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# Dirac Point and Edge States in a Microwave Realization of Tight-Binding Graphene-like Structures

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We present a microwave realization of finite tight-binding graphene-like structures. The structures are realized using discs with a high index of refraction. The discs are placed on a metallic surface while a second surface is adjusted atop the discs, such that the waves coupling the discs in the air are evanescent, leading to the tight-binding behavior. In reflection measurements the Dirac point and a linear increase close to the Dirac point is observed, if the measurement is performed inside the sample. Resonances due to edge states are found close to the Dirac point if the measurements are performed at the zigzag-edge or at the corner in case of a broken benzene ring.

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Due to its electronic shell structure carbon forms structures like diamonds, fullerenes and nanotubes, which all have very specific and fascinating properties, mechanically as electronically. Another realization is graphene, a one-atom-thick allotrope of carbon, which has a structure of the honeycomb lattice, i. e. two combined triangular lattices. Due to the specific symmetry of the lattice the gapless band structure has a conical singularity at two  $k$ -vectors [1], called nowadays the Dirac points. The name comes from the fact that the reduced equations around the Dirac points corresponds to the Dirac equation of a massless particle resulting in a band structure of the form of two cones touching at the tips [2, 3]. Recently it was shown that this single layer can be realized and investigated experimentally [4, 5]. The main ingredient for this relation is the symmetry. Thus a realization via a photonic crystal [6, 7] was proposed [8] and realized experimentally in the microwave regime [9]. In the latter case, the authors measured the transmission through a triangular scattering system using two dimensional slabs and investigating the transport properties in dependence of the incident angle. They found evidence of the Dirac point in the scattering, leading to a pseudodiffusive regime, even though a perfectly ordered, though finite structure had been investigated. Another experimental transport realization has been published recently [10].

In this paper we want to investigate “graphene” in the tight-binding regime [11] with only next nearest neighbor coupling. This is a crucial ingredient for not only Dirac points to exist but also to have a circular cone in the band structure close to them. We realize a microwave experiment using dielectric discs with a high index of refraction in a hexagonal lattice. The discs are weakly coupled by evanescent modes as will be explained later.

So far microwave experiments have been used to show different kind of effects originally coming from condensed

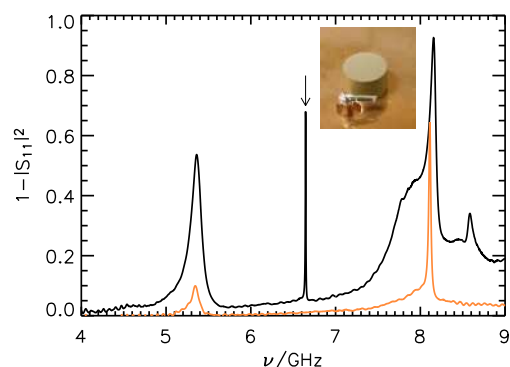


FIG. 1: (color online) Spectrum of a single disc for the TE antenna (dark, black) and the TM antenna (light, yellow). The TE<sub>1</sub> resonance of the disc, which is used for further investigations is marked. The inset shows the TE antenna with a single disc on the metallic copper plate. The top plate is not shown.

matter physics like the Hofstadter butterfly [12], localisation in disordered systems [13, 14], and transport in case of correlated disorder [15].

In the proposed microwave setup is possible to investigate defects, edges, disorder etc. as the realized “graphene” is a macroscopic object. Thus it is possible to study problems experimentally which are not accessible in real graphene at the moment. Additionally it is possible to have a look into the system and study e. g. the density of states or transport properties as well. In this paper we will concentrate on the local density of states (LDOS) and its characteristics at edges or corners.

As a starting point we investigate a single disc, then the coupling of two of them and proceed via a benzene ring to graphene flakes. We measure the reflection  $S_{11}$  of an antenna situated close to a disc of  $r_d=4$  mm radius,  $h_d=5$  mm height and an index of refraction  $n_d \approx 6$ . The antenna is bended (see inset of Fig. 1) to excite both transverse magnetic (TM) and transverse electric (TE) modes, whereas a standard dipole antenna can only ex-

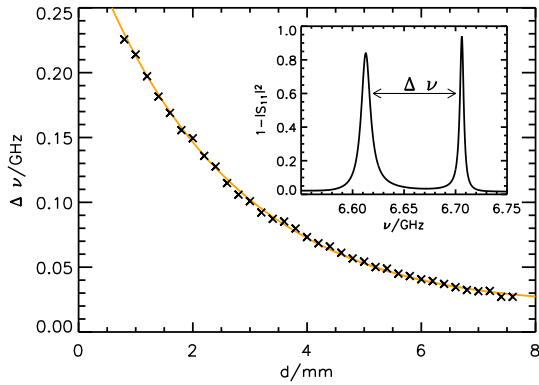


FIG. 2: (color online) Splitting of the eigenfrequencies of two discs as a function of the distance caused by evanescent coupling. The experimental data (crosses) can be described by a Bessel function  $K_0$  (see Eqs (2) and (3)) where  $\kappa_0 \approx 1.82$  GHz and  $\gamma \approx 0.288$  mm $^{-1}$  and a small offset  $\Delta\nu_0 \approx 0.022$  GHz. The inset shows the spectrum and splitting for  $d=3.3$  mm.

cite the TM modes. The disc is placed on a metallic plate (see inset of Fig. 1), a second plate is covering the whole setup at height  $h = 16$  mm. The same discs have been used in Ref. [14] to find localization in disordered microwave structures. In contrast to previous experiments the top plate is not touching the discs but it is 11 mm above the disc. The distance is chosen such that the resonance is still sharp but the eigenfrequency is not sensitive on small deviations of the distance to the top plate. In Fig. 1 the measured spectrum, more precisely  $1 - |S_{11}|^2$ , of a single disc is shown. The dark black curve presents the spectrum measured by the antenna shown in the inset coupling both to the transverse electric (TE) and transverse magnetic (TM) mode (see inset of Fig. 1), whereas the light yellow curve was measured with a pure dipole antenna only coupling to the TM mode. The  $TE_{01}$ -resonance which is used solely for the further investigations is marked. For all measurements we took care that the resonances below and above the investigated frequency range were still separated from the frequency range of interest. The eigenfrequency of the TE resonance decreased with increasing height of the plate, demonstrating that it is a  $TE_{01}$ -resonance of the disc.

Now we investigate the coupling of two discs. Between the discs the  $TE_1$  mode can be described approximately by only a  $z$  component of the magnetic field  $\vec{B} = (0, 0, B_z)$  and perpendicular components of the electric field. Let us assume for the moment that both the bottom and top plate touch the discs from above and below. Then the  $z$  dependance can be separated, yielding for the lowest TE-mode close to an individual disc

$$B_z(x, y, z, k) = B_0 \times \begin{cases} \sin\left(\frac{\pi}{nh}z\right) J_0(k_\perp r) & r < r_d \\ \alpha \sin\left(\frac{\pi}{h}z\right) K_0(\gamma r) & r > r_d \end{cases} \quad (1)$$

where  $k_\perp = \sqrt{k^2 - \left(\frac{\pi}{nh}\right)^2}$ ,  $\gamma = \sqrt{\left(\frac{\pi}{h}\right)^2 - k^2}$ , and  $r$  is the

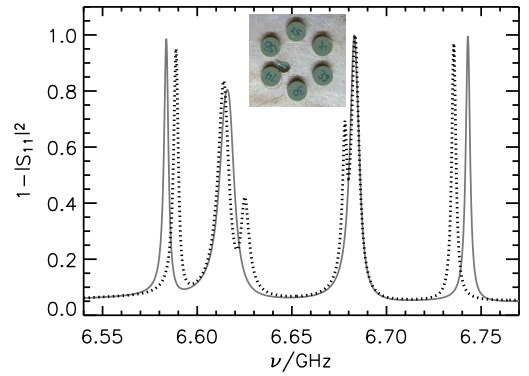


FIG. 3: (color online) Spectrum of a single benzene cell. The dotted line corresponds to a spectrum of a perturbed benzene cell, where the lifting of the degeneracy can be seen.

distance from the centre of the disc.  $J_0$  and  $K_0$  are Bessel functions and  $\alpha$  is a constant to be determined from the continuity equations at the surface. At the resonance frequency of the disc the wave number for  $r > r_d$  is thus purely imaginary leading to an evanescent coupling between the discs. Since in reality there is a gap between the top plate and the discs, this can be correct only approximately. But still we use Eq. (1) with an effective  $\gamma$  to be determined from the experiment. A direct coupling to the  $TM_0$  and  $TM_1$  mode is strongly suppressed due to the continuity conditions of the fields. From Eq. (1) we obtain for the coupling between two discs

$$\kappa(d) = \kappa_0 \left| K_0 \left( \gamma \left[ d + \frac{r_d}{2} \right] \right) \right|^2 \quad (2)$$

where  $d$  is the distance between the discs ( $d=0$  means the discs are touching) and  $\kappa_0$  is a constant. The frequency splitting is given by

$$\Delta\nu(d) = \sqrt{4\kappa(d)^2 + (\Delta\nu_0)^2}, \quad (3)$$

where  $\Delta\nu_0$  is taking into account a small eigenfrequency difference of the discs and the differing coupling of the antenna to the discs, which actually dominates. The resonance splitting is shown in Fig. 2 exhibiting an agreement with the predicted behavior.

As a next test of the tight-binding approximation, we performed a measurement with three discs in a row as a function of the distance and found the coupling constant to be consistent with the two disc measurement. The next nearest neighbor coupling showed up to be about 6% of the nearest neighbor coupling. As graphene is made of coupled benzene cells we investigated the benzene cell (see inset of Fig. 3) next. In Fig. 3 the spectrum (dark solid line) is shown. Benzene has dihedral symmetry  $D_{6h}$  with two singlets and two doublets, where the singlets have the extremal energies. Correspondingly we observe four resonances. The degeneracy of the doublets can be lifted by a perturbation, e.g. by moving one disc (see

dotted line). Again we performed a measurement varying the distance and found a good agreement with the coupling constant obtained by the two disc measurement.

Before we come to the final measurements we have to establish a connection between our measured quantity, the reflection  $S_{11}$ , and the density of states (DOS) or more precisely the local density of states (LDOS). The starting point is the relation between the Green function and the local density of states  $\mathcal{L}(E) = -\frac{1}{\pi}\text{Im}G$  [16]. Here we illustrate a short hand argument the connection between the reflection and the LDOS. The total current through the antenna is given by  $T = 1 - |S_{11}|^2 = \oint ds \mathbf{j}$ , where  $\mathbf{j} = \text{Im}\Psi^*\nabla\Psi$  is the current density. The wavefunction of the field excited by a point-like antenna at  $\mathbf{R}$  is proportional to the Green function,  $\psi = G(\mathbf{R}, E)$  [17],

$$\begin{aligned} \oint ds \mathbf{j} &= \frac{1}{2} \oint ds (\Psi\nabla\Psi^* - \Psi^*\nabla\Psi) \\ &= -|B(\mathbf{R}; E)|^2 \text{Im}G(\mathbf{R}; E). \end{aligned} \quad (4)$$

where the proportionality coefficient  $B(\mathbf{R}; E)$  depends on the details of the coupling. Thus finally we find

$$1 - |S_{11}(E)|^2 = \pi|B(\mathbf{R}; E)|^2 \mathcal{L}(E), \quad (5)$$

where  $\mathcal{L}(E)$  is the LDOS of the unperturbed system. To take into account the perturbation by the measurement antenna, in the correct derivation the Green function  $G$  has to be replaced by a properly renormalized Green function  $\xi_\beta$ , where  $\beta$  corresponds to the scattering length of the antenna [17, 18]. The coefficient  $|B(\mathbf{R}; E)|^2$  is defined by the antenna properties which are slowly varying with energy and thus can be set to  $|B(\mathbf{R}; E_D)|^2$  in the vicinity of the Dirac point  $E_D = k_D^2$ . With  $\mathcal{L}(E) \sim |E - E_D| \sim |k - k_D|$  we expect

$$1 - |S_{11}(E)|^2 \sim |k - k_D|. \quad (6)$$

Now we can proceed to the discussion of our experimental findings for the graphene layers. We generated different graphene flakes and averaged over flakes and antenna positions. By this averaging procedure the LDOS is turned into the DOS. The result is shown in Fig. 4. At the frequency corresponding to the eigenfrequency a single disc (vertical dotted line) a spectral gap is observed which corresponds to the Dirac point in an infinite system. Additionally a symmetric linear increase of the DOS is found close to the Dirac point. The overall decrease of  $1 - |S_{11}|^2$  with frequency has its origin in the fact that the measuring antenna is not placed at a disc center, but somewhere in between the discs, leading to a frequency dependent decrease of  $B$  at the antenna due to Eq. (1).

Now let's have a look what happens if we move the antenna to the outside, e.g. to the zigzag edge. This measurement is presented in Fig. 5. A state close to the Dirac point is observed, which cannot be seen in Fig. 4, even though it is the same system. As the measured

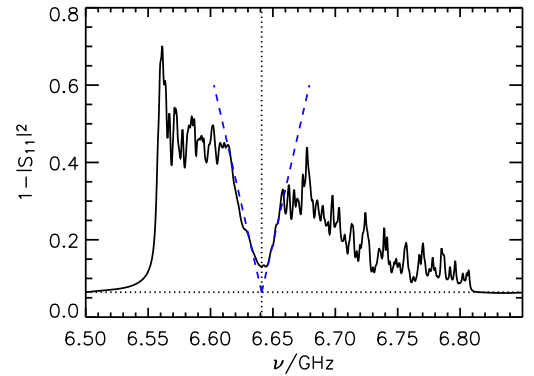


FIG. 4: Spectrum of a graphene flake, where the spectrum was averaged over three different flake forms and different interior antenna positions. The dashed vertical line marks the resonance position of a single disc corresponding approximately to the Dirac point. The dotted horizontal line corresponds to the background. The dashed lines give a guide to the eye to see the linear increase of the DOS.

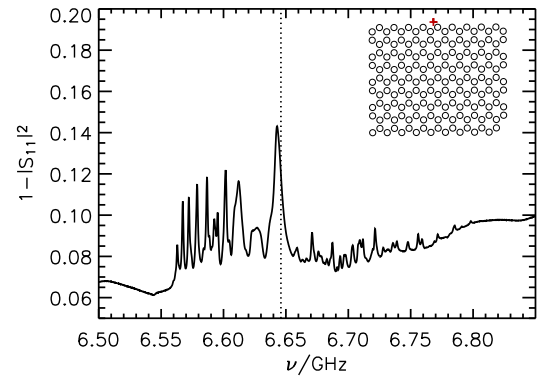


FIG. 5: Spectrum of a graphene flake, where the antenna is placed at the zigzag boundary. The inset shows the actual lattice and the cross marks the position of the measuring antenna. The dashed vertical line is the resonance position of a single disc.

quantity is related to the LDOS and not the DOS it is possible to test the states locally and not only globally. We also moved the antenna to the armchair edge and did not observe a state close to the Dirac point. This is in accordance with the findings for a zigzag-edged infinitely long graphene ribbons where the effect of edge states was discussed [3, 19], and where the localization of the edge states was estimated to about one to two atomic layers. If we move the antenna to the lower left corner where there is an isolated disc not contained in a full benzene ring, again a single state is observed at the Dirac point. By removing the disc at the corner the state disappears.

To interpret our experimental findings we additionally performed numerical calculations by determining eigenvalues and eigenvectors of the Hamiltonian

$$H = E_D \cdot 1 + \kappa C, \quad (7)$$

where  $E_D$  is the gap energy,  $\kappa$  the coupling constant, and  $C$  the connectivity matrix with elements  $C_{nm} = 1$

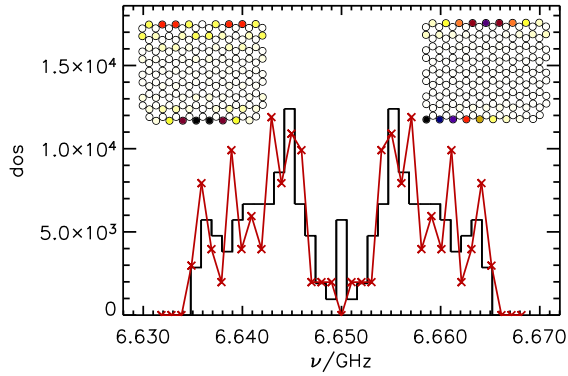


FIG. 6: (color online) DOS from a numerical calculation of the systems shown in the top right corner (histogram). Additionally the right inset shows an grey scale indication of an edge state found close to the Dirac frequency. In the left inset an typical eigenfunction of an edge state is presented, where the disc in the lower left corner was removed. The crosses correspond to a system with periodic boundary conditions taking 178 discs into account.

or 0 for nearest and non-nearest neighbors, respectively. The resulting DOS is shown in Fig. 6. The histogram corresponds to the experiment findings presented in Fig. 5, where the edge states are strongly observed but internal states are suppressed. One observes a peak of the DOS at the Dirac point. Analyzing the wavefunctions at this energy we found only edge states similar to the one presented in the left inset of Fig. 6 as well as a single corner state observed if an individual disc not part of a full benzene cell is added (see right inset). By moving the measurement antenna towards the center of the system, corner or edge states cannot be seen as their probability inside is very small and the measured quantity is related to the LDOS. In addition we performed the calculation with periodic boundary conditions. Now the edge states have disappeared, and the triangular gap at the Dirac point is clearly seen (crosses in Fig. 6). This is similar to the situation of an internal antenna, i. e. it corresponds to Fig. 4, where the Dirac point is found as well. In the numerics with periodic boundary conditions edge states can not exist, whereas in the experiment the existing edge states are not observable, due to their small amplitude inside the flake. Finally by removing the corner disc leaving the lattice only with complete benzene cells the corner state did vanished, again in accordance with the experimental findings.

In this paper we presented the first microwave realization of graphene in the tight-binding approximation. Using high index of refraction discs inside two metallic plates, where the discs resonances are only coupled evanescently, a tight-binding realization became possible. The Dirac point and a linear increase of the DOS close to it has been found as well as edge and corner states. This opens the possibility to investigate further properties of graphene-like transport, dependance on disorder,

defects, graphene quantum dots [20] or even to realize a Dirac oscillator [21].

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- [1] P. R. Wallace, *Phys. Rev.* **71**, 622 (1947).
- [2] K. S. Novoselov, A. K. Geim, S. V. Morozov, D. Jiang, M. I. Katsnelson, I. V. Grigorieva, S. V. Dubonos, and A. A. Firsov, *Nature* **438**, 197 (2005).
- [3] A. H. C. Neto, F. Guinea, N. M. R. Peres, K. S. Novoselov, and A. K. Geim, *Rev. Mod. Phys.* **81**, 109 (2009).
- [4] K. S. Novoselov, A. K. Geim, S. V. Morozov, D. Jiang, Y. Zhang, S. V. Dubonos, I. V. Grigorieva, and A. A. Firsov, *Science* **306**, 666 (2004).
- [5] A. K. Geim and K. S. Novoselov, *Nature Materials* **6**, 183 (2007).
- [6] C. M. Soukoulis, ed., *Photonic Band Gaps and Localization*, NATO ASI Series B: Physics, Vol. 308 (Plenum Press, New York, 1993).
- [7] J. M. Lourtioz, H. Benisty, V. Berger, J. M. Gerard, D. Maestre, and A. Tchelnokov, *Photonic Crystals - Towards Nanoscale Photonic Devices* (Springer, Berlin, 2005).
- [8] R. A. Sepkhanov, J. Nilsson, and C. W. J. Beenakker, *Phys. Rev. B* **78**, 045122 (2008).
- [9] S. R. Zandbergen and M. J. A. de Dood, *Phys. Rev. Lett.* **104**, 043903 (2010).
- [10] S. Bittner, B. Dietz, M. Miski-Oglu, P. O. Iriarte, A. Richter, and F. Schaefer, Preprint (2010), arXiv:1005.4506.
- [11] A. Matulis and F. M. Peeters, *Am. J. Phys.* **77**, 595 (2009).
- [12] U. Kuhl and H.-J. Stöckmann, *Phys. Rev. Lett.* **80**, 3232 (1998).
- [13] A. A. Chabanov, M. Stoytchev, and A. Z. Genack, *Nature* **404**, 850 (2000).
- [14] D. Laurent, O. Legrand, P. Sebbah, C. Vanneste, and F. Mortessagne, *Phys. Rev. Lett.* **99**, 253902 (2007).
- [15] U. Kuhl, F. M. Izrailev, and A. A. Krokhn, *Phys. Rev. Lett.* **100**, 126402 (2008).
- [16] E. Akkermans and G. Montambaux, eds., *Mesoscopic Physics of electrons and photons* (Cambridge University Press, Cambridge, 2006).
- [17] T. Tudorovskiy, U. Kuhl, and H.-J. Stöckmann, Preprint (2009), arXiv:0910.3079.
- [18] T. Tudorovskiy, R. Höhmann, U. Kuhl, and H.-J. Stöckmann, *J. Phys. A* **41**, 275101 (2008).
- [19] F. Muñoz-Rojas, D. Jacob, J. Fernández-Rossier, and J. J. Palacios, *Phys. Rev. B* **74**, 195417 (2006).
- [20] F. Libisch, C. Stampfer, and J. Burgdörfer, *Phys. Rev. B* **79**, 115423 (2009).
- [21] E. Sadurní, T. Seligman, and F. Mortessagne, *New J. of Physics* **12**, 053014 (2010).