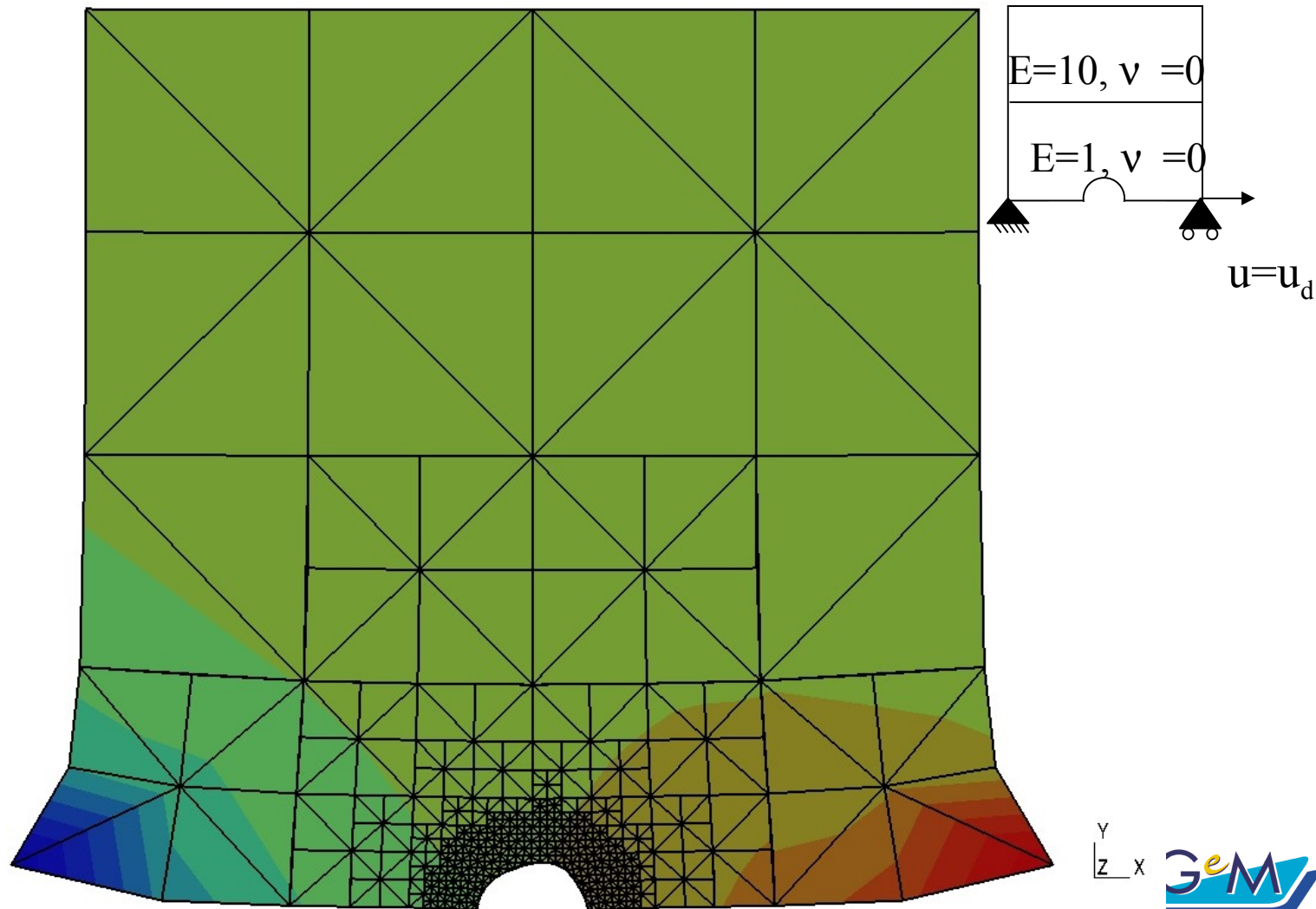
Branching crack.

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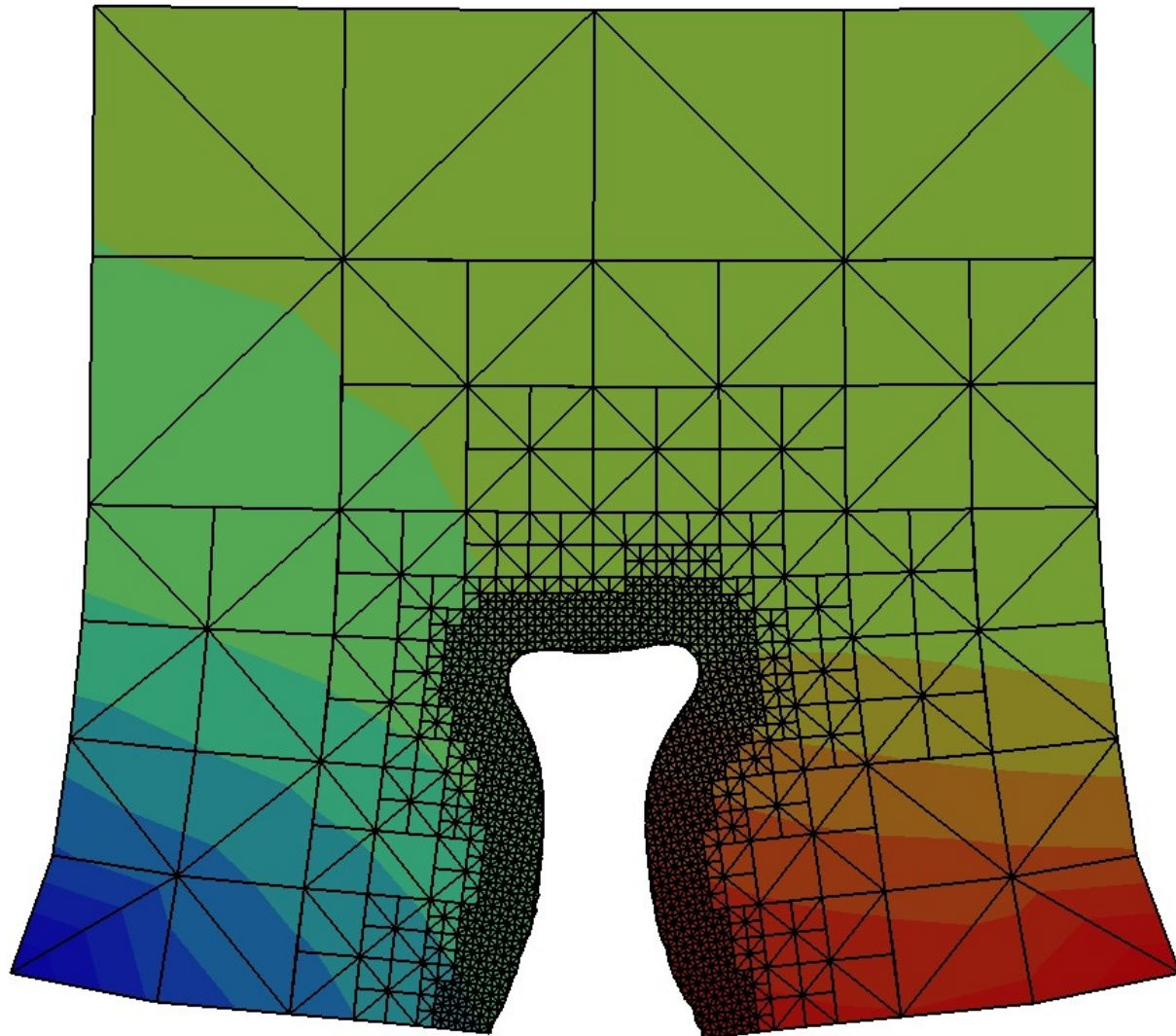
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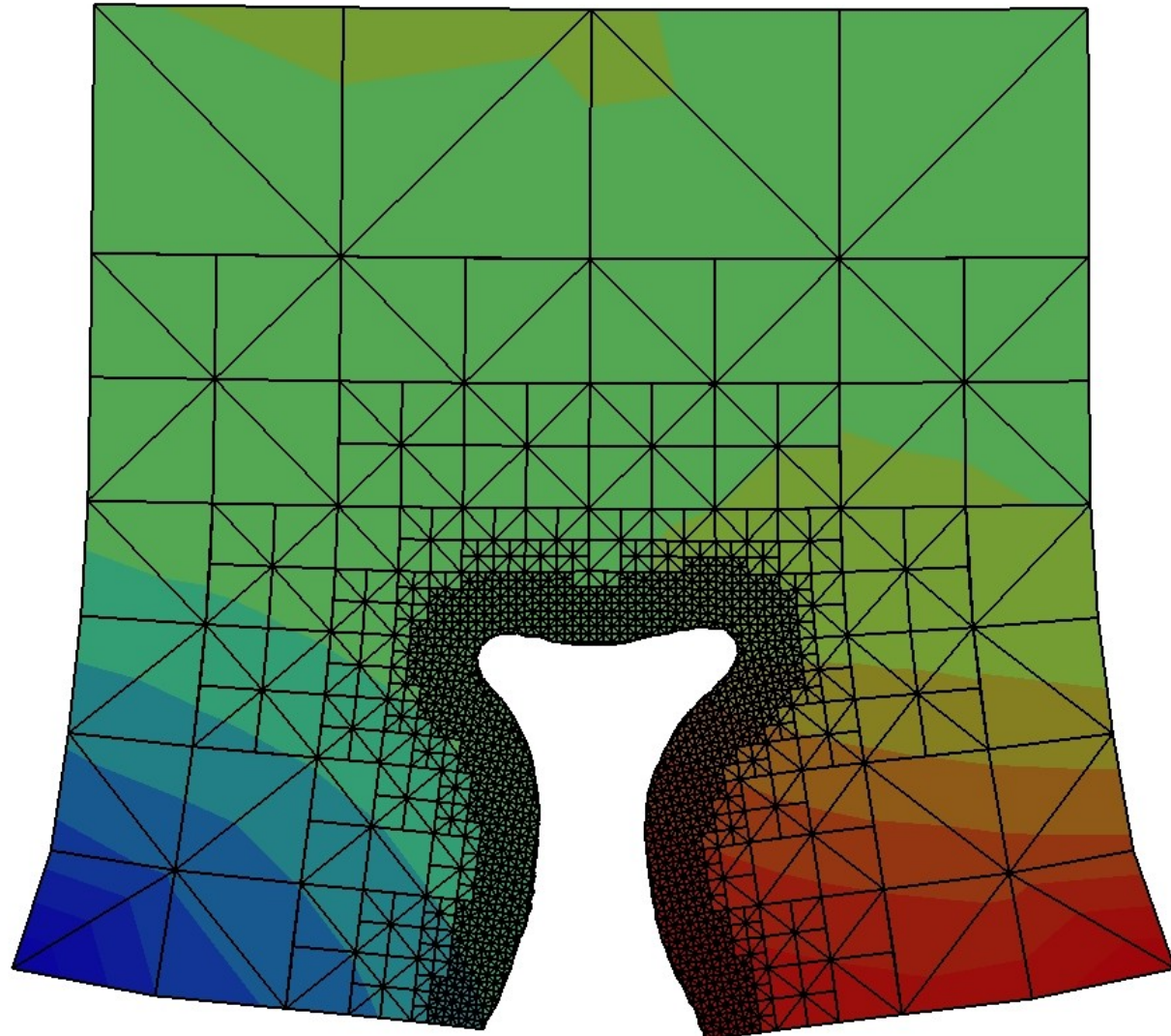
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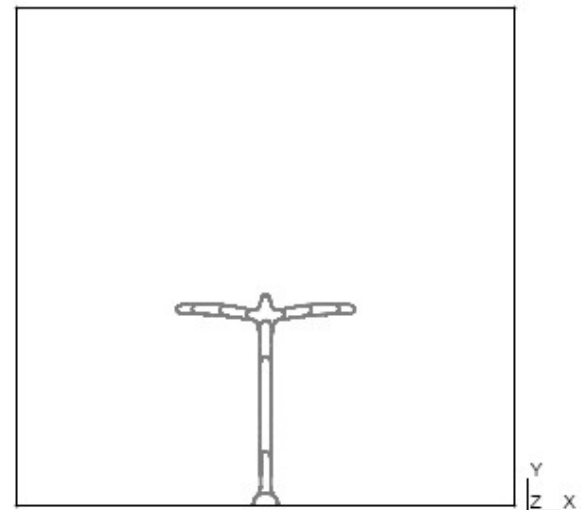
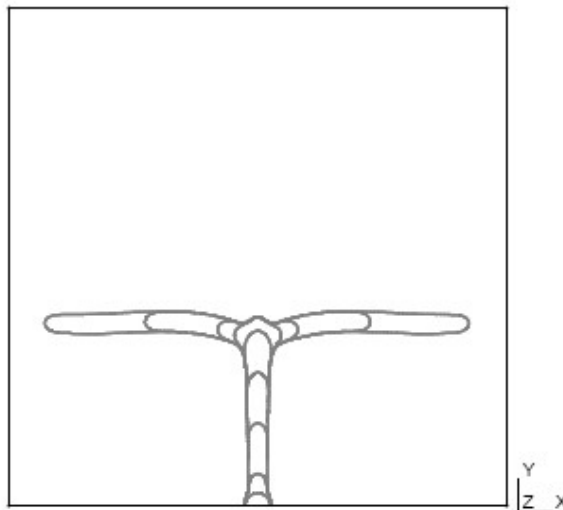
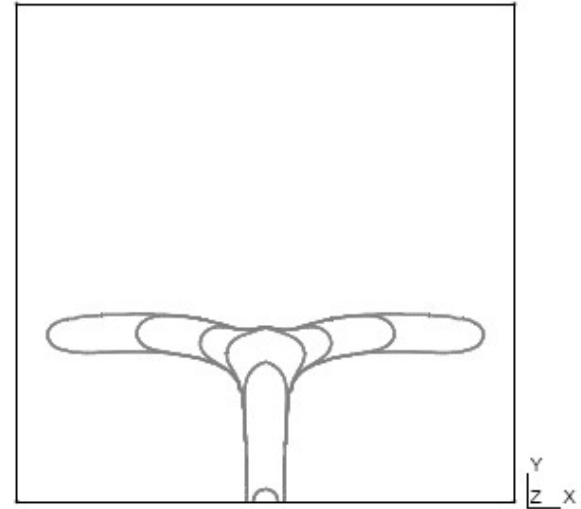
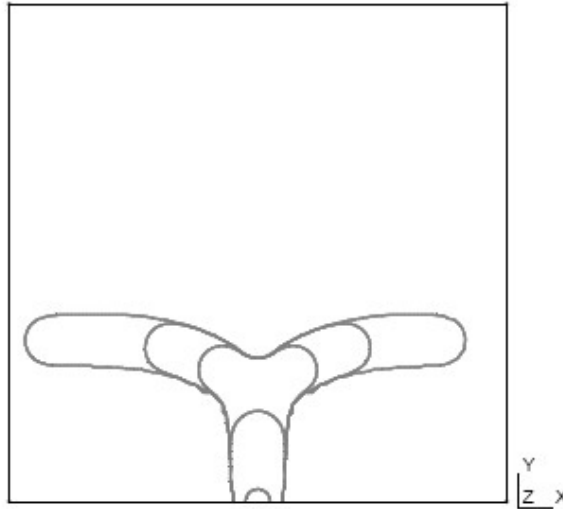
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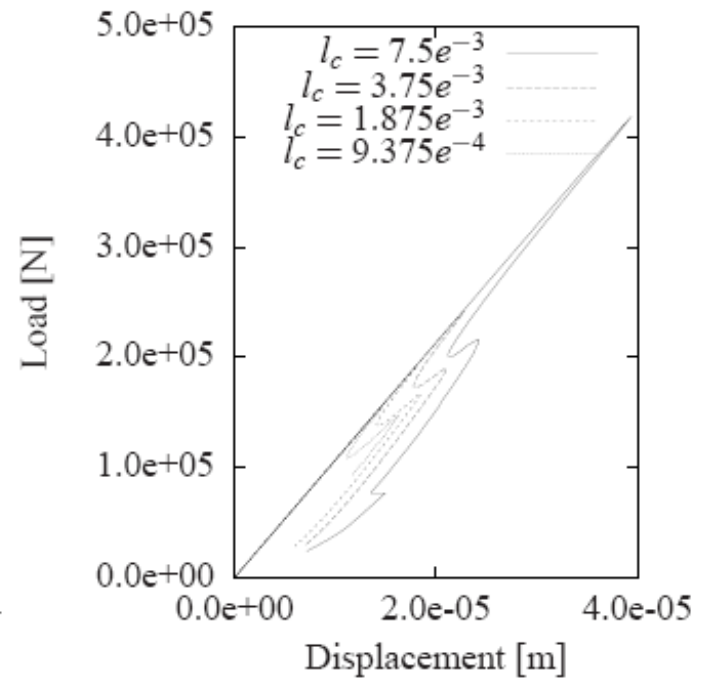
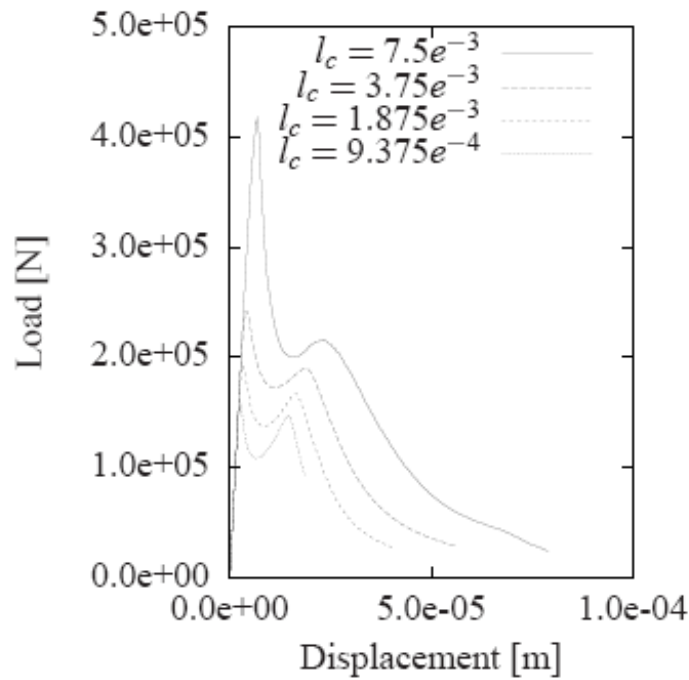
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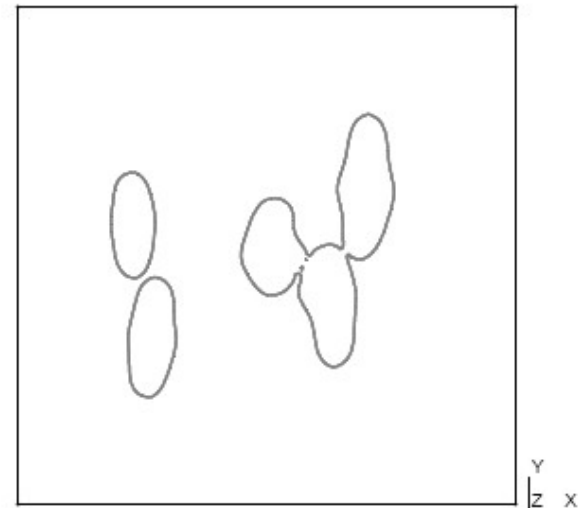
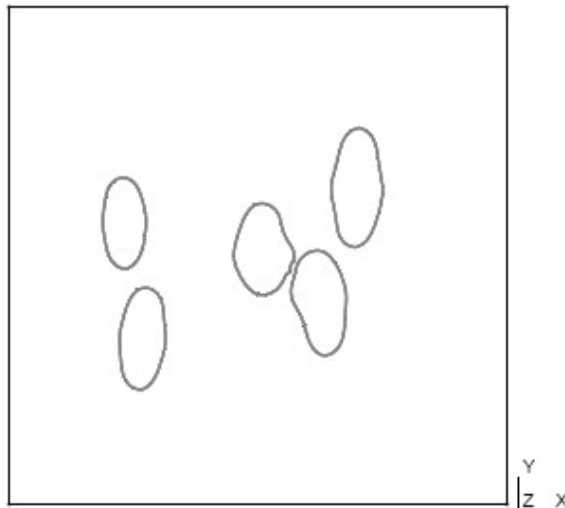
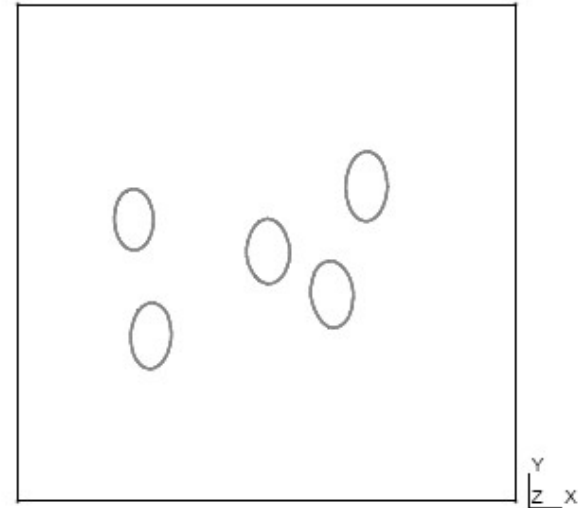
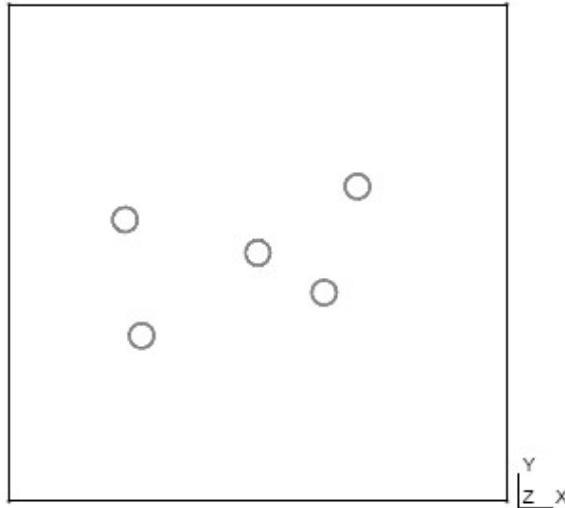
Coalescence.

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Conclusions on the first model.

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- + Topological changes are possible and handle automatically.
- + Characteristic length have direct effect on crack thickness.
- + Simple implementation compared to non-locals models.
- Damage is brutal. No transition between sound and fully damaged model.
- No criteria to initiate damage.

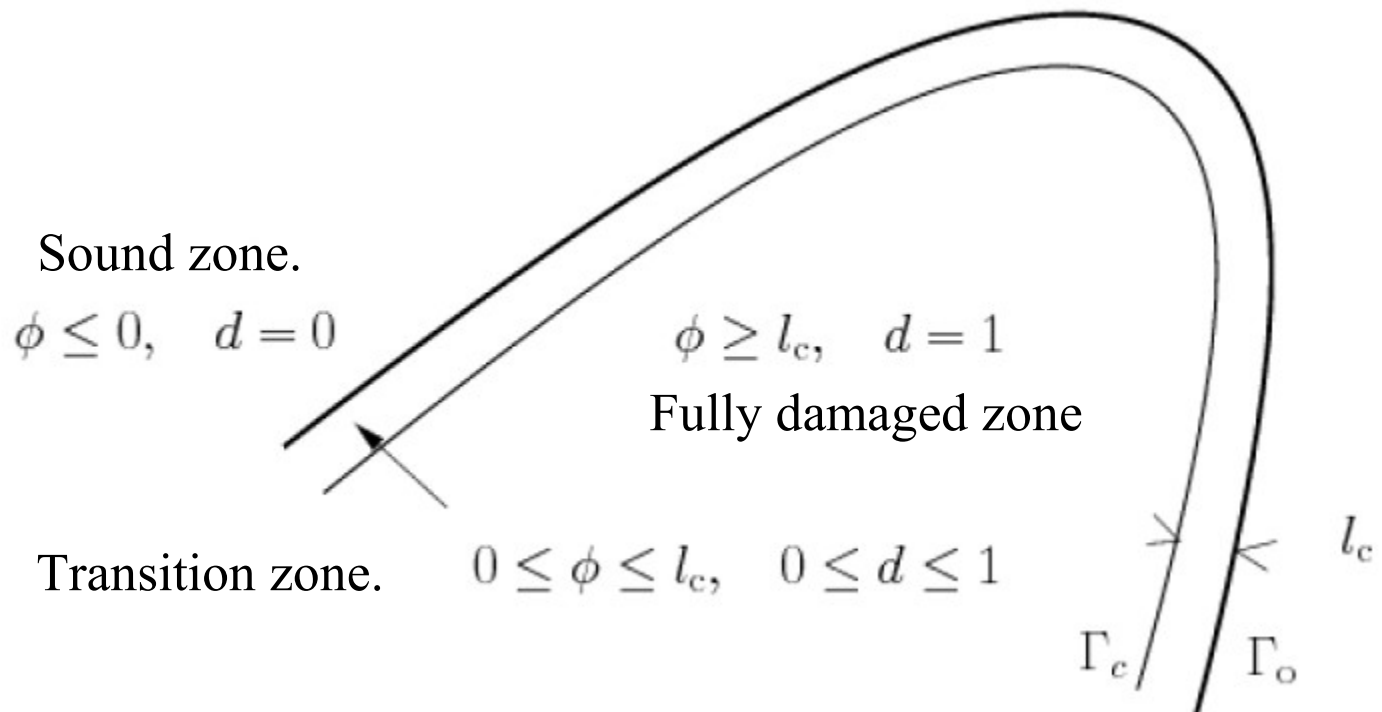
Thick Level Set (TLS) approach to damage.

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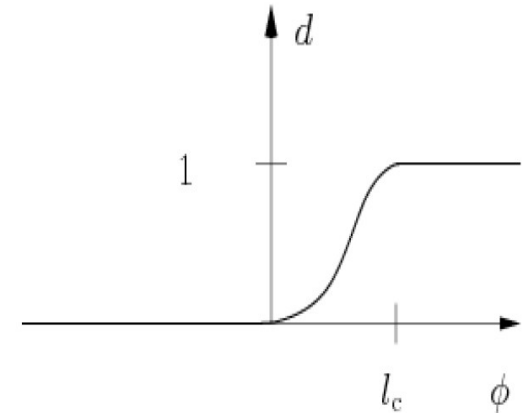
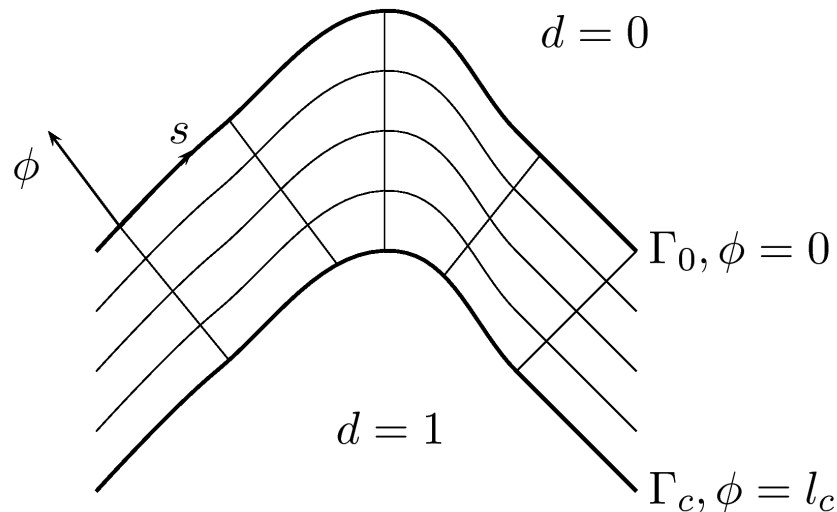
Damage “Kinematics” in the Band.

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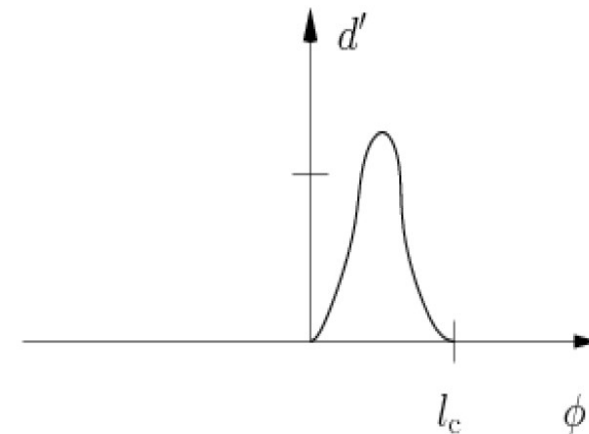
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Model hypothesis and input : the damage evolution is a function of the distance to the front.



Computing the speed.

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Mechanical problem.

$$\varphi = \varphi(\epsilon, d) = \frac{1}{2}(1 - d)\epsilon : \mathbf{E} : \epsilon$$

$$\sigma = \frac{\partial \varphi}{\partial \epsilon} = (1 - d)\mathbf{E} : \epsilon$$

$$Y = -\frac{\partial \varphi}{\partial d} = \frac{1}{2}\epsilon : \mathbf{E} : \epsilon$$

Damage constrain kinematics.

$$\|\nabla\phi\| = 1$$

$$\nabla\phi \cdot \nabla v_n = 0$$

$$v_n \geq 0$$

$$\frac{\partial \phi}{\partial t} + v_n \|\nabla\phi\| = 0$$

$$\dot{d}(\phi) = d'(\phi)v_n$$

How to choose the normal velocity ?

Computing the speed.

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We propose the following approach :

-Take a classical local damage model.

-Choose the velocity that minimize the behavior error under the kinematics constrain

$$\dot{d} = \frac{\partial \psi^*(Y)}{\partial Y} \quad Y = \frac{\partial \psi(\dot{d})}{\partial \dot{d}}$$

$$\psi(\dot{d}) \text{ convex, } \psi(\dot{d}) \geq 0, \quad \psi(0) = 0$$

$$\psi^*(Y) + \psi(\dot{d}) - Y\dot{d} \geq 0$$

$$\dot{d}(Y) = k \left\langle \frac{Y}{Y_c} - 1 \right\rangle_+^n$$

$$\psi(\dot{d}) = Y_c \dot{d} + \frac{k Y_c}{1/n+1} \left(\frac{\dot{d}}{k} \right)^{1/n+1} + I_+(\dot{d})$$

Evolution law, using
dissipation potential and
it's dual.

And $\dot{d} = 0$ if the damage
evolution law is verified

In our examples we will pick
a simple power law.

Computing the speed: from the local law to the front speed.

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$$\inf_{\dot{d}=d'(\phi)v_n} \int_{\Omega_c} \psi^*(Y) + \psi(\dot{d}) - Y \dot{d} \, d\Omega \quad \dot{d} = \frac{\partial d}{\partial \phi} \frac{\partial \phi}{\partial t} = d' v_n.$$

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After some manipulation, the probleme reduces to :

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$$\inf_{v_n} \int_{\Omega_{OC}} \psi(d' v_n) - Y d' v_n d\Omega$$

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This is the probleme that we solve numerically on the band.

No further manipulation are needed.

The following developement

are there just to understand better the physics of the model and it's relationship with other model.

Energy Variation – link with no-local model.

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$$E(\mathbf{u}, \phi) = \int_{\Omega_c} \varphi(\boldsymbol{\epsilon}(\mathbf{u}), d(\phi)) \, d\Omega - \int_{\partial\Omega_T} \mathbf{T} \cdot \mathbf{u} \, dS$$

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$$\frac{\partial E(\mathbf{u}, \phi)}{\partial \phi} = \int_{\Omega_{oc}} Y d'(\phi) \, d\Omega$$

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$$= \int_{\Gamma_0} \int_0^l Y(\phi, s) d'(\phi) (1 - \phi/\rho(s)) \, d\phi \, ds$$

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$$= \int_{\Gamma_0} \boxed{g(s)} \, ds \quad \text{Driving force.}$$

- Similar to non local where Y is integrated over a sphere.

$$\bar{Y} = \int_{\text{sphere}} Y w \, d\Omega$$

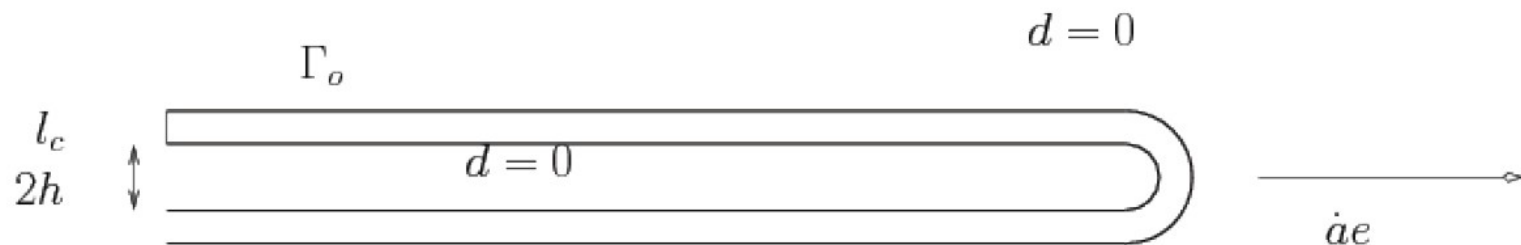
Link with fracture mechanics

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Suppose the damaged zone propagate to the right, by preserving its shape, with speed \dot{a}

Equivalent to have the normal speed $v_n = \dot{a} e \cdot n$

Which can be injected into the dissipation potential.

Link with fracture mechanics

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$$\begin{aligned} \Psi(\dot{a}) &= G_c \dot{a} + \frac{KG_c}{n'+1} \left(\frac{\dot{a}}{K}\right)^{n'+1} + I_+(\dot{a}) & G &= \int_{\Gamma_0} g e \cdot n \, d\Gamma \\ \Psi^*(G) &= \frac{KG_c}{n+1} \left\langle \frac{G}{G_c} - 1 \right\rangle_+^{n+1} & G_c &= \int_{\Gamma_0} \bar{Y}_c e \cdot n \, d\Gamma \\ \dot{a} &= K \left\langle \frac{G}{G_c} - 1 \right\rangle_+^n & K &= \left(\frac{\int_{\Gamma_0} \bar{Y}_c e \cdot n \, d\Gamma}{\int_{\Gamma_0} \bar{k}^{-n'} \bar{Y}_c (e \cdot n)^{n'+1} \, d\Gamma} \right)^n \end{aligned}$$

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Further more, if the tip of the crack is circular,
and the damage is a linear function of the level set in the band, we obtain :

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$$\begin{aligned} G_c &= Y_c (h + l_c) \\ K &= kl_c \left(\frac{2}{\int_{-\pi/2}^{\pi/2} (\cos \theta)^{n'+1} \, d\theta} \right)^n \end{aligned}$$

Numerical aspects : Computing the speed.

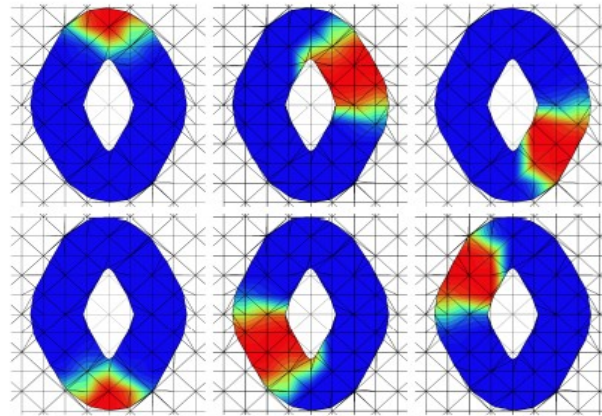
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First method : The constraint $\nabla\phi \cdot \nabla v_n = 0$ is imposed directly by constructing the approximation space on top of the parametrisation of the band (Hermite polynomial defined on s). -> need to parametrize the band.



The constraint on the positivity of the velocity, is imposed with an augmented lagrangian approach.

$$\inf_{v_n} \sup_{\lambda} \int_{\Omega_{oc}} \psi(d'v_n) - Yd'v_n + \frac{1}{2\beta} \langle \lambda + \beta v_n \rangle_-^2 - \frac{1}{2\beta} \lambda^2 d\Omega$$

Numerical aspects : Computing the speed.

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Variational approach :

The speed is directly discretised on the mesh (P1 mini or P2). The constraint $\nabla\phi \cdot \nabla v_n = 0$ is imposed using augmented lagrangian.

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$$\inf_{v_n} \sup_{\lambda} \inf_q \int_{\Omega_{OC}} \psi(d'v_n) - Yd'v_n + \frac{1}{2\beta} \langle \lambda + \beta v_n \rangle_-^2 - \frac{1}{2\beta} \lambda^2 \\ + q(\nabla v_n \cdot \nabla \phi) + \alpha_n (\nabla v_n \cdot \nabla \phi)^2 + \alpha_s (\nabla v_n \cdot \mathbf{s})^2 d\Omega$$

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Numerical aspects : Computing the speed.

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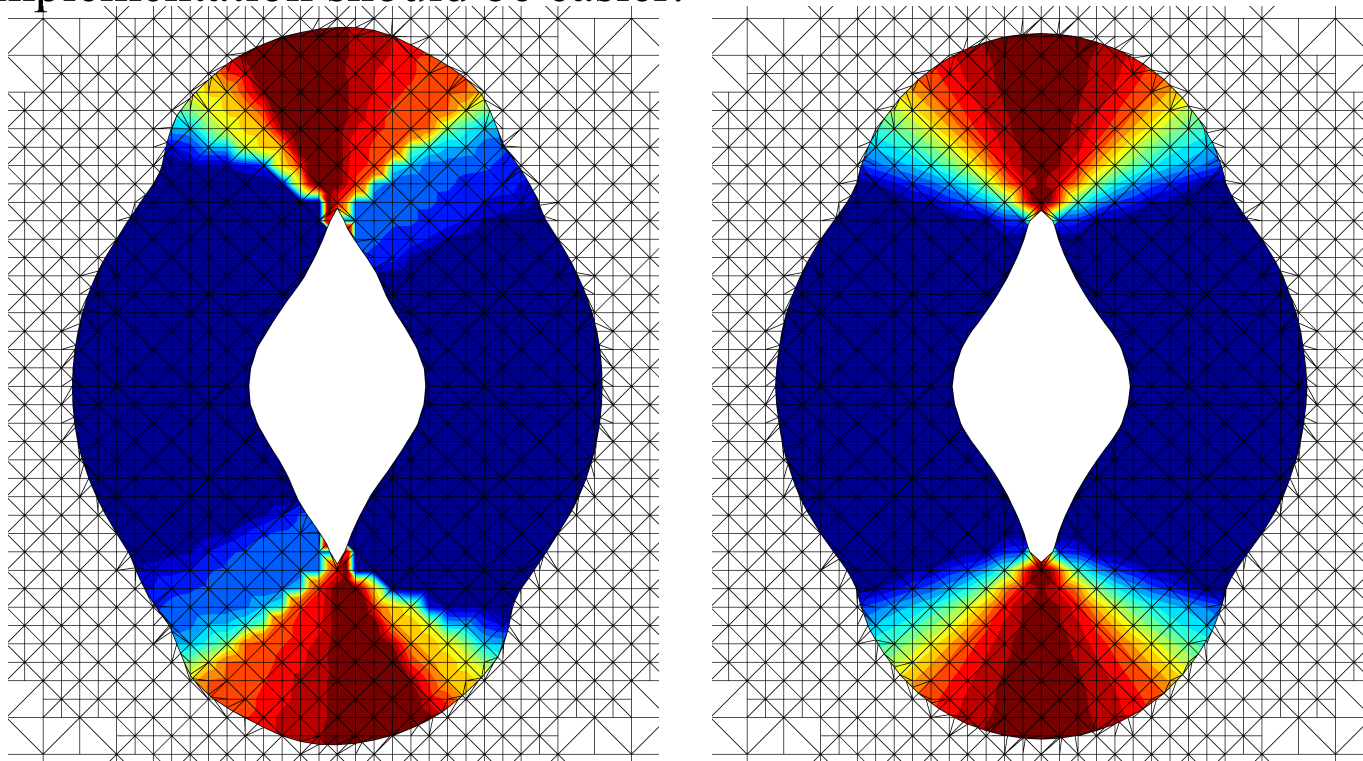
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Both method give similar results.

First method : parametrizing the band can be difficult in 3D.

Second methode : problem to solve for the velocity is bigger, but 3D implementation should be easier.



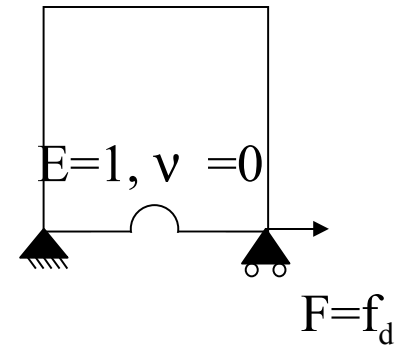
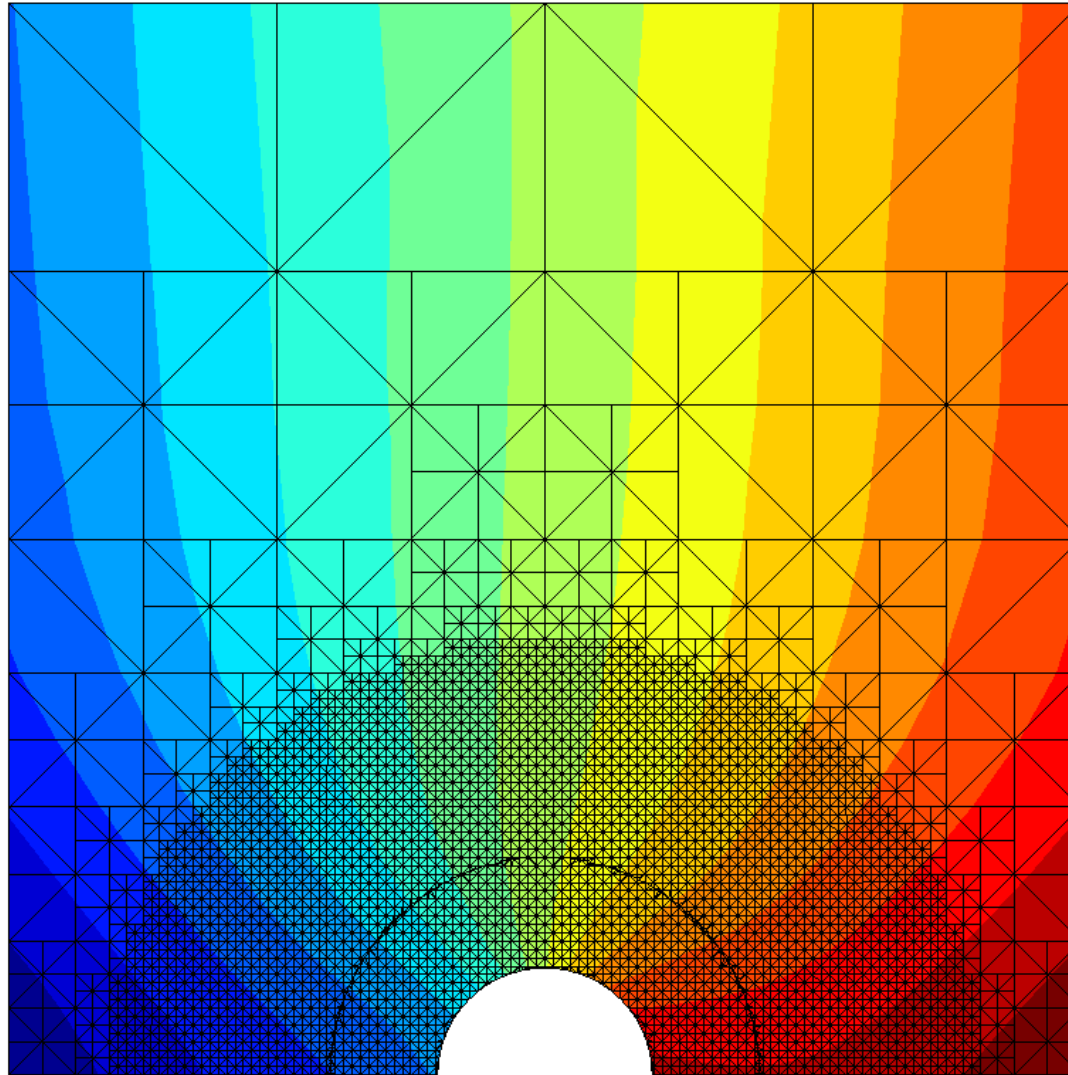
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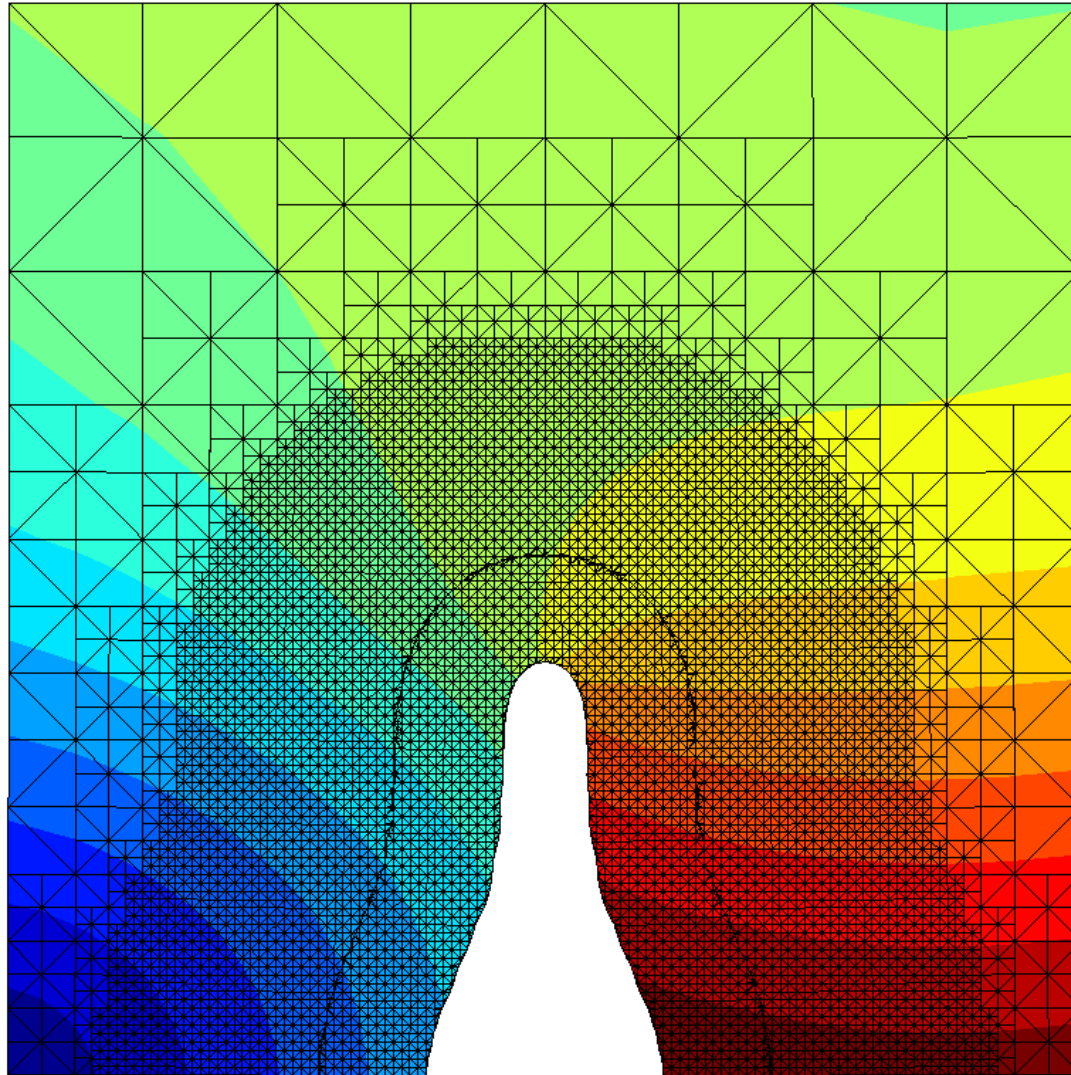
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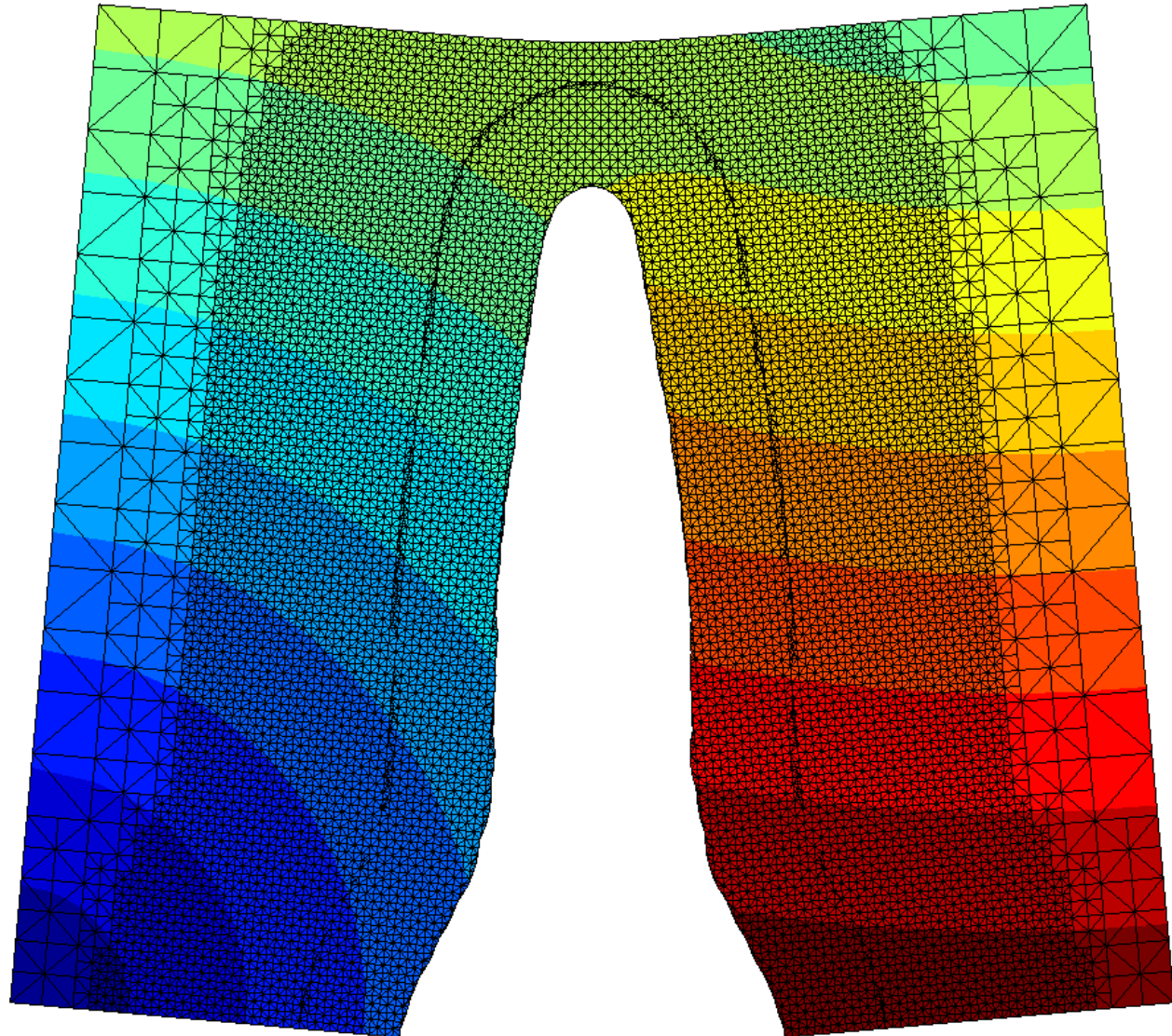
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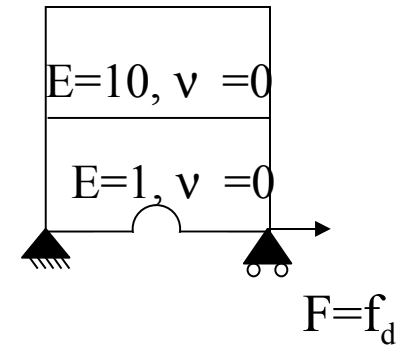
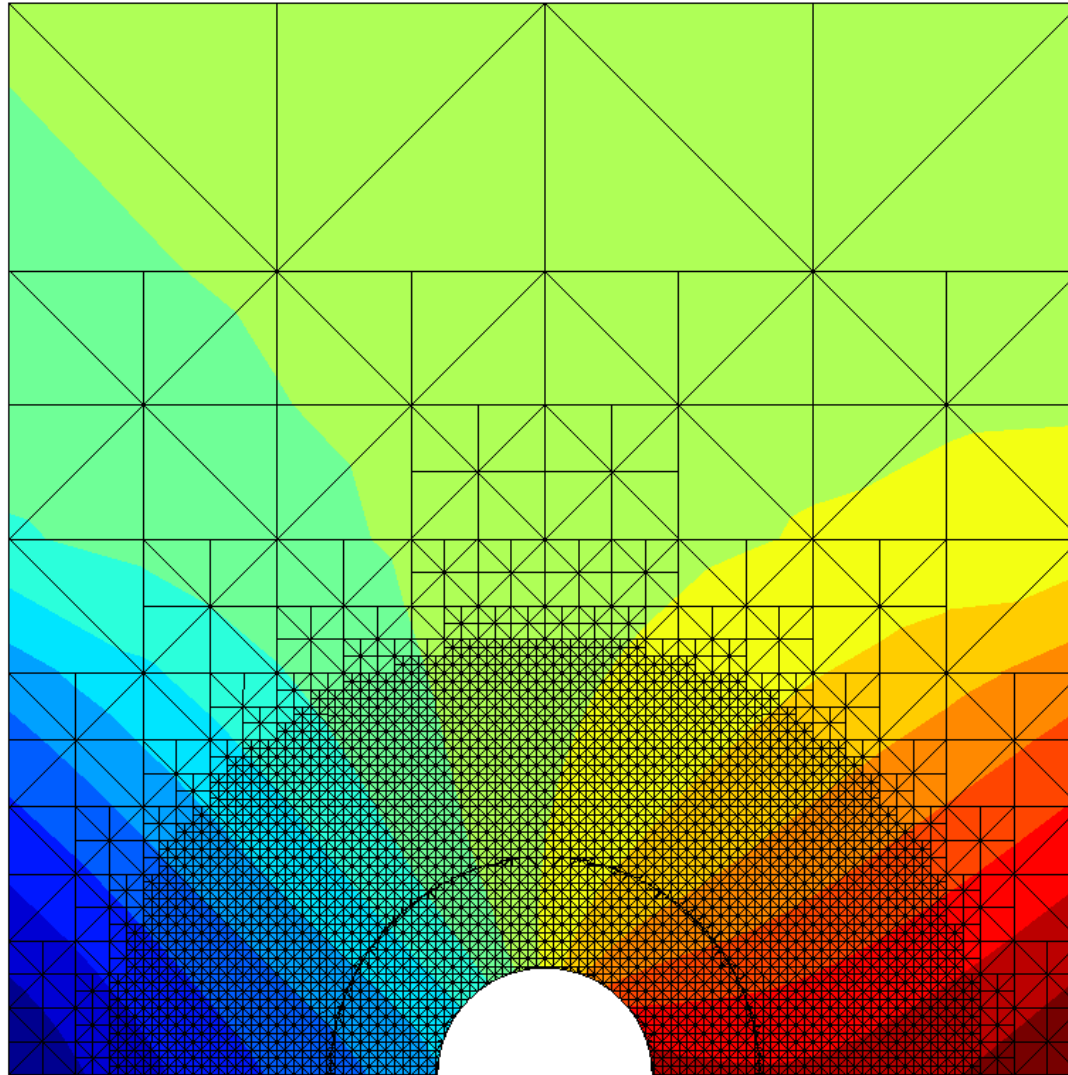
Branching crack.

Level set
Approach to
damage.

I- Introduction

II -SLS
Approach.

III - TLS
Approach.



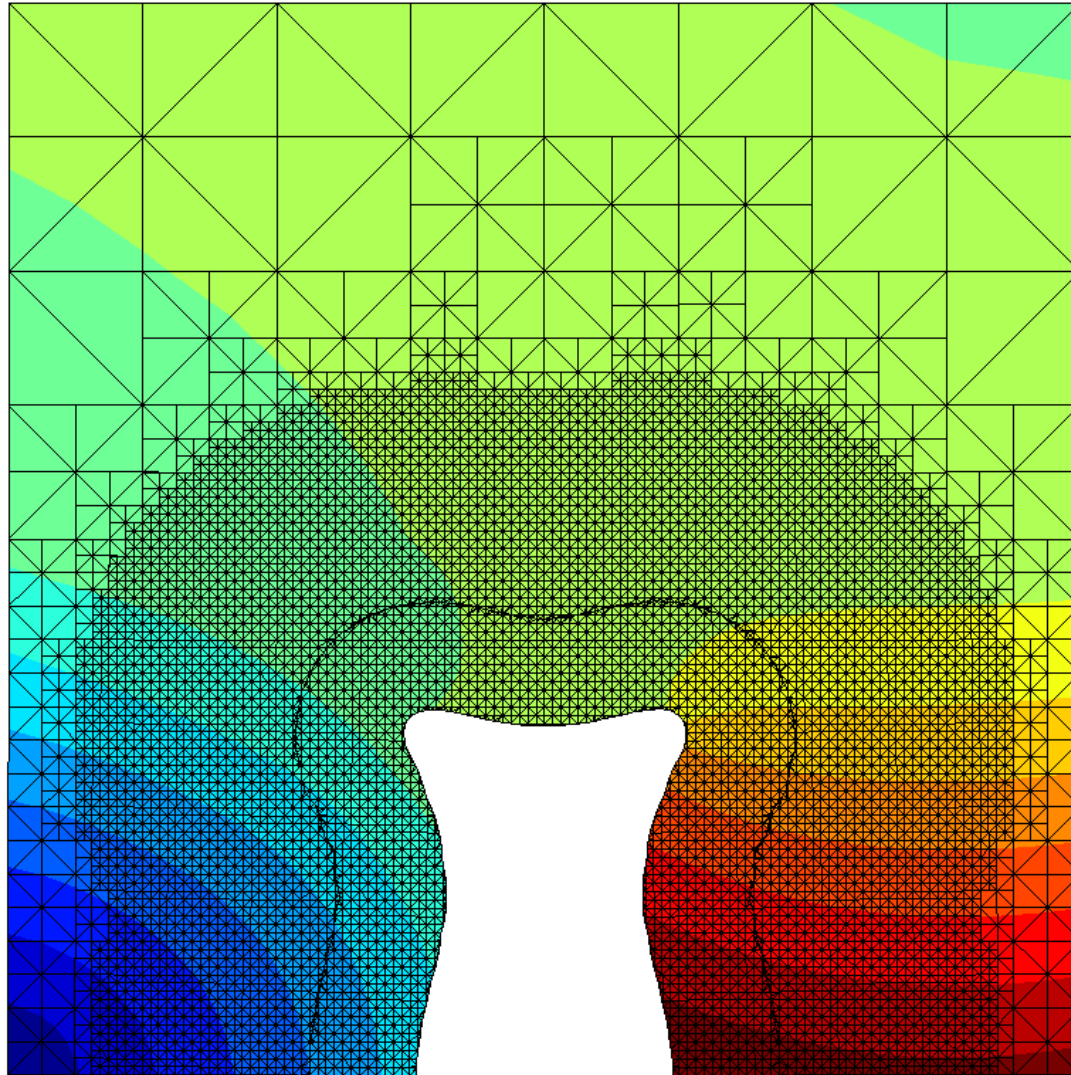
Branching crack.

Level set
Approach to
damage.

I- Introduction

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III – TLS
Approach.



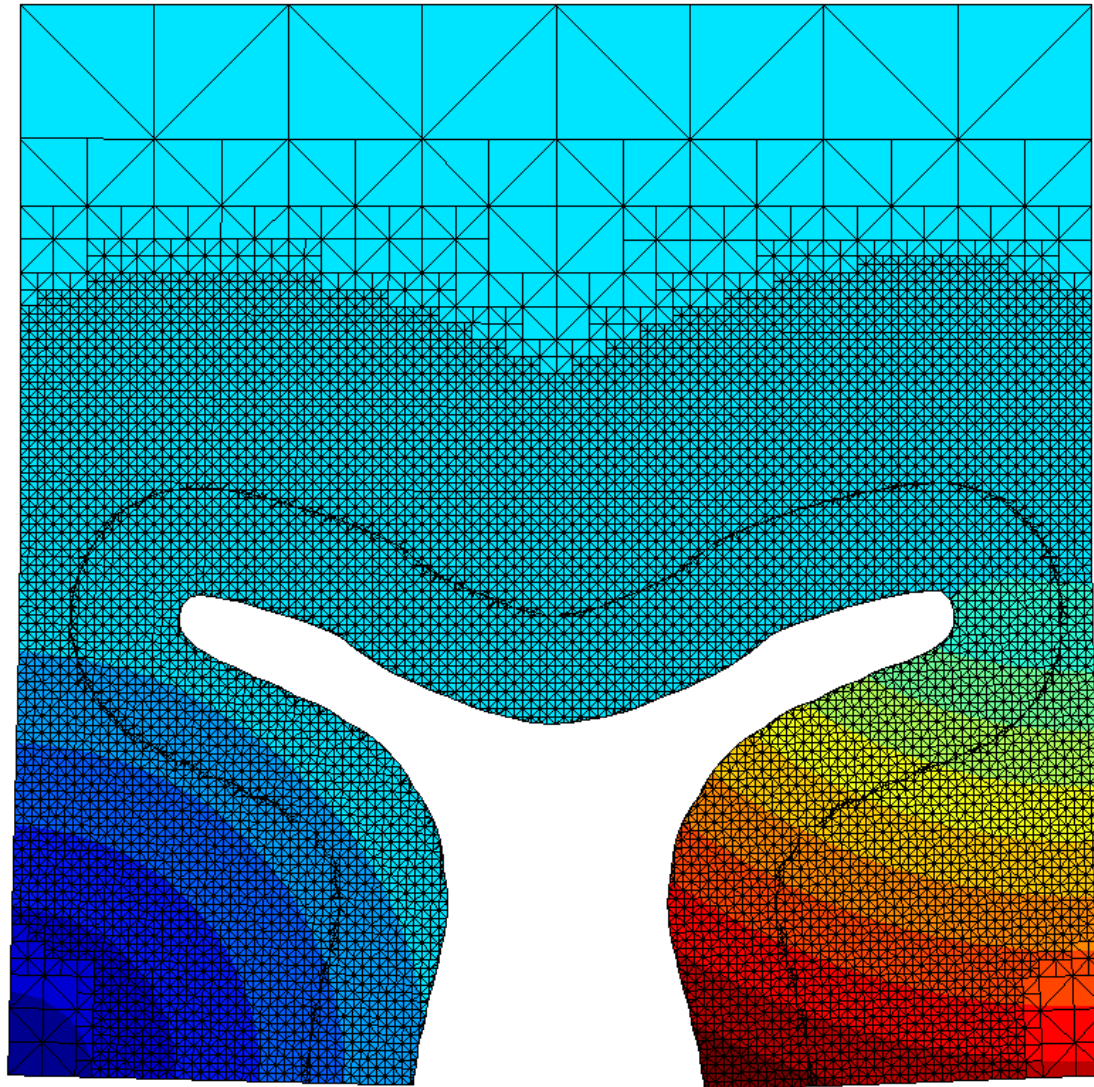
Branching crack.

Level set
Approach to
damage.

I- Introduction

II -SLS
Approach.

III – TLS
Approach.



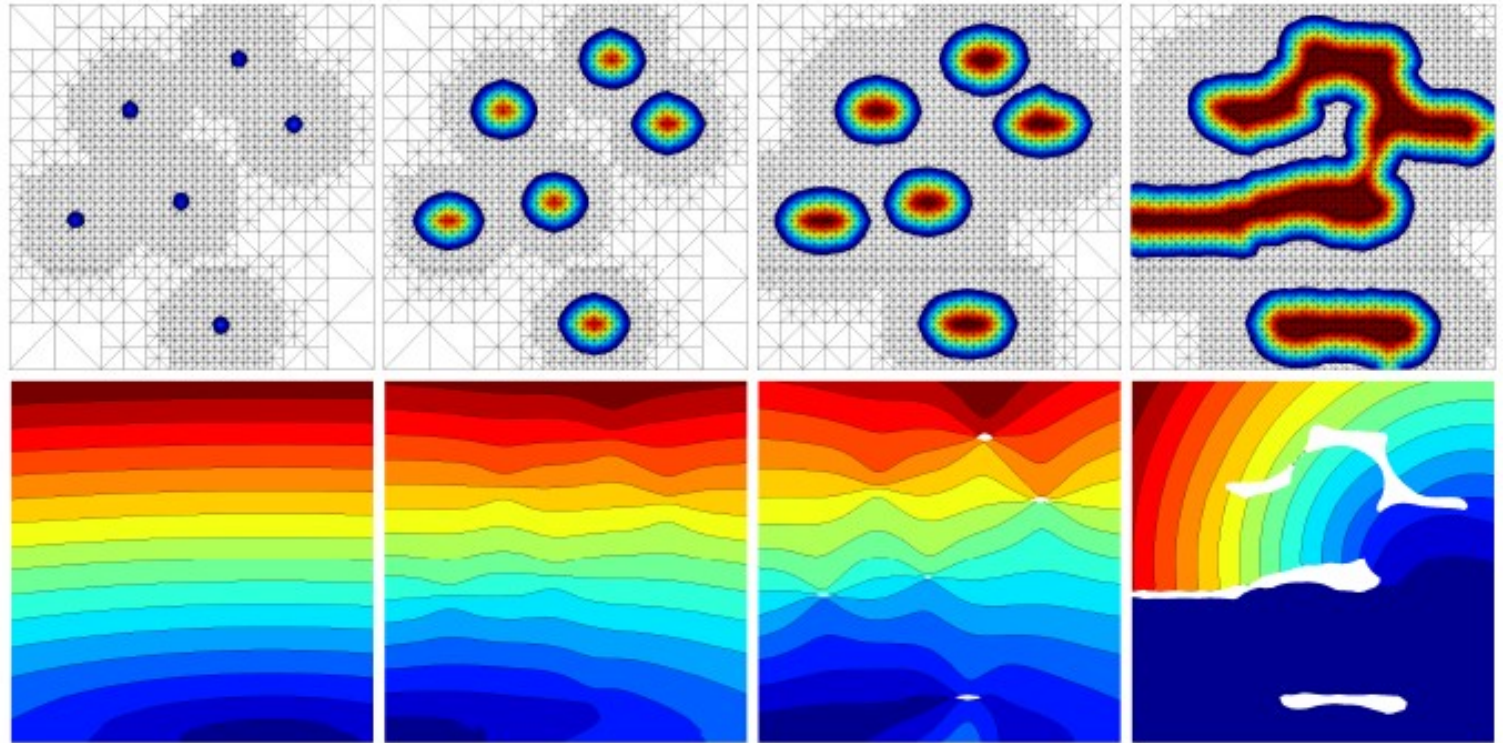
Defect Coalescence.

Level set
Approach to
damage.

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III - TLS
Approach.



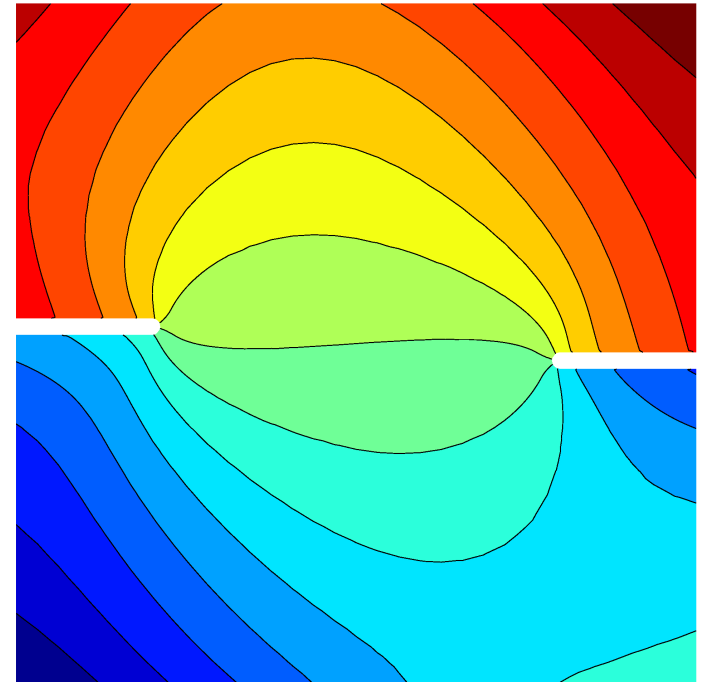
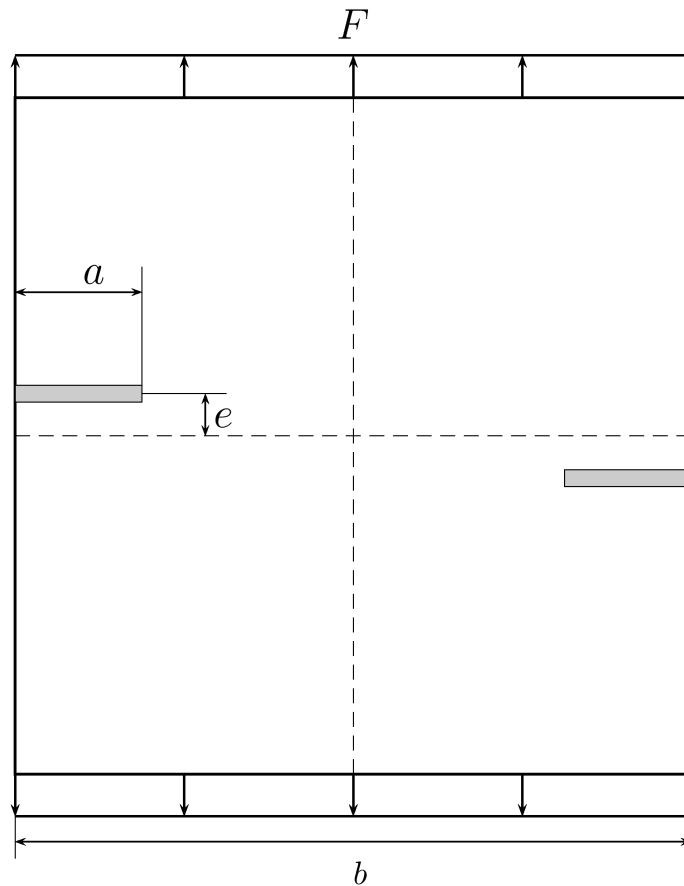
Two cracks.

Level set
Approach to
damage.

I- Introduction

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III - TLS
Approach.



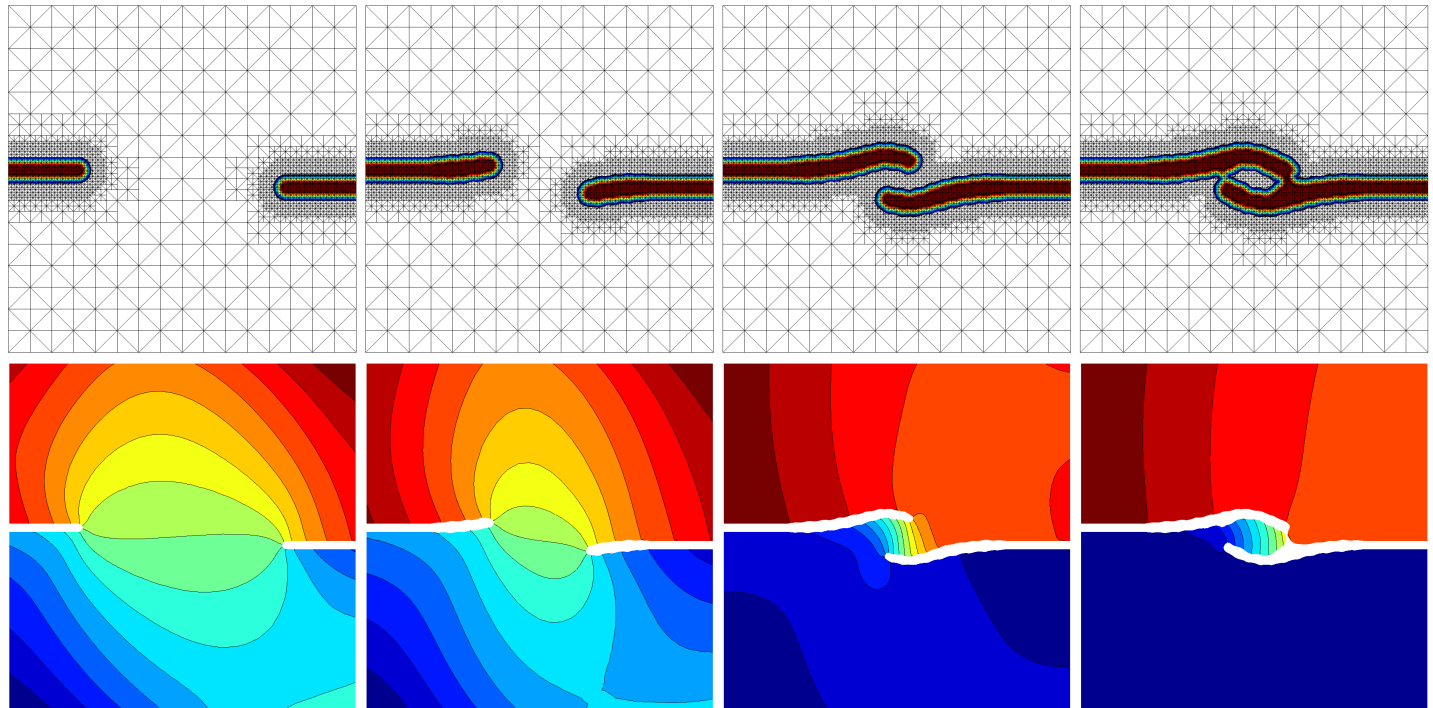
Two cracks.

Level set
Approach to
damage.

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Approach.



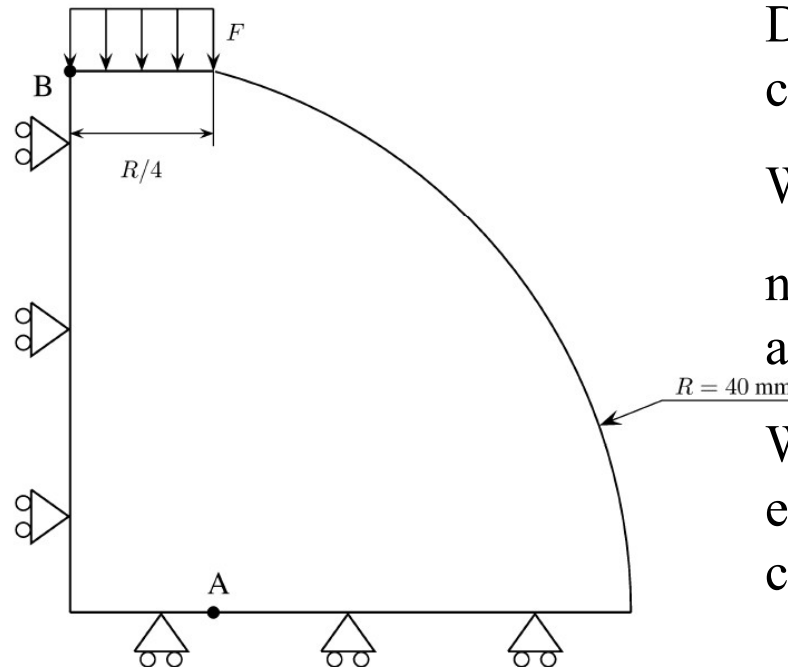
Brazilian test.

Level set
Approach to
damage.

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Approach.



Difficulty : Zones are in
compression.

With the previous damage model :
no difference between compression
and traction.

We need to modify the potential
energy if we want to have the
crack develop vertically.

Non symmetrical behaviour

Level set
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damage.

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Potential energy :

$$\varphi(\epsilon, d) = \mu(1 - d) \langle \epsilon \rangle_+ : \langle \epsilon \rangle_+ + \mu(1 - hd) \langle \epsilon \rangle_- : \langle \epsilon \rangle_- \\ + \frac{\lambda}{2}(1 - d) \langle \text{tr} \epsilon \rangle_+^2 + \frac{\lambda}{2}(1 - hd) \langle \text{tr} \epsilon \rangle_-^2$$

Plane strain case :

$$\varphi(\epsilon, d) = \frac{E}{2(1 + \nu)} \left((1 - \alpha_1 d) \epsilon_1^2 + (1 - \alpha_2 d) \epsilon_2^2 \right) + \\ \frac{\nu E(1 - \alpha d)}{2(1 - 2\nu)(1 + \nu)} (\epsilon_1 + \epsilon_2)^2$$

with :

$$\alpha_j = h \quad \text{if} \quad \epsilon_j < 0 \\ = 1 \quad \text{if} \quad \epsilon_j \geq 0 \\ \alpha = h \quad \text{if} \quad \text{tr}(\epsilon) < 0 \\ = 1 \quad \text{if} \quad \text{tr}(\epsilon) \geq 0$$

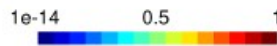
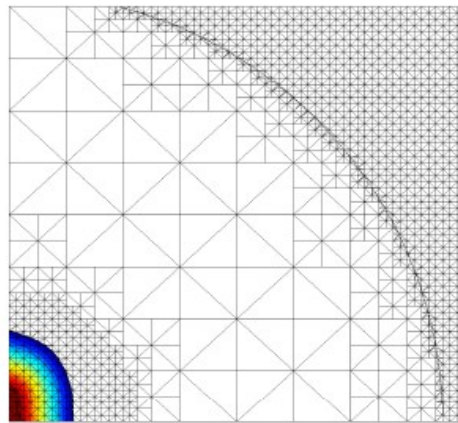
Brazilian test.

Level set
Approach to
damage.

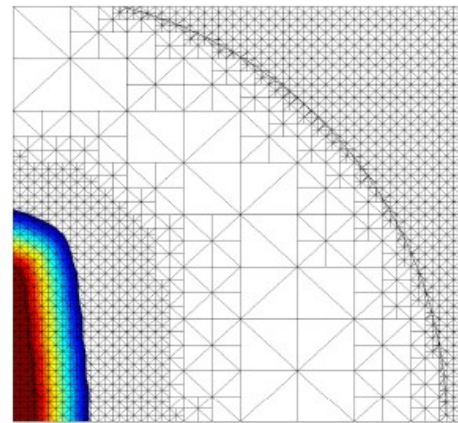
I- Introduction

II -SLS
Approach.

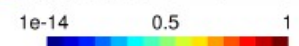
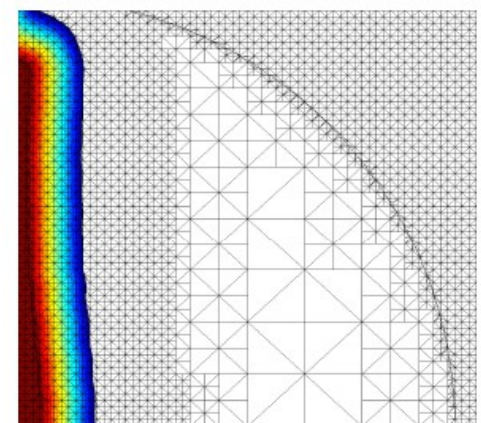
III - TLS
Approach.



(1) $t = 6.6$ s



(2) $t = 9.1$ s



(3) $t = 16.4$ s

Conclusion.

Level set
Approach to
damage.

I- Introduction

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Approach.

- One Level set to represent damage. Allow for topological changes in the crack pattern naturally.
- Crack evolution direction can be handled without a specific criteria. Just compute the speed on the front.
- For the TLS approach we get :
 - Any local damage law in potential form can be included in principle.
 - Natural transition from local damage model to fracture mechanics and non local models.
 - At the very beginning of the damage process, the law is in fact local : the length on which the driving force is computed tends to zero. At the limit, we recover $Y = Y_c$.