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STATIONS

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# **A Monocentric City With Discrete Transit Stations**

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## **ABSTRACT**

We extend the monocentric model by considering a discrete number of accessible mass transit stations. Households combine two modes for their daily home-to-work trip: a first mode for terminal access to stations and a second (long haul) mode which consists in radial mass transit axes. The urban equilibrium, i.e. city size and households' distribution, is derived as a function of the mass transit network and the distribution of land housing capacity. Then at the urban equilibrium the land rent is peaked at transit stations and decreases with the travel cost from the city center rather than with the distance to it. Accordingly, the housing lot size increases with the travel cost from the city center. These features distinguish our framework from previous monocentric models. Our analysis is based on the assumptions that land-owners are absent and city is open (the households' level of utility is given and the population size is endogenous). For numerical illustration, the model is calibrated to a selected rail network in the Paris area. A sensitivity analysis of the urban structure and land-use equilibrium is conducted with respect to the key model parameters.

# 1 Introduction

The monocentric model is a centerpiece in urban economics because it explains for the disaggregate location of individual households in an urban area, in relation to a single employment place where they get income and to which they have to travel. Still, the basic formulation of the model has received considerable criticism for its lack of realism [1]. In most research works, the transport network is considered in an abstract way on the basis of a travel cost function. When it is considered explicitly as a set of radial axes that extend from a given location to the city center, it is assumed that the axes are accessible from everywhere [2, 3], which amounts to assume that there is a continuum of stations available over the axis. This paper addresses the issue of access stations by imposing a limited number of access points to mass transit.

The explicit description of transit stations is important to make the monocentric model more relevant to empirical studies, particularly so to those which involve mass transit. Indeed, the monocentric model has been used mainly for theoretical purposes. [4, 5] are among the few papers that have used the model in an empirical setting.

This paper assumes that mass transit is available along a finite number of radial axes that connect to the city center, through a limited number of stations located along these axes. Every household makes its daily trips to the city center in two legs: a first leg by private transportation from the house location to a station, and a second leg by transit from that station to the city center.

This extended model has a twofold objective. First, as an extension to the basic model, it provides insight in the role of the access stations and their effects on the urban structure. Second, the model can be applied to (stylised) empirical studies: hereafter the urban equilibrium is simulated in a stylised case which mimics the Greater Paris Area.

By comparison to the basic monocentric model, we obtain a city shape characterized by irregularities in their borders. Indeed, the urban area is stretched around access points (stations) and axes of mass transit. Land rent and household density are no longer monotonic with respect to distance from city center as in the basic model, but rather peaked at transit stations.

The paper is organized as follows. The next section introduces the model and the notation. The solution of the model and the features of urban equilibrium are dealt with in section 3, including a sensitivity analysis with respect to key parameters. The case of the stylised Paris area is addressed in section 4. Finally, section 5 concludes.

## 2 The model

Each household makes a daily home-to-work trip. Every trip includes two stages: the household first uses a private transportation (walk, car, bike,) to reach a mass transit station, from where he then transits by train to reach the city center where all jobs are located.

Public transport lines consist of radial axes between the city center and the peripheral areas. Each household uses public transport on the axis close to its housing location. The specific way to reach a transit axis will be discussed below. A household's location is specified by polar coordinates  $(r, \theta)$ , in which  $r$  is the distance along a straight line from the city center to the housing location, and  $\theta$  indicates the angle between the closest transit axis and that line.

## 2.1 Households as microeconomic consumers

Every household is modelled as a consumer of both a housing service at a given location and travel between that location and the city center. The household chooses the quantities of a composite good,  $z$ , and of housing area,  $s$ , so as to derive a utility which it wants to maximize under his budget constraint that involves the composite good's cost,  $1 \cdot z$  at unit price of 1, plus the housing cost,  $R \cdot s$  at unit price  $R(r, \theta)$  that is the land rent, plus travel cost  $T(r, \theta)$ , the sum of which is faced to the household income,  $Y$ . Assuming homogeneous households that have same income and same utility function  $u(z, s)$  that is specified as a Cobb-Douglas function with parameters  $\alpha, \beta > 0$  such that  $\alpha + \beta = 1$ , the household's microeconomic behaviour amounts to the following mathematical program:

$$\begin{cases} \max_{z, s, r, \theta} u = \alpha \ln(z) + \beta \ln(s) \\ \text{subject to } z + R(r, \theta)s + T(r, \theta) \leq Y \end{cases} \quad (1)$$

The price that the household is willing to pay at location  $(r, \theta)$ , known as the bid rent function, is given by [6]

$$\Psi(r, \theta, u) = \alpha^{\alpha/\beta} \beta (Y - T(r, \theta))^{1/\beta} e^{-u/\beta}, \quad (2)$$

and the bid-max lot size

$$S(r, \theta, u) = \alpha^{-\alpha/\beta} (Y - T(r, \theta, u))^{-\alpha/\beta} e^{u/\beta}. \quad (3)$$

At equilibrium the land rent is equal to the bid rent given in (2). This expression shows that the land rent decreases as the transport cost increases. The term  $Y - T(r, \theta)$  reflects the part of the income which is available to spend on housing and the composite good. Notice that a higher utility can be reached if land rent decreases (when  $T(r, \theta)$  is kept unchanged).

## 2.2 Transport technologies

We consider two technologies of travel from the housing location to the mass transit station. The first possibility is through a circular path of radius  $r$  to the public transport axis and then directly along that axis to the center, as in [2] but distinguishing between two radial legs, one leg by private mode from projected point to station and the other by transit from station to center. The second possibility is a straight line between the origin and destination station.

Transport cost under technology  $i$  ( $i=1,2$ ) is denoted by  $T_i(r, \theta)$ . The unitary travel costs are  $c_v$  and  $c_t$  respectively for private and public transport. There are  $n_a$  public transport axes which are symmetric and separated by angle  $\theta_a = 2\pi/n_a$  from their neighbours. Each axis has a fixed number  $n_s$  of access stations # $i$  numbered in increasing order of distance  $s_i$  from the city center. Finally the typical dwell time of a train in a station makes an additional delay, assumed fixed to  $g$  in cost units, to the travellers who are already on board.

For technology 1, assume that

$$g \leq (s_{i+1} - s_i)(c_v - c_t). \quad (4)$$

Condition (4) ensures that station  $s_{i+1}$  remains attractive at least for the household living close to it. If the condition is not satisfied this station will not be used by any traveller. Under technology

1, transport cost at  $(r, \theta)$ , with  $s_i \leq r \leq s_{i+1}$ , is given by

$$T_1(r, \theta) = \begin{cases} c_v[r\theta + (r - s_i)] + s_i c_t + (i-1)g & \text{if } r \leq r_{1,i}^m \\ c_v[r\theta + (s_{i+1} - r)] + s_{i+1} c_t + i g & \text{if } r > r_{1,i}^m \end{cases}, \quad (5)$$

where  $r_{1,i}^m$  is the location of the user who is indifferent to transit by station  $s_i$  or station  $s_{i+1}$ :

$$r_{1,i}^m = \frac{s_i + s_{i+1}}{2} + \frac{g + (s_{i+1} - s_i)c_t}{2c_v}. \quad (6)$$

For technology 2, assume that  $g$  is sufficiently small (in a sense to make clear below).

Define  $x_i = \sqrt{r^2 + s_i^2 - 2r s_i \cos(\theta)}$ , and denote by  $r_{2,i}^m$  the solution of

$$c_v x_i = c_v x_{i+1} + c_t (s_{i+1} - s_i) + g \quad (7)$$

in  $r$ . Transport cost at  $(r, \theta)$ , with  $s_i \leq r \leq s_{i+1}$ , is given by

$$T_2(r, \theta) = \begin{cases} c_v \sqrt{r^2 + s_i^2 - 2r s_i \cos(\theta)} + s_i c_t + (i-1)g & \text{if } r \leq r_{2,i}^m \\ c_v \sqrt{r^2 + s_{i+1}^2 - 2r s_{i+1} \cos(\theta)} + s_{i+1} c_t + i.g & \text{if } r > r_{2,i}^m \end{cases} \quad (8)$$

The inconvenience with technology 2 is that Eq. (7) does not have an explicit solution in  $r$ .

## 2.3 Housing supply

The third and last subset of assumptions in the model pertains to housing supply. Here the landowners are assumed absent from the city. The supply of housing is characterised in terms of housing surface available at any given location  $(r, \theta)$ , measured as a spatial density  $L(r, \theta)$  hence as a plain number of no physical dimension. Thus  $L(r, \theta).r dr d\theta$  is the amount of surface available to households on the piece of land  $r dr d\theta$  around point  $(r, \theta)$ . Denoting by  $n(r, \theta)$  the spatial density of households at that point (in units of #/m<sup>2</sup>), then  $n(r, \theta)S(r, \theta) = L(r, \theta)$  where  $S(r, \theta)$  is the lot size.

On assuming a radial symmetry i.e.  $L(r, \theta) = \tilde{L}(r)/(2\pi r)$  then  $\tilde{L}(r)$  corresponds to the usual radial density of housing area available at distance  $r$  from the city center in the basic model. In the case of a constant density  $L(r, \theta) = \ell$ , then at equilibrium  $n(r, \theta) = \ell/S(r, \theta, u)$  which decreases as we move away from the city center but also from the stations of mass-transit.

In the Application section it is assumed that the central part of the city is devoted to firms and employment, i.e.  $L(r, \theta) = 0$  at small  $r$ .

## 3 Urban equilibrium

### 3.1 Solution scheme

To simplify, let us assume that the utility level,  $u$ , is known at equilibrium: this corresponds to the ‘‘open city’’ assumption where households migrate between cities to maximize their utility, and may easily be relaxed by solving for  $u$  on assuming a given number of

households.

Notice that at the city boundary, land rent is equal to the agricultural rent, denoted  $R_A$ , i.e.

$$\Psi(Y - T(r_f, \theta_f), u) = R_A. \quad (9)$$

Substituting (5) into (2) and solving for  $r_f$  and  $\theta_f$ , we obtain that

$$c_v(1 + \theta_f)r_f = s_n(c_v - c_t) + Y - e^u R_A^\beta \alpha^{-\alpha} \beta^{-\beta} - (n-1)g. \quad (10)$$

Using (3), the corresponding housing area is derived as

$$S(r, \theta, u) = a^{-\alpha/\beta} (Y - T(r, \theta))^{-\alpha/\beta} e^{u/\beta}. \quad (11)$$

The model can be solved along the following three steps:

1. Specify parameter values.
2. Find the market area for each station:
  - (a) For  $\theta = 0$ , use (7) to find the limits of the station market area along the transport axis. Denote the interval as  $(r_{\min}, r_{\max})$ .
  - (b) For each  $r \in (r_{\min}, r_{\max})$  find the corresponding  $\theta$  from (10), truncated at the median angle  $\theta_a / 2$  that separates the axis from its closest neighbour.
3. Compute aggregates (as shown below). At this stage integration over a constant domain is required.

The second stage in this procedure is the most sensitive one. For each station there are three possibilities: either it does not compete with any other station, or it competes with stations along the same mass transit axis, and/or it competes with stations along an adjacent axis. From each household location there are two possibilities to get to the transit station: either it is located further away from the center and thus it moves in the same direction when it uses successively the private and mass transit modes, or it uses the two modes in opposite directions. All these subtleties must be addressed in the solution process.

### 3.2 Sensitivity analysis

The discretization of the monocentric model introduces new variables. Let us focus on the impact of the discomfort parameter,  $g$ , borne by the travelers boarded in the train at each station. About the city size, notice that an increase in  $g$  increases the transport cost at each location and in particular at the edge of the city:  $(r_f, \theta_f)$ . Since the transport cost  $T(r, \theta)$  is monotonely increasing in both arguments, we have  $dr_f/dg < 0$  and  $d\theta_f/dg < 0$ . For example, with technology 1, differentiate (10) with respect to  $g$  to get

$$c_v r_f d\theta_f + c_v(1 + \theta_f) dr_f = -(n-1)dg \quad (12)$$

So, under a higher stop discomfort the city becomes smaller, i.e.  $dr_f/dg < 0$  and  $d\theta_f/dg < 0$ .

The same increase in  $g$  decreases the land rent on the basis of Eq. (2) in which the transport cost is increased. Eq. (3) then implies that the lot size increases with the transport cost. So, as  $g$  increases the housing area increases. The intuition for this result is that since land rent decreases a household gets a larger housing area.

If the number of stations was determined endogenously to maximize the total households'

welfare in the city, then an increase in  $g$  would induce a lower optimal number of stations, and thus smaller average commuting distance by train.

The two other travel cost parameters, i.e.  $c_v, c_t$ , play a role similar to  $g$ . The only difference is that the impact of  $g$  depends on the number of stations between the household location and the CBD while the other travel cost parameters apply as coefficients on the distance.

The sensitivity analysis for the remaining parameters is the same as in the basic monocentric model. Since both the housing and composite goods are normal, an increase in the income increases the size of the city. The agricultural rent  $R_A$  reflects the opportunity cost of land, and an increase in its value reduces the size (and the population) in the city. In this case, the housing surface remains unchanged in the urban region (notice that (11) does not depend on  $R_A$ ). This would change with a closed city, since with the same population and a reduced lot size, the housing area would have to decrease. Concerning the utility level, notice that (2) and (9) yield  $\alpha^\alpha \beta^\beta (Y - T(r_f, \theta_f)) = e^u R_A^\beta$ . So, an increase in the utility level requires a decrease in  $T(r_f, \theta_f)$  (along the boundary condition (9)), and thus leads to a smaller city.

### 3.3 Urban aggregates

Given the housing supply, the transport network and the population of households, an urban equilibrium yields the exact shape of the city. This section is focused on the impact of the mass transit network on the main features of the city that pertain to, respectively: total population, total travel cost and total differential rent.

Here the main difficulty is to set the integration boundaries correctly. Assuming there are  $n_a$  symmetric axes and  $n_s$  stations along each axis, let  $\Omega(i, n_s, n_a)$  denote the market area (i.e. the domain of catchment) of station  $\#i$  located at  $s_i$  along a given axis. The area  $\Omega(i, n_s, n_a)$  can be determined using Eq. (6) and taking into account the symmetry between the transit axes.

The total population,  $\bar{N}$ , is computed as

$$\bar{N} = n_a \sum_{i=1}^{n_s} \int_{\Omega(i, n_s, n_a)} n(r, \theta) r d\theta dr . \quad (13)$$

A more explicit expression for (13) may not exist even with the simple Cobb-Douglas utility function. It remains possible, however, to qualitatively describe  $\bar{N}$  as a function of the key parameters of the model.

As the city size increases and households locate further away from the city center, travel costs increase (and should have a negative impact on the total production in the city). The total travel cost in the city, denoted  $TT$ , can be computed similarly to (13):

$$TT = n_a \sum_{i=1}^{n_s} \int_{\Omega(i, n_s, n_a)} T_1(r, \theta) n(r, \theta) r d\theta dr . \quad (14)$$

About the differential rent  $R - R_A$ , its total value associated to the whole city amounts to:

$$TDR = n_a \sum_{i=1}^{n_s} \int_{\Omega(i, n_s, n_a)} (R(r, \theta) - R_A) S(r, \theta) n(r, \theta) r d\theta dr .$$

Lastly, as concerns the shape of the city boundary, from (10) it is obvious that the distance of the urban boundary from the city center,  $r_f$ , varies with the angle to the transit axis,



$\theta_f$ . Define  $X = Y - e^u R_A^\beta \alpha^{-\alpha} \beta^{-\beta}$  and let  $m$  denote the last populated station along the axis. The difference between the highest and the lowest values of  $r_f$  is given by

$$(r_f)_{\max} - (r_f)_{\min} = \frac{(r_f)_{\max}}{1 + n_a / \pi} = \frac{X + s_m(c_v - c_t) - (m-1)g}{c_v(1 + n_a / \pi)} \quad (15)$$

where  $(r_f)_{\max}$  and  $(r_f)_{\min}$  denote the radius of the city for  $\theta_f = 0$  and  $\theta_f = \theta_a = 2\pi/n_a$ , respectively. As  $n_a$  increases, the difference in (15) decreases.

## 4 Applications

Let us illustrate the urban forms that can be obtained in our framework by addressing two cases. The first case is intended to compare alternative urban forms induced by different transport technologies 1 and 2. The second case simulates the commuter rail network in Paris.

### 4.1 Transport technology and urban form

To illustrate the model features, let us consider a simple numerical example with parameter values given in Table 1. The coefficients in the utility function reflect the fact that a French household typically spends about 25% of its income on housing and the remaining 75% on other goods. These values are taken from the French national statistical institute, the Insee, as in a related research [5]. Travel costs split into three components, namely: (1) private travel from house to station; (2) public transportation; (3) a penalty incurred at train stops in stations. These parameters have been assigned values that yield a 1 to 4 ratio of transit cost to private car cost by unit of distance, which is reasonable for home-to-work travel in the Paris area.

There are four perpendicular mass transit axes. On each axis there are four access points (stations). We have a symmetric configuration, but its extension to an irregular shape would be straightforward. Figure 1 depicts both the urban form and the spatial distribution of land rent across the city. As the colour gets darker the land rent is higher. A white colour corresponds to the non-urban area (with land rent  $R_A$ ). The stations are located at the darkest points where the land rent is peaked. It is easy to identify the influence region for each station on the graphic. The regions' shapes as well as the outer boundaries of the city depend on the corresponding transport technology. Technology 1 induces a city with a circular (wave-like) structure which does not appear under technology 2.

Technology 2 would be the more efficient one since each passenger uses the shortest distance to reach the nearest station, while a longer trip is needed under technology 1. Thus, under the constrained technology 1 the travel cost related to the first stage of the daily home-to-work trip is higher yielding a smaller available income, net of transport cost, to the households. This may be seen as a form of inefficiency related to a constrained transportation system (technology 1). In quantitative terms, the less efficient city based on technology 1 has a smaller population, by about 20%, and a smaller land area (by about 21%). In monetary terms, we find that the population and geographical size of the less efficient city is almost matched by technology 2 and a household income reduced by about one thousand euros per year.

This comparison emphasizes the importance of the interaction between land use and transport infrastructure.

However the assumption of a straight line by the private mode from every home to a

transit station may not be realistic since this cannot correspond easily to a network structure. Technology 1 which involves both radial and circular moves on the private mode is more akin to a realistic network structure. We plan to elaborate more evidence on this point in future research.

[ Table 1 about here ]

[ Figure 1 about here ]

## 4.2 Simulating the commuter rail network in Paris

The second case is purported to simulate more realistically the Paris area. We model the commuter rail network by replicating the eastern part of the “RER A” line from station “Châtelet” (near the center) to station “Marne-la-Vallée” (peripheral). The radius of the central business district is 1km: all the land in the CBD is used for professional activities and transportation (which is assumed costless). Transit access is provided along each rail axis at fourteen stations, whose locations range from 3 to 35km from the city centre. Only Travel technology 1 is considered for private transport, as it is more realistic.

The network, including mass transit stations and their influence regions is depicted in Fig. 2 which is limited to the first quadrant. The CBD region is placed at the origin of the coordinate system and the transit stations can be identified along the x and y-axis. The city boundary is indicated for the base case (bold line) and also for an alternative case of higher household income (thin dotted line), which yields a larger city where both the population size and the lot size are higher.

[ Figure 2 about here ]

Along a transit axis the city is stretched up to more than 40km from the centre. As we move away from the axis the urbanized area gets thinner on average but remains concentrated around the stations. The station locations have an important impact on the final shape of the city: it seems that some stations are not located optimally as they strongly compete with neighbouring stations (see, for example, the stations between 20 and 25km). This insight might be explored by an in-depth geographical study of the region in order to formulate a consistent evaluation.

Land rent is plotted in Fig. 3 for two cases: along a public transport axis ( $\theta = 0$ ) and away from that axis ( $\theta = \pi/18$ ). It is higher around transit stations and not monotone decreasing from the city centre as in the basic monocentric model. The average land rent is higher around stations that are closer to the CBD because of smaller transport costs. Land rent is always higher than agricultural rent (or opportunity cost of land)  $R_A$ . Note that along the line  $\theta = \pi/18$  the urban area is not continuous. For higher values of  $\theta$  the urbanized area is smaller and does not extend around more than one or two stations.

Lot size has a shape that is similar, but symmetric, to land rent. Fig. 4 provides the average lot size as a function of the distance from the city centre: for each  $r$ , we integrated the lot sizes over the urbanized area from  $\theta = 0$  to  $\theta_f$ , the solution of Eq. (10). Around mass transit stations we have smaller lot sizes hence higher urban densities, but on average there is a decreasing trend in density as we move away from the city centre.

[Figure 3 about here]

[Figure 4 about here]

Let us next conduct a sensitivity analysis with respect to key parameters in the model, namely: the discomfort parameter,  $g$ , and the location of some stations. Four scenarios are considered: (i) base case; (ii) an increase (twice the base value) in the discomfort parameter; (iii) a displacement of station 2; (iv) a displacement of station 3. These may be seen as a form of spatial competition between stations.

Numerical results are summarized in Tables 2 and 3. The first column in each table designates the scenario. By scenario line, the four next cells give the market share of stations 1 to 4 on which we focus our attention, concerning population in Table 2 and housing area in Table 3.

In the base case the total population in the city is about 2.120 million households. The first four stations are patronized by about one third of the total population. Station 3 has the largest market share due to a relatively large distance between it and its two neighbouring stations. In scenario (ii), an increase in the discomfort parameter  $g$  increases the demand for the first stations since households are motivated to get access at stations closer to the CBD. The intuition for this impact is obvious. Notice that the increase in the demand for station 1 is determined out of competition with station 2 only, whereas station 2 also competes with station 3. Station 2 loses some users to station 1 but gains some from station 3. Station 2 access costs are larger than in the base case even for the users that patronize it initially, whereas the initial users of station 1 have unchanged costs.

[ Table 2 about here ]

[ Table 3 about here ]

Concerning housing area, the market shares comply with those in the distribution of the population, but the differences are more important. Indeed, around station 2, the average lot size is larger than around station 1 (cf. Figure 4). It is easy to see from the two tables the trade-off made by the household: to locate either near the city centre in a smaller lot, or far away from the centre but in a larger lot. An increase in  $g$ , hence in transport cost, induces smaller smaller population but a larger urban area, and hence a larger housing area per household. This stems from Eq. (2): an increased transport cost makes the bid rent decrease everywhere, a fact that enables every household to get more land. The second scenario may not necessarily imply a higher average surface since the higher travel cost reduces the household net income. Still, as shown by Eq. (3), when the utility level is kept at a given level the overall impact on the housing area is positive.

The last two scenarios pertain to the spatial competition between the transit stations. In both cases the competition is tighten between stations 2 and 3. In scenario (iii), where station 2 moves away from the city centre, the number of passengers who transit through station 1 increase substantially as the station itself covers a larger urban area. At the same time transit through stations 2 and 3 decreases. The situation for station 4 is almost unchanged (its relative market share increases due to the decrease in the number of households in the city).

The last scenario corresponds to a re-location of station 3 closer to the city centre. This is similar to the last scenario except that the variation is higher in absolute value. As in scenario

three, transit through stations 2 and 3 is reduced substantially due to the tougher competition (the impact is particularly important on station 2). This benefits to station four, and the total number of households slightly decreases (and induces a small increase in the market share of station 1).

## 5 Conclusion

In this paper, the monocentric model has been extended by considering transit radial axes which discrete stations as the main mode of travel. Each household makes its daily home-to-work trip by using a sequence of two transport modes: a private mode from house to mass transit station and public transport from the station to the business location. The related urban equilibrium has been derived, with land rents that are peaked at transit stations and decrease with respect to the travel cost to the city center rather than with respect to the distance to it. A complementary effect pertains to the household lot size. The city boundary displays a starlike pattern, with one arm by transit axis and broader extension around access stations.

Although radial axes of mass transit had been considered earlier, our model is the first to consider discrete stations for access to a transit axis, yielding results that differ significantly from the basic model. More precisely, by making the transport conditions less abstract and adapting the classic monocentric analysis, we bring its outcomes closer to an empiric pattern that pertains to many cities. The analysis shows the important interaction between the transport system and the induced urban land form.

Further work could be directed to two targets. First target is to model capacity limitations at stations and congestion in the related passenger flows, yielding further delay to the passengers. Second target pertains to household heterogeneity to study the impact of mass transit on equity and accessibility [7,8]: it would be sufficient to differentiate two income classes to study potential effects of segregation.

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Variable	Value	Unit
$\alpha$	0,75	-
$\beta$	0,25	-
$Y$	38,000	€year
$c_v$	1,000	€km/year
$c_t$	200	€km/year
$g$	30	€station/year

TAB. 1: NUMERICAL EXAMPLE

<b>Scenario</b>	<b>Station 1</b>	<b>Station 2</b>	<b>Station 3</b>	<b>Station 4</b>	<b>Total number of households</b>
Base case	4.74%	6.55%	15.28%	11.69%	2 117 652
$g=60$	4.86%	6.65%	15.45%	11.86%	2 035 295
Station 2 at 5km (instead of 4.5km)	5.82%	5.41%	15.22%	11.71%	2 118 258
Station 3 at 6km (instead of 7.2km)	4.78%	3.77%	14.24%	14.96%	2 108 039

TAB. 2: DISTRIBUTION OF POPULATION AROUND THE FIRST FOUR STATIONS

<b>Scenario</b>	<b>Station 1</b>	<b>Station 2</b>	<b>Station 3</b>	<b>Station 4</b>
Base case	2.89%	4.44%	12.86%	11.55%
$g=60$	2.94%	4.51%	13.04%	11.72%
Station 2 at 5km (instead of 4.5km)	3.58%	3.69%	12.86%	11.56%
Station 3 at 6km (instead of 7.2km)	2.89%	2.46%	11.63%	14.86%

TAB. 3: SHARE OF CITY AREA ACROSS THE FIRST FOUR STATIONS



Impact of transport technology on urban form

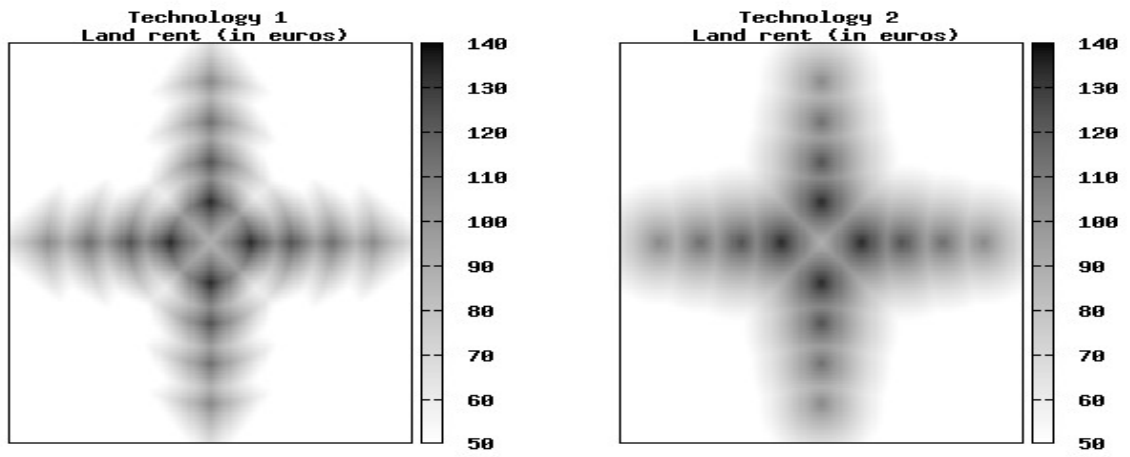


FIG. 1. IMPACT OF TRANSPORT TECHNOLOGY ON URBAN FORM

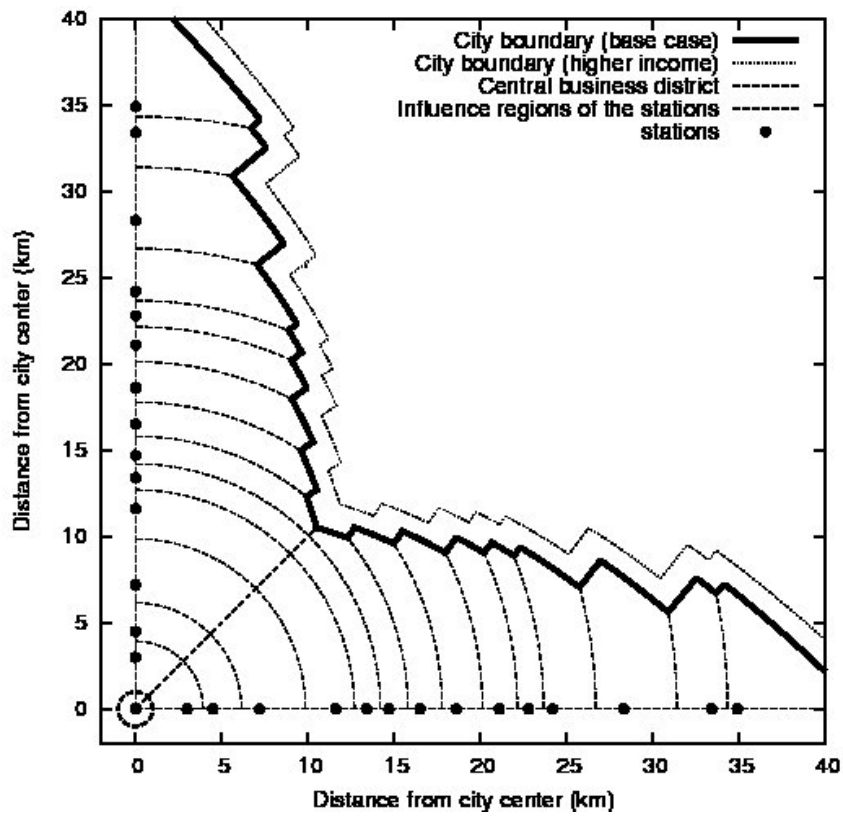


FIG. 2. CITY BOUNDARY

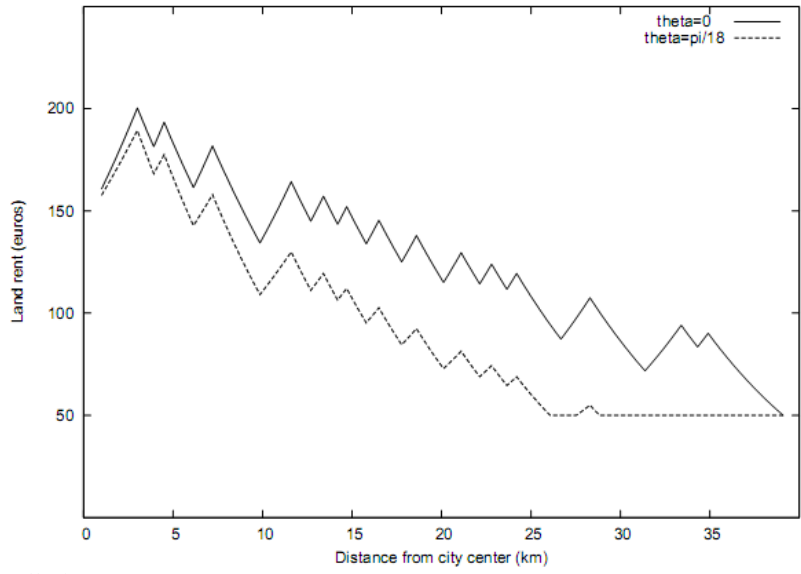


FIG. 3. LAND RENT

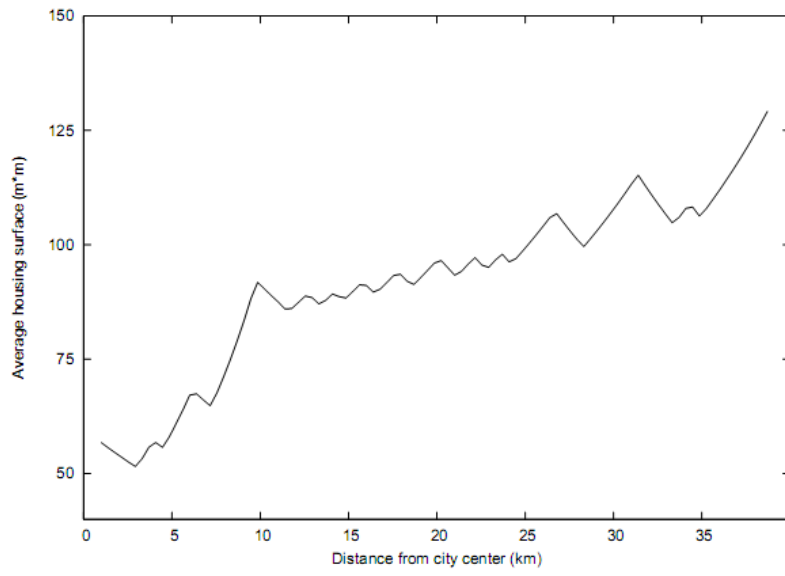


FIG. 4. HOUSING AREA