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A straightforward determination of fluid viscosity and density using microcantilevers: analytical and experimental studies

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Abstract

Vibrating microcantilevers can be used to measure density and viscosity of surrounding fluids. The classical procedure involves experimental acquisition of the deflection spectrum of the beam, but a systematic calibration step is mandatory for obtaining viscosity and density. In the present study, a method is proposed to facilitate these measurements for Newtonian fluids with only one calibration step in the cantilever lifetime. Our approach is based on approximating the deflection spectrum with a second-order transfer function and determining a simple analytical relationship between viscosity, density and the parameters of the transfer function. Fluid parameters determination results are shown for validation and discussed.

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Keywords: microrheology; microcantilevers; viscosity; density

1. Introduction

In-situ (and even embedded) accurate and rapid estimation of viscosity and density of liquid medium is increasingly needed in various fields (for optimizing oil combustion, in food industry, medicine...). Conventional (cone and plate or Couette) rheometers are not suited for this kind of applications, and only probe viscosity at low frequencies (under few hundreds Hertz), whereas viscosity is often frequency-dependent: this is an issue for real-time viscosity checking of fast flowing fluids. Consequently, a need for microrheometer emerged. Microelectromechanical systems (MEMS), *e.g.* microcantilevers, have been used to probe viscosity and density of gases and liquids [1-6].

Though, whatever the MEMS structure, some issues always remain: long iterative calculation, inaccurate estimation (especially for viscosity), low viscosity range or low frequency range, lack of analytical model, complex calibration procedure... For example, in a recent study [6], density is accurately probed but the error on the estimation of the viscosity remains relatively high over a small viscosity range from 1 to 7cP.

In the first part, we describe the dedicated cantilever structures used to investigate these remaining issues and the experimental set-up for obtaining deflection spectra. The second part briefly summarizes the theoretical state-of-the-art equations and introduces the hypothesis of our own which allows an analytical determination of fluid viscosity and density of Newtonian fluids. Discussion about these results and conclusion follow.

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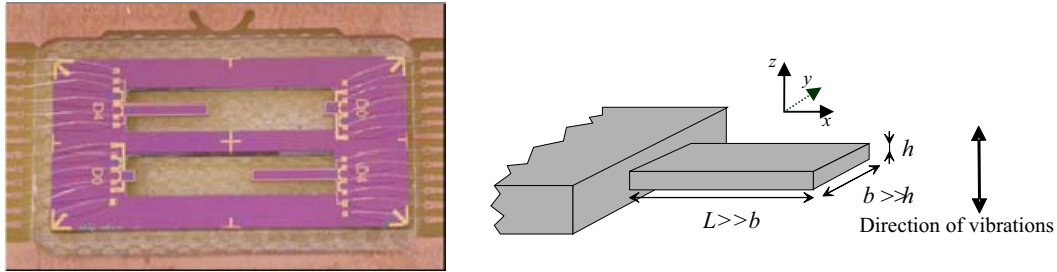


Fig. 1: Example of microcantilevers which have been used

2. Set-up and measurements

The cantilevers were fabricated using standard silicon microtechnologies with a process provided by ESIEE group, Paris, France. The geometries of the cantilever used in this study are $4000 \times 600 \times 100 \mu\text{m}^3$ (referred as D4) and $6000 \times 600 \times 108 \mu\text{m}^3$ (referred as D6). An example of the cantilever used here and its geometry is depicted in Fig. 1.

Based on the vibrating phenomena of cantilevers, induced by the alternating current in the cantilever and the surrounding magnetic field, the deflection spectra (amplitude and phase vs. frequency) are acquired. Measurements of cantilever vibration amplitude (and phase) as functions of frequency f (or radial frequency $\omega = 2\pi f$) were carried out using a vibrometer (MSA-500 Micro-System Analyser from Polytech) in a clean room facility (at IMS laboratory) where temperature was measured to be 19.4°C during the experiments. Examples are depicted in Fig. 2 for two cantilevers with different lengths immersed in some Newtonian fluids: silicone oils and dodecane.

Actual values at 19.4°C were measured using a “classical” rheometer; they are given as references in Table 2. Spectra obtained when the fluid is the air are also shown.

These experimental curves are fitted with a second-order transfer function in order to obtain static value, damping factor (ζ) and *eigenfrequency* (f_0), simply using the *curve fitting tool* of Matlab over the frequency range of measurements. These values are shown in Table 1. This approach is to be compared to other teams’ work where quality factor and *resonant peak frequency* (when existing, for low viscosities) are measured.

3. Analytical determination of viscosity and density

The real equation describing deflection spectra of a cantilever excited at its free-end is much more complex than a second-order transfer function, due to the expression of the hydrodynamic force F_{lin_fluid} exerted by the fluid on the

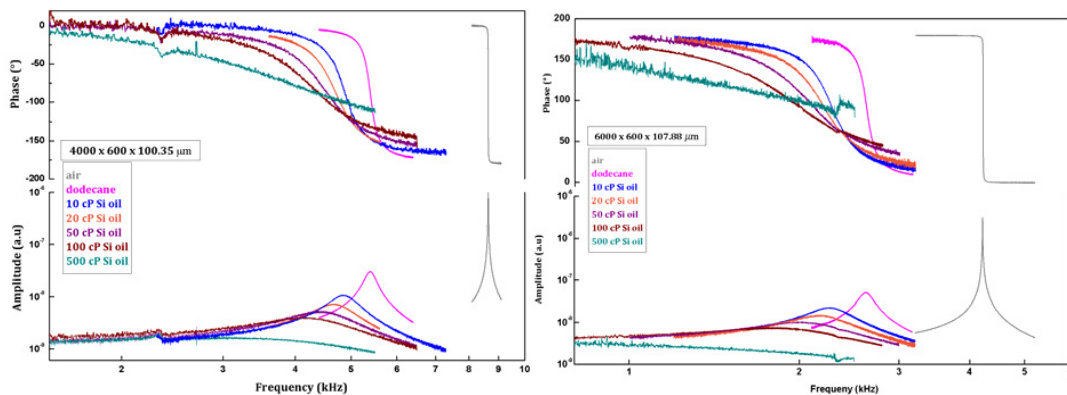


Fig. 2: Deflection spectra (up: phase, down: amplitude) of a $4000 \times 600 \times 100 \mu\text{m}^3$ (left) and a $6000 \times 600 \times 108 \mu\text{m}^3$ (right) cantilever in different fluids (silicon oils have a density of 1000 kg.m^{-3} ; density of dodecane is 749 kg.m^{-3}).

Table 1. Eigenfrequency [Hz] and damping ratio obtained by curve fitting amplitude deflection spectra with a second-order transfer function for different cantilevers and different viscosities (dod states for dodecane), with a 95% confidence bounds.

		f_0	$f_{0 \min}$	$f_{0 \max}$	Ξ	ξ_{\min}	ξ_{\max}
D4	dod	5397	5397	5397	0.02206	0.02205	0.02206
D4	10cP	4868	4867	4868	0.05603	0.05590	0.05617
D4	20cP	4695	4695	4696	0.08047	0.08040	0.08054
D4	50cP	4499	4496	4501	0.1259	0.1253	0.1265
D4	100cP	4306	4304	4309	0.1809	0.1802	0.1816
D4	500cP	3959	3946	3973	0.436	0.4326	0.4395
D6	dod	2623	2623	2623	0.02688	0.02682	0.02694
D6	10cP	2274	2274	2274	0.08196	0.08188	0.8204
D6	20cP	2195	2195	2196	0.1165	0.1165	0.1169
D6	50cP	2072	2071	2073	0.1775	0.1771	0.1778
D6	100cP	1933	1931	1934	0.2637	0.263	0.2644
D6	500cP	1783	1766	1799	0.7605	0.7438	0.7772

cantilever. But, based on state-of-the-art standard equations related to rheology [7] and by using approximations of Maali et. al. [8], for the case of rectangular beams, F_{lin_fluid} can be written very simply in the frequency domain:

$$F_{lin_fluid} = \frac{\pi \rho_f b^2}{4} \omega^2 \left[\left(a_1 + a_2 \frac{\delta}{b} \right) - j \left(b_1 \frac{\delta}{b} + b_2 \left(\frac{\delta}{b} \right)^2 \right) \right] w(\omega, x) \quad (\text{with } \delta = \sqrt{\frac{\eta}{\pi \rho_f f}}) \tag{1}$$

$a_1=1.0553$, $a_2=3.7997$, $b_1=3.8018$ and $b_2=2.7364$ are referred as ‘‘Maali’s parameters’’, $w(\omega, x)$ is the amplitude of vibration at coordinate x of the cantilever, ρ_f and η are respectively mass density and viscosity of the fluid, and b is the width of a cantilever.

Then, an analytical expression of viscosity and density (2-3) can be obtained, depending only on Maali’s parameters, dimensions, density ρ of the cantilever, resonant frequency in vacuum $f_{0,vacuum}$, damping factor and eigenfrequency in the given fluid. Except the ‘‘second-order transfer function approximation’’, the only other assumption we made is that the hydrodynamic force is supposed to be constant and equal to its value at the eigenfrequency over the frequency range concerned by the fitting.

$f_{0,vacuum}$ is the only parameter requiring a calibration step (actually, if the thickness of the cantilever was known with certainty, this would not be necessary since it only depends from cantilever geometry and material properties).

$$\rho_f = \frac{2h\rho}{\pi b f_0^2 a_1 (a_2 b_1 - a_1 b_2)} \left(\frac{(a_2 b_1 - 2a_1 b_2)(f_{0,vacuum}^2 - f_0^2) - 2\xi a_2^2 f_{0,vacuum}^2}{a_2^2 \xi f_{0,vacuum}^2 - (a_2 b_1 - 2a_1 b_2)(f_{0,vacuum}^2 - f_0^2)} \right) \tag{2}$$

Table 2. Density ρ (kg.m^{-3}) and viscosity η (cP) calculated analytically from values of table 1 and error when comparing to the reference values measured with a classical rheometer.

		Actual viscosity [cP] (@19.4°C)	Viscosity η [cP]	Error on viscosity	Mass density ρ [kg.m^{-3}]	Error on mass density
D4	dod	1.67	1.89	13.2	686	-8.5
D4	10cP	10.6	12.8	20.6	855	-14.5
D4	20cP	22.3	27.5	23.3	884	-11.6
D4	50cP	54.2	72.6	33.9	870	-13
D4	100cP	111	163	46.9	829	-17.1
D4	500cP	553	1160	110	491	-50.9
D6	dod	1.67	1.51	-9.6	748	-0.3
D6	10cP	10.6	15.3	44.3	984	-1.6
D6	20cP	22.3	32.9	47.5	979	-2.1
D6	50cP	54.2	83.8	54.6	956	-4.4
D6	100cP	111	208	87.2	892	-10.8
D6	500cP	553	1840	233	347	-65.3

$$\eta = \frac{2a_1 b_1 h \rho}{f_0 (b_1 a_2 - a_1 b_2)} \frac{\left[\frac{b_1 (f_{0,vacuum}^2 - f_0^2) + 2\xi a_2 f_{0,vacuum}^2}{-\sqrt{b_1^2 (f_{0,vacuum}^2 - f_0^2)^2 + 4\xi f_{0,vacuum}^2 [a_2^2 \xi f_{0,vacuum}^2 - (a_2 b_1 - 2a_1 b_2)(f_{0,vacuum}^2 - f_0^2)]}} \right]^2}{(a_2 b_1 - 2a_1 b_2)(f_{0,vacuum}^2 - f_0^2) - 2a_2^2 \xi f_{0,vacuum}^2 + a_2 \sqrt{b_1^2 (f_{0,vacuum}^2 - f_0^2)^2 + 4\xi f_{0,vacuum}^2 [a_2^2 \xi f_{0,vacuum}^2 - (a_2 b_1 - 2a_1 b_2)(f_{0,vacuum}^2 - f_0^2)]}} \quad (3)$$

Table 2 shows results obtained with the values given in table 1. As seen, error on density is under 20% for viscosities up to 100cP; error on viscosity ranges from 10% to 55% for viscosities up to 50cP.

Unaccurate results for viscosities over 100cP are interpreted as a failure of our two simplifying hypotheses since for high viscosities higher modes of resonance interfere with the first one. It shows the limits of our work.

4. Discussion and conclusion

For the cantilever named D4, results concerning viscosity are still relatively correct at 100cP whereas D6 seems to be less accurate; this means that cantilever's geometry has an influence. This is under investigation.

Three tracks were already investigated to improve these results:

- Being more accurate about "Maali's parameters" since it is worth noting that these parameters were obtained by simple fitting of hydrodynamic function over a broad range of Reynolds number.
- Incorporating very recent study by Sader et. al. [9] which improves the expression of the hydrodynamic function taking into account a non-zero aspect ratio of the beam.
- Investigating other fitting methods than Matlab embedded fitting tool (such as: changing the frequency range or fitting on phase data instead of amplitude data).

None of these tracks brought significant improvements up to now, meaning (as far as we know) that discrepancy is inherent to our simplifying hypothesis. Even if this could sound disappointing, we would like to point out that these results show a small error over a wide range of viscosities as compared to other works. It is also the first time to our knowledge that an analytical expression for viscosity is given.

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