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► **To cite this version:**

Carlo Rizzo, Arnaud Dupays, Rémy Battesti, Mathilde Fouché, Rikken G.L.J.A.. Inverse Cotton-Mouton effect of the Vacuum and of atomic systems. EPL - Europhysics Letters, 2010, 90 (6), pp.64003. 10.1209/0295-5075/90/64003 . hal-00480562

HAL Id: hal-00480562

<https://hal.science/hal-00480562>

Submitted on 4 May 2010

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Inverse Cotton-Mouton effect of the Vacuum and of atomic systems

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PACS **nn.mm.xx** – First pacs description
 PACS **nn.mm.xx** – Second pacs description
 PACS **nn.mm.xx** – Third pacs description

Abstract. - In this letter we calculate the Inverse Cotton-Mouton Effect (ICME) for the vacuum following the predictions of Quantum ElectroDynamics. We compare the value of this effect for the vacuum with the one expected for atomic systems. We finally show that ICME could be measured for the first time for noble gases using state-of-the-art laser systems and for the quantum vacuum with near-future laser facilities like ELI and HiPER, providing in particular a test of the nonlinear behaviour of quantum vacuum at intensities below the Schwinger limit of 4.5×10^{33} W/m².

The advent of laser sources in the 1960s has opened the way to non-linear optics thanks to the rapid increase in the light intensities which reached 10^{19} W/m² in the 1980s, and that can be nowadays as high as 10^{25} W/m² [1]. Near-future laser facilities like the Extreme Light Infrastructure (ELI) [2] and the High Power laser Energy Research system (HiPER) [3] should deliver 10^{29} W/m² approaching the Schwinger limit of 4.5×10^{33} W/m² [1]. At this intensity optical nonlinearities of quantum vacuum should be experimentally accessible and quantum vacuum studies are one of the main motivations to further increase laser intensity [4].

In 1999 measurements of quantum electrodynamics processes in an intense electromagnetic wave, have been reported by Bamber *et al.* [5]. Nonlinear Compton scattering and electron-positron pair production have been observed in collisions between a laser beam of intensity up to 5×10^{21} W/m² and electrons of energy close to 50 GeV. The electric field strength of the laser in the electron rest frame corresponded to a few percents of the Schwinger limit.

Recently, an experiment coupling a very intense transverse pulsed magnetic field with an intense laser source has been performed [6]. The goal was to detect a possible oscillation of photons into massive particles. The maximum value of the pulsed magnetic field was about 10 T

over 0.36 m, pulse duration was a few milliseconds. The laser source intensity was about 10^{19} W/m², corresponding to about 1500 J, over 5 ns focussed on a spot of 100 μ m diameter. These two pulsed facilities proved to work ideally together, opening new possibilities for studies of non linear optics effects where a strong magnetic field and a powerful light source are necessary. One of these effects is the inverse Cotton-Mouton effect (ICME in the following), a non linear optical effect that in principle exists in any medium. In the presence of a transverse magnetic field, a linearly polarized light induces a magnetization in the medium in which it propagates [7]. The optically induced magnetization depends linearly on the transverse magnetic field amplitude. ICME, as its name indicates, is related to the much more studied Cotton-Mouton effect (CME in the following) i.e. the linear birefringence induced by a transverse magnetic field [8] in a similar fashion as the Faraday effect and the inverse Faraday effect are related [7]. ICME and CME can be explained as a mixing of four waves, two static fields, and two photonic fields. The CME depends on the square of the amplitude of the transverse magnetic field. To measure it, intense magnetic fields are necessary. ICME depends on the transverse magnetic field amplitude, and to the light intensity. To measure such an effect one needs to couple a powerful laser beam to an intense magnetic field transverse with respect to the light

wave vector as in the experiment of ref. [6].

As far as we know, experimental observations of ICME are very rare. In Ref. [9] measurements of ICME in atomic gases are cited [10, 11] for the case of resonant optical pumping. Measurements of this kind of effect, called in ref. [7] induced magnetization by resonant excitation, can be traced back to the sixties [12].

A measurement of the magnetization in a magnetically ordered crystal illuminated by a laser beam in the presence of a static magnetic field has been reported in ref. [13]. A linearly polarized beam from a neodymium laser ($\lambda=1,064 \mu\text{m}$) with a pulse duration of 20 ns was focussed on a film of $10\mu\text{m}$ thickness of $(\text{Lu,Bi})_3(\text{Fe,Ga})_5\text{O}_{12}$ immersed in a magnetic field ranging from a few 10^{-4} T to $3 \cdot 10^{-3}$ T. Measurements have been conducted using laser energy between 4 and 20 mJ, beam spot diameter was 1.3 mm, corresponding to intensity between 1.5×10^{11} W/m² and 7.5×10^{11} W/m². The magnetization of the order of 10^{-8} T was measured by a planar three-turn coil on the surface of the sample. The measured magnetization did not depend on the laser polarization. The authors of ref. [13] have called the phenomenon that they have observed ICME which is questionable since their static magnetic field is parallel to the direction of propagation of light. This kind of geometry is usually called Faraday configuration and it is associated in general to Faraday effects.

As far as we know a complete study of ICME for molecules does not exist in literature. In ref. [9] one finds a theoretical expression for the magnetization \mathbf{M}_{ICM} related to the ICME in the case of atoms. \mathbf{M}_{ICM} is proportional to the elements of second hypermagnetizability tensor $\eta_{\alpha\beta,\gamma\delta}$ on which also CME depends.

In this letter we calculate the inverse Cotton-Mouton effect for the quantum vacuum following the predictions of Quantum ElectroDynamics. We compare the value of this effect for the vacuum with the one expected for atomic systems. We finally show that ICME could be measured for the first time for noble gases using state-of-the-art laser systems and in the case of quantum vacuum with near-future laser facilities like ELI [2] and HiPER [3], providing in particular a test of the nonlinear behaviour of quantum vacuum at intensities below the Schwinger limit.

Optical non linearities in the propagation of light in vacuum have been predicted since 1935 by the work of Euler and Heisenberg [14, 16]. In particular vacuum in the presence of a static magnetic field should behave as an uniaxial birefringent crystal [17]. This phenomenon is in all the aspects similar to what is generally known as Cotton-Mouton effect [8]. The CME of quantum vacuum has not yet been observed in spite of several experimental attempts (see [15] and references within). In the 1936 paper by Heisenberg and Euler [16] the complete study of the phenomenon can be found, together with the general expression of the non linear effective Lagrangian of the light-light interaction.

The form of the effective lagrangian L_{HE} of the light-light interaction is determined by the fact that the lagrangian has to be relativistically invariant and therefore can only be a function of the Lorentz invariants F, G:

$$F = (\epsilon_0 E^2 - \frac{B^2}{\mu_0}) \quad (1)$$

$$G = \sqrt{\frac{\epsilon_0}{\mu_0}} (\mathbf{E} \cdot \mathbf{B}) \quad (2)$$

where ϵ_0 is the vacuum permittivity, μ_0 is the vacuum permeability and E and B are the electromagnetic fields. Up to fourth order in the fields, L_{HE} can be written as $L_{\text{HE}} = L_0 + L_{\text{EK}}$ where L_0 is the usual Maxwell's term and L_{EK} is the first order non linear term first calculated by Euler and Kockel [14].

L_{EK} is valid in the approximation that the fields vary very slowly over a length equal to the reduced electron Compton wavelength $\lambda = \frac{\hbar}{m_e c}$ during a time $t_e = \frac{\lambda}{c}$:

$$\frac{\hbar}{m_e c} |\nabla E(B)| \ll E(B) \quad (3)$$

$$\frac{\hbar}{m_e c^2} \left| \frac{\partial E(B)}{\partial t} \right| \ll E(B) \quad (4)$$

with \hbar the Planck constant divided by 2π , m_e the electron mass and c the speed of light in vacuum.

Moreover E and $\frac{B}{\sqrt{\epsilon_0 \mu_0}}$ have to be smaller than the critical field $E_{\text{cr}} = \frac{m_e^2 c^3}{e \hbar}$ i.e. $B \ll 4.4 \times 10^9$ T and $E \ll 1.3 \times 10^{18}$ V/m. with e the elementary charge. The laser intensity which corresponds to an electric field associated to the light wave equal to E_{cr} is 4.5×10^{33} W/m². This intensity value is what is usually called the Schwinger limit.

L_{HE} can be written as

$$L_{\text{HE}} = \frac{1}{2} F + a(F^2 + 7G^2) \quad (5)$$

where $L_0 = \frac{1}{2} F$ and $L_{\text{EK}} = a(F^2 + 7G^2)$. The value of a given by Euler-Kockel [14] is:

$$a = \frac{2\alpha^2 \hbar^3}{45 m_e^4 c^5} \quad (6)$$

with α the fine structure constant. This corresponds to $a = 1.7 \times 10^{-30}$ m³/J.

We are interested in the magnetization $\mathbf{M} = \frac{\mathbf{B}}{\mu_0} - \mathbf{H}$. The field \mathbf{H} can be obtained thanks to the relations:

$$\mathbf{H} = - \frac{\partial L_{\text{HE}}}{\partial \mathbf{B}} \quad (7)$$

which gives:

$$\mathbf{H} = - \frac{1}{2} \frac{\partial F}{\partial \mathbf{B}} - 2aF \frac{\partial F}{\partial \mathbf{B}} - 14aG \frac{\partial G}{\partial \mathbf{B}} \quad (8)$$

and finally:

$$\mathbf{H} = \frac{\mathbf{B}}{\mu_0} + 4a \frac{\mathbf{B}}{\mu_0} F - 14a \sqrt{\frac{\epsilon_0}{\mu_0}} \mathbf{E} G. \quad (9)$$

In the case of the propagation of an electromagnetic plane wave, to which the fields E_ω and B_ω are associated, in the presence of a static magnetic field B_0 , one can write $B = B_\omega + B_0$ and $E = E_\omega$, with $\epsilon_0 E_\omega^2 - \frac{B_\omega^2}{\mu_0} = 0$ and $\sqrt{\frac{\epsilon_0}{\mu_0}}(\mathbf{E}_\omega \cdot \mathbf{B}_\omega) = 0$.

Finally one gets:

$$F = -\frac{1}{\mu_0}[B_0^2 + 2(\mathbf{B}_0 \cdot \mathbf{B}_\omega)] \quad (10)$$

$$G = \sqrt{\frac{\epsilon_0}{\mu_0}}(\mathbf{E}_\omega \cdot \mathbf{B}_0). \quad (11)$$

The magnetization can be written as:

$$\begin{aligned} \mathbf{M} &= 4a \frac{\mathbf{B}_0 + \mathbf{B}_\omega}{\mu_0^2} (B_0^2 + 2\mathbf{B}_\omega \cdot \mathbf{B}_0) \\ &+ 14a \frac{\epsilon_0}{\mu_0} \mathbf{E}_\omega (\mathbf{E}_\omega \cdot \mathbf{B}_0). \end{aligned} \quad (12)$$

In the case of interest laser intensities are such that B_ω , $E_\omega/c \gg B_0$, and the magnetization corresponding to the ICME has to depend linearly on the external magnetic field amplitude and quadratically on the electromagnetic fields associated to light wave. Extracting from the previous equation the terms of that type, one therefore obtains:

$$\mathbf{M}_{\text{ICM}} = 14a \frac{\epsilon_0}{\mu_0} \mathbf{E}_\omega (\mathbf{E}_\omega \cdot \mathbf{B}_0) + 8a \mathbf{B}_\omega \frac{\mathbf{B}_\omega \cdot \mathbf{B}_0}{\mu_0^2}. \quad (13)$$

Let's now recall that the square of the laser fields E_ω^2 and B_ω^2 are related to the laser intensity I by:

$$\epsilon_0 E_\omega^2 = \frac{I}{c} = \frac{B_\omega^2}{\mu_0}. \quad (14)$$

Two cases are possible ($\mathbf{E}_\omega \parallel \mathbf{B}_0$, $\mathbf{B}_\omega \perp \mathbf{B}_0$) or ($\mathbf{E}_\omega \perp \mathbf{B}_0$, $\mathbf{B}_\omega \parallel \mathbf{B}_0$). In the first case one gets:

$$\mathbf{M}_{\text{ICM}\parallel} = 14a \epsilon_0 E_\omega^2 \frac{\mathbf{B}_0}{\mu_0} = 14a \frac{I \mathbf{B}_0}{c \mu_0}. \quad (15)$$

In the second case one obtains:

$$\mathbf{M}_{\text{ICM}\perp} = 8a \frac{B_\omega^2 \mathbf{B}_0}{\mu_0} = 8a \frac{I \mathbf{B}_0}{c \mu_0}. \quad (16)$$

In both cases \mathbf{M}_{ICM} is parallel to \mathbf{B}_0 . It is worth to stress that the fact that $\mathbf{M}_{\text{ICM}\parallel} \neq \mathbf{M}_{\text{ICM}\perp}$ confirms that under the effect of an external magnetic field, vacuum should become non isotropic, and its magnetic susceptibility should depend on light polarization. Actually, electric polarizability should also become non isotropic and finally the index of refraction should depend on light polarization. This is the cause of the CME of quantum vacuum [17].

As said before, an ICME set up consists of a powerful laser and of an intense transverse magnetic field. The most powerful lasers are usually pulsed, the same applies to magnetic field generation [18]. A set up coupling these two instruments have been recently realized in the framework

of the search for photon oscillations into massive particles [6]. A 10^{19} W/m² laser pulse was focused in a vacuum region where a transverse magnetic field of more than 10 T was present.

Let's take these numerical values to have a reasonable estimate of the magnetization to be measured in the case of the ICME of the quantum vacuum:

$$\mathbf{M}_{\text{ICM}\parallel} \approx 8 \times 10^{-18} \text{T} \quad (17)$$

and

$$\mathbf{M}_{\text{ICM}\perp} \approx 4.5 \times 10^{-18} \text{T} \quad (18)$$

where we have used the relation $\mu_0 \mathbf{M}(\text{A/m}) = \mathbf{M}(\text{T})$.

Measurements of a magnetization induced by a laser beam are also performed in the framework of the inverse Faraday effect (IFE). A circularly polarized laser beam creates in a medium a magnetization proportional to the energy density associated to the electromagnetic wave [7]. This effect is related to the Faraday effect as the ICME is related to the Cotton-Mouton effect. In the case of IFE measurements sensitivity in magnetization of the order of 10^{-10} T has been reached [19]. The same kind of sensitivity should be reached in the case of ICME.

The values in Eqs. (17) and (18) are still below the sensitivity reported in [19], but new laser sources like the Extreme Light Infrastructure (ELI) [2] and the High Power laser Energy Research system (HiPER) [3] are supposed to reach intensities exceeding 10^{29} W/m² increasing the expected ICME of vacuum at levels that should be detectable. In particular, taking also advantage of progress in transverse pulsed magnetic field [18] and using a field of at least 30 T, a laser intensity of 5×10^{25} W/m², well below the possibilities of new facilities, will be sufficient to open up direct studies of quantum vacuum with powerful laser systems.

In the following, for the sake of comparison, let's calculate the expected ICME in the case of atoms and in particular noble gases.

Our calculation of the magnetization corresponding to the ICME in atoms is based on the Buckingham and Pople general theory of molecular polarizabilities in the presence of a strong magnetic field [20]. In the framework of this theory the atomic magnetic moment can be written as:

$$\mu_{\text{at}} = -\frac{dU}{d\mathbf{B}} \quad (19)$$

where U is the atomic energy in a strong external magnetic field. U can be expanded in a power series of the electromagnetic fields [8]. The upper limit of validity of such an approximation is not discussed in literature, but the comparison between measurements and theoretical values obtained using this expansion indicates that it is certainly valid for magnetic fields of several Teslas (see e.g. ref. [8]). We will assume in the following that it is also valid for higher fields.

It is important to stress that, as shown in ref. [21], in the case of CME the effect of the interaction of the magnetic field associated with the propagating wave with the atomic or molecular system is very small compared to the main effect induced by the electric field of the wave and is usually neglected. We will assume in the following that the same applies to ICME. This is not the case for the quantum vacuum as clearly shown by Eqs. (15) and (16).

We are looking for an atomic magnetic moment which depends linearly on the external magnetic field and quadratically on the electric field. Because of Eq. 19, this kind of induced magnetic moment can only be obtained by derivating the term of the U series quadratic in the electric and magnetic fields:

$$U_\eta = -\frac{1}{4}\eta_{\alpha\beta,\gamma\delta}E_\alpha E_\beta B_\gamma B_\delta \quad (20)$$

where η is the second hypermagnetizability tensor, Einstein summation is assumed and $(\alpha, \beta, \gamma, \delta) = x, y, z$.

To obtain the magnetization \mathbf{M}^{at} we have to multiply the atomic magnetic moment μ_{at} by the atom density which for ideal gases is equal to P/kT , where P is the gas pressure, k the Boltzmann constant, and T the temperature. Taking into account the two possible cases as for the quantum vacuum, we finally obtain:

$$\mathbf{M}_{\text{ICM}\parallel,\perp}^{\text{at}} = \frac{1}{2} \frac{P}{kT} \eta_{\parallel,\perp} E_\omega^2 \mathbf{B}_0 \quad (21)$$

where $\eta_{\parallel(\perp)}$ is the component of the η tensor parallel (perpendicular) to \mathbf{B}_0 . These two components are related to the Cotton-Mouton effect in atoms [8] since the magnetic induced birefringence $\Delta n = n_{\parallel} - n_{\perp}$ is proportional to $(\eta_{\parallel} - \eta_{\perp})$. In both cases \mathbf{M}_{ICM} is parallel or antiparallel to \mathbf{B}_0 depending on the sign of the η component. Our theoretical result is equivalent to the one given in Ref. [9].

Formula (21) can also be written as:

$$\mathbf{M}_{\text{ICM}\parallel,\perp}^{\text{at}} = \frac{1}{2} \frac{P}{kT} \eta_{\parallel,\perp} Z_0 I \mathbf{B}_0 \quad (22)$$

where $Z_0 = \sqrt{\frac{\mu_0}{\epsilon_0}} = 377 \Omega$ is the vacuum impedance.

To get a numerical estimation of $\mathbf{M}_{\text{ICM}}^{\text{at}}$ in Tesla units, let's write Eq. (22) as follows:

$$\mathbf{M}_{\text{ICM}\parallel,\perp}^{\text{at}} \simeq 5.1 \times 10^{-28} \frac{P}{T} \eta_{\parallel,\perp} I B_0 \quad (23)$$

where P is given in atm, T in K, $\eta_{\parallel,\perp}$ in atomic units (au in the following), I in W/m^2 , B_0 in T, and the resulting $\mathbf{M}_{\text{ICM}\parallel,\perp}^{\text{at}}$ is also given in T. Let's also recall that 1 $\eta(\text{au})$ is equal to $2.98425 \times 10^{-52} \text{C}^2 \text{m}^2 \text{J}^{-1} \text{T}^{-2}$ [8].

Theoretical values of $\eta_{\parallel,\perp}$ for noble gases can be found in Ref. [22]. In table 1 we summarize our results obtained assuming that $P = 1$ atm, $T = 300$ K, and that $I = 10^{19} \text{W}/\text{m}^2$, $B = 10$ T like in Ref. [6].

Comparing results of table 1 with results for quantum vacuum given by Eqs. (17) and (18), one obviously finds

Table 1: Expected values of ICME magnetization for noble gases for $P = 1$ atm, $T = 300$ K, $I = 10^{19} \text{W}/\text{m}^2$, $B = 10$ T.

gas	$\eta_{\parallel}(\text{au})$	$\mathbf{M}_{\text{ICM}\parallel}^{\text{at}}(\text{T})$	$\eta_{\perp}(\text{au})$	$\mathbf{M}_{\text{ICM}\perp}^{\text{at}}(\text{T})$
He	-1.213	-2.1×10^{-10}	-2.1668	-3.8×10^{-10}
Ne	-2.040	-3.5×10^{-10}	-4.254	-7.4×10^{-10}
Ar	-18.84	-3.2×10^{-9}	-41.21	-7.1×10^{-9}
Kr	-38.11	-6.6×10^{-9}	-86.72	-1.5×10^{-8}
Xe	-83.10	-1.4×10^{-8}	-200.85	-3.4×10^{-8}

that the effect in gases is many orders of magnitude bigger than the one predicted for quantum vacuum. On the other hand in the case of gases one cannot increase the laser intensity arbitrarily because of gases ionization. Laser ionization of noble gases has been studied in Ref. [23] at $\lambda = 1.053 \mu\text{m}$. A systematic scan of intensities from $10^{17} \text{W}/\text{m}^2$ to $10^{20} \text{W}/\text{m}^2$ was performed. Ionization appears at different intensities depending on the noble gas. For Helium and Neon ionization begins around $10^{19} \text{W}/\text{m}^2$, for Argon and Krypton around $10^{18} \text{W}/\text{m}^2$, and for Xenon around $10^{17} \text{W}/\text{m}^2$. Ion production rate is of the order of a few tens of ions at the intensities given before for a gas pressure of a few 10^{-9} atm. The consequent ion current could somewhat perturb the ICME measurement.

Result shown in table 1 places Helium at the limit of which is detectable with existing facilities, and it also shows that ICME of other noble gases like Ne and Ar could be observed for the first time. It is also important to notice that ICME could allow to measure η_{\parallel} and η_{\perp} separately, while CME gives only access to the difference of the two.

In conclusion, in this letter we show that both ICME of quantum vacuum and ICME of atomic species can be measured using near-future or existing laser facilities opening the way to the observation of a new phenomenon in dilute matter.

We thank A. Ben-Amar Baranga and A. Rizzo for carefully reading the manuscript and very useful discussions.

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