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Robust H_∞ Fault Diagnosis for Multi-Model Descriptor Systems: A Multi-Objective Approach

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***Abstract*— A large class of engineering systems are modeled by coupled Differential and algebraic Equations (DAEs), called also singular or descriptor systems. Due to the singular nature of the algebraic equations, descriptor systems do not satisfy the standard state-space description and require special techniques. So far, the literature has concentrated mostly on the numerical analysis and control aspects of descriptor or DAEs systems. This paper investigates the problem of faults detection in nonlinear DAE systems described by a multi-model with the design of an Unknown Input Observer. The stability and robustness properties of the fault diagnosis scheme are investigated in term of LMIs. A simulation example illustrating the ability of the proposed fault diagnosis architecture to detect multiple faults is presented.**

***Index Terms*— Nonlinear descriptor systems, multi-model, fault diagnosis, LMIs.**

I. Introduction

It is well recognized that many practical dynamical systems are subject to various environmental changes, unknown disturbances, and changing operating conditions, thus *sensors/ actuators/ components* failure and faults in those systems are inevitable. Since any *faults/ failures* in a dynamical system may lead to significant performance degradation, serious system damages, and even loss of human life, it is essential to be able to detect and identify faults and failures in a timely manner so that necessary protective measures can be taken in advance. To that end, fault diagnosis of dynamic systems has received much attention and significant progress has been made in recent years in searching for model-based diagnosis techniques. Much attention has been devoted to the development of robust fault-detection methods under external disturbances for continuous time systems modeled via Ordinary Differential Equations (ODE)[14], [4], [12].

In contrary, rare are the methods dealing with the systems consisting of both Differential and Algebraic Equations (DAE), called also, differential-algebraic systems, implicit differential equations, singulars or descriptor systems. The field of the DAE systems was the witness of much activity of research in a variety of axis.

Indeed, many work of fault diagnosis for descriptor systems have been developed [6], [8], [1].

However, robust fault detection is still an open problem for further research. One of the most important approaches for robust fault detection is the use of disturbance decoupling principles. The aim of this approach is to completely cancel the disturbance effect from the residual. Nonetheless, the complete elimination of disturbance effects may not be possible due to the lack of design freedom. Moreover, it may be problematic, in some cases, because the fault effect may also be eliminated. Hence an appropriate criterion for robust residual design should be to take into account the effects of both disturbance and faults. There is a compromise between sensitivity to faults and robustness to disturbance. Robust residual generation can be then considered as a *multi-objective optimization problem*, i.e, the maximization of fault effects and the minimization of disturbance effects. The multi-objective problem in time domain for ordinary systems is studied by Chen et al [2]. On the other hands, the residual generator design can become very delicate even impossible according to the type and the complexity of the employed model, from where the importance to have a mathematical model of the system where it's at the same time, simple and precise. The multi-model approach is a powerful technique of modeling of nonlinear systems which makes it possible to get a good compromise between the precision and the complexity of the model [13]. The multi-models are recognized for their capacity to take into account the changes in the operating mode of the system and to reproduce its behavior with precision in a broad operating range. Moreover, they offer mathematical properties which can be made profitable during the design of the observer. These properties hold in particular, the extension to the nonlinear systems, a certain results obtained for the linear systems and this, without to have carried out a specific analysis of non-linearity of the system.

The multi-model representation was generalized to the singular systems with [15] and [10]. The stability and the design of the controllers for multi-model singular systems are characterized by the LMIs in [17], the formalism of the singular systems is employed also by [16] for the H_∞ filter design. The problem of Fault Diagnosis and Isolation (FDI) of LPV descriptor systems was studied in [5].

This paper considers the problem of the robust faults diagnosis for the nonlinear process modeled by a multi-model descriptor system. The presented method is an extension of the work of Ichalal et al. [7] for the ordinary nonlinear systems. First of all, the following process consists in obtaining a more interesting representation of the nonlinear dynamic systems. This representation is based on the interpolation of the linear local models, representing the local behaviors of the nonlinear system, by weighings functions. Then, an unknown inputs multi-observer approach is developed for robust residual generator and fault diagnosis which minimizes the sensitivity to the disturbances and maximizes the sensitivity to the faults. The existence and stability conditions of the residual generator are also studied.

The paper is organized as follows, section II formulates the problem of this note. The design of the residual generation is considered in section III. The study of stability is also introduced. It is based on the design of LMIs. An illustrative example is introduced into the section IV to validate the design suggested.

II. Problem Formulation

Consider the following continuous-time nonlinear descriptor system:

$$\begin{cases} E\dot{x}(t) = F(x(t), u(t), d(t)) \\ y(t) = Cx(t) \end{cases} \tag{1}$$

where $x(t) \in \mathbf{R}^n$ is the singular state vector, $u(t) \in \mathbf{R}^p$ $(p \leq n)$ is the input vector, $d(t) \in \mathbf{R}^q$ is a disturbance vector and $y(t) \in \mathbf{R}^m$ is the output vector. $F(.)$ is continuous and indefinitely derivable nonlinear function. $rank(E) = r$ and E is a singular matrix with constant parameters. The linearization of the function $F(.)$ by Taylor series around h operating points(x_i, u_i) gives a set of local linear singular model as follow:

$$\begin{cases} E\dot{x}(t) = A_i x(t) + B_i u(t) + R_i d(t) + \Delta x_i \\ y(t) = Cx(t) \end{cases} \tag{2}$$

$A_i, B_i,$ and C are jacobian matrices relating to the ith operating point. R_i is a disturbance matrix and Δx_i is a vector depend on the ith operating point. Then, the nonlinear system represented by (1) can be written according to the multi-model structure [12] (with h local models) as presented below:

$$\begin{cases} E\dot{x}(t) = \sum\limits_{i=1}^{h} h_i(\xi(t))(A_i x(t) + B_i u(t) + R_i d(t) + \Delta x_i) \\ y(t) = Cx(t) \end{cases} \tag{3}$$

The $h_i(\xi(t))$ are the weighting functions that measure the relative contribution of each local model to build the global model. The function $h_i(\xi(t))$ checks the property of the convex sum.

$$\begin{cases} \sum\limits_{i=1}^{h} h_i(\xi(t)) = 1 \\ 0 \leq h_i(\xi(t)) \leq 1 \end{cases} \tag{4}$$

The multi-model structure makes it possible to represent any non-linear behavior. It provides a mean of generalization of the tools developed for the linear systems to the nonlinear systems. In order to do fault diagnosis of nonlinear systems by using the multi-model approach, one considers the singular multi-model system subject to the fault $f(t)$ and the disturbances $d(t)$ given by:

$$\begin{cases} E\dot{x}(t) = \sum\limits_{i=1}^{h} h_i(\xi(t))(A_i x(t) + B_i u(t) + R_i d(t) + F_i f(t) + \Delta x_i) \\ y(t) = Cx(t) + Hf(t) \end{cases} \tag{5}$$

where $f(t) \in \mathbf{R}^{nf}$ is a fault vector. It is of interest to notice that since H is non null, the approach is more generic than for instance in [5]. The singular multi-model (5) is observable [3] if and only if:

$$\text{rank} \begin{bmatrix} sE - A_i \\ C_i \end{bmatrix} = \text{n} \tag{6a}$$

$$\text{rank} \begin{bmatrix} E & A_i \\ 0 & E \\ 0 & C \end{bmatrix} = n + rank(E), \forall\, i = 1, \ldots, h \tag{6b}$$

The fault diagnosis of nonlinear singular systems were rarely been approached. Arun et al. [1], present a method of faults diagnosis based on the use of the online estimators for the dynamic systems described by nonlinear differentials-algebraic equations. In Marx et al. [10], a design method of unknown inputs observers for Takagi-Sugeno descriptor systems was proposed. In this work, the authors supposed that the activations functions depend on the measurable variables. The obtained observer was used in faults detection and isolation. The majority of the dedicated works for fault diagnosis in singular systems concerns the residual generation by the nonsingular unknown inputs observers. In this work, one proposes a method of faults detection for nonlinear singular systems represented by a multi-model. The residual generation is built according to a standard H_∞ design.

III. The residual generation design

In this section, one addresses the design of a residual generator based on an unknown input observer for nonlinear singular systems represented by a multi-model. Let us consider the following singular multi-model given by (5), the global model of the residual generator is defined by:

$$\begin{cases} \dot{z}(t) = \sum\limits_{i=1}^{h} h_i(\xi(t))(N_i z(t) + G_i u(t) + L_i y(t) + \Delta z_i) \\ \hat{x}(t) = z(t) + T_2 y(t) \\ r(t) = M(y(t) - \hat{y}(t)) \end{cases} \tag{7}$$

where $z(t) \in R^n$ is the observer state vector and $r(t) \in R^{nf}$ is the residual vector. The unknown matrices N_i, G_i, Δz_i, L_i, T_2 and M are of appropriate dimensions. The objective is to find out the gains of the residual generator in order to maximize the transfer of the faults towards the error. The estimation error is defined as: $e(t) = x(t) - \hat{x}(t)$

$$e(t) = x(t) - z(t) - T_2 y(t)$$
$$e(t) = (I_n - T_2 C)x(t) - z(t) - T_2 H f(t)$$

Let the matrix $T_1 \in R^{n\times n}$ such that:

$$T_1 E = I_n - T_2 C \tag{8}$$

So, for $\begin{bmatrix} E \\ C \end{bmatrix}$ is full rank column, we have,

$$\begin{bmatrix} T_1 \; T_2 \end{bmatrix} \begin{bmatrix} E \\ C \end{bmatrix} = I_n \quad \text{then} \tag{9}$$

$$\begin{bmatrix} T_1 \; T_2 \end{bmatrix} = \begin{bmatrix} E \\ C \end{bmatrix}^{+} \tag{10}$$

Now, we assume that the faults are with slow variation, i.e $\dot{f}(t) \simeq 0$. Then, using (5) and (7), the dynamics error is:

$$\begin{aligned}
\dot{e}(t) &= T_1 E \dot{x}(t) - \dot{z}(t) \\
\dot{e}(t) &= \sum_{i=1}^{h} h_i(\xi(t)) \{T_1(A_i x(t) + B_i u(t) + R_i d(t) + F_i f(t) \\
&\quad + \Delta \mathrm{x_i}) - (N_i z(t) + G_i u(t) + L_i y(t) + \Delta z_i)\} \\
\dot{e}(t) &= \sum_{i=1}^{h} h_i(\xi(t)) \{N_i e(t) + (T_1 A_i - N_i T_1 E - L_i C) x(t) \\
&\quad + (T_1 B_i - G_i) u(t) + T_1 R_i d(t) \\
&\quad + (T_1 F_i - L_i H) f(t) + T_1 \Delta x_i - \Delta z_i\}
\end{aligned} \tag{11}$$

Moreover, the residual equation can be written such that:

$$\begin{aligned}
r(t) &= M(y(t) - \hat{y}(t)) \\
r(t) &= \sum_{i=1}^{h} h_i(\xi(t))(M\{Cx(t) + Hf(t) - C\hat{x}(t)\})
\end{aligned}$$

By using the static error equation and the relations (5), the residual is written as:

$$r(t) = \sum_{i=1}^{h} h_i(\xi(t))(\{MCe(t) + MHf(t)\}) \tag{12}$$

Then, if the constraints

$$T_1 A_i - N_i T_1 E - L_i C = 0 \tag{13a}$$

$$T_1 B_i - G_i = 0 \tag{13b}$$

$$\Delta z_i = T_1 \Delta x_i \tag{13c}$$

$$\tag{13d}$$

are checked, so the relations (11) and (12) become:

$$\begin{aligned}
\dot{e}(t) = \sum_{i=1}^{h} h_i(\xi(t)) \{N_i e(t) + T_1 R_i d(t) \\
+ (T_1 F_i - L_i H) f(t)\}
\end{aligned} \tag{14a}$$

$$r(t) = \sum_{i=1}^{h} h_i(\xi(t))(\{MCe(t) + MHf(t)\}) \tag{14b}$$

By using the equalities (8) and (13a);

$$N_i = T_1 A_i + K_i C \tag{15a}$$

$$\text{where} \quad K_i = N_i T_2 - L_i \tag{15b}$$

From where, the relations (14a) and (14b) are written as follows:

$$\begin{aligned}
\dot{e}(t) = \sum_{i=1}^{h} h_i(\xi(t)) \{(T_1 A_i + K_i C) e(t) + T_1 R_i d(t) \\
+ (T_1(F_i - A_i T_2 H) - K_i(C T_2 H - H)) f(t)\}
\end{aligned} \tag{16a}$$

$$r(t) = \sum_{i=1}^{h} h_i(\xi(t))(\{MCe(t) + MHf(t)\}) \tag{16b}$$

Therefore, the transfer matrices from the disturbance to the residual and from the fault to the residual (16) are given by:

$$r(t) = \sum_{i=1}^{h} h_i(\xi(t))(G_{rd}^i d(t) + G_{rf}^i f(t)) \tag{17}$$

A state-space realization of the ith transfer matrices G_{rd}^i and G_{rf}^i is denoted as:

$$G_{rf}^i = \left(\frac{(T_1 A_i + K_i C) |}{MC \quad |} \frac{\Upsilon_i}{MH} \right) \tag{18a}$$

$$G_{rd}^i = \left(\frac{(T_1 A_i + K_i C) |}{MC \quad |} \frac{T_1 R_i}{0} \right) \tag{18b}$$

where $\Upsilon_i = T_1(F_i - A_i T_2 H) - K_i(C T_2 H - H)$

Both faults and disturbances affect the residual and discrimination between these two effects is difficult. To reduce false and missed alarm rates, the effect of faults on the residual should be maximized and the effect of disturbances on the residual should be minimized. To avoid this conflict, the effect of the fault $f(t)$ on the residual $r(t)$ can be expressed like a problem of minimization. Indeed, by introduction of a weighting parameter W_f^i, the problem is reduced to a minimization of the effect of the fault on the residual error:

$$\tilde{r}(t) = \sum_{i=1}^{h} h_i(\xi(t))(r(t) - W_f^i f(t)) \tag{19}$$

W_f^i is a weighting parameter which makes it possible to take account the knowledge of the fault distribution. The detection and isolation of the fault depend on the selected structure of W_f^i . Indeed, the estimated faults are obtained when $W_f^i = I$ and the problem of detection is considered when $W_f^i \in R^{1\times nf}$.

The problem of faults detection and isolation can be formulated into the form of multi-objective optimization problem which consists in determining the matrices K_i and M which minimize $a\gamma_f + (1-a)\gamma_d$ under the following constraints:

$$\left\| G_{rf}^i - W_f^i \right\|_\infty < \gamma_f \tag{20a}$$

$$\left\| G_{rd}^i \right\|_\infty < \gamma_d \tag{20b}$$

$$\text{the system (16) is stable} \tag{20c}$$

where $a \in [0 \quad 1]$. Note that, the condition (20a) measures the deviation of the faults compared to the residual with the parameter of weighting functions, while the condition (20b) represents the robust decoupling of the disturbances compared to the residual.

A. Linear Matrices Inequalities Formulation

In this subsection, the main idea is to express the constraints (20) into the form of a LMIs problem optimization where W_f^i is a dynamic weighting parameter defined by:

$$W_f^i = \left(\frac{A_{if} |}{C_{if} \,|} \frac{B_{if}}{D_{if}} \right) \tag{21}$$

where $W_f^i \in \Im$. $\Im$ is the whole of the stable transfer matrices such as [11]:

$$\left\| W_f^i \right\|_- = \inf_{w \in R} (W_f^i(jw)) \geq 1 \tag{22}$$

Then, the expression $G^i_{rf} - W^i_f$ can be rewritten in the following form:

$$G^i_{rf} - W^i_f = \left(\begin{array}{cc|c} (T_1A_i + K_iC) & 0 & \Upsilon_i \\ 0 & A_{if} & B_{if} \\ \hline MC & -C_{if} & MH - D_{if} \end{array}\right) \quad (23)$$

The residual generator (7) problem design is solved by the following theorem.

Theorem 1: [7] For $a \in [0 \quad 1]$ a real positive parameter and $W^i_f \in \Im$ a weighing function, the residual generator (7) exists, if there exist two definite positive matrices $Q_1 = Q_1^T$ and $Q_2 = Q_2^T$, gain matrices K_i, M and positive reals scalars γ_f and γ_d solution of the following optimization problem

$$\min_{M,\, K_i,\, Q_1,\, Q_2, \gamma_f,\, \gamma_d} a\gamma_f + (1-a)\gamma_d \quad (24)$$

subject to:

$$\begin{pmatrix} \Pi_i & 0 & \Psi_i & (MC)^T \\ 0 & A^T_{if}Q_2 + Q_2A_{if} & Q_2B_{if} & -C^T_{if} \\ \Psi^T_i & B^T_{if}Q_2 & -\gamma_f^2 I & \bar{D}^T_i \\ MC & -C_{if} & \bar{D}_i & -I \end{pmatrix} < 0 \quad (25a)$$

$$\begin{pmatrix} \Pi_i & Q_1T_1R_i & (MC)^T \\ (T_1R_i)^TQ_1 & -\gamma_d^2 I & 0 \\ MC & 0 & -I \end{pmatrix} < 0 \quad (25b)$$

where

$$\Pi_i = (T_1A_i)^TQ_1 + Q_1(T_1A_i) + \Omega_iC + (\Omega_iC)^T \quad (26a)$$

$$\Psi_i = (Q_1T_1(F_i - A_iT_2H) - \Omega_i(CT_2H - H)) \quad (26b)$$

$$\bar{D}_i = MH - D_{if} \quad (26c)$$

$$\Omega_i = Q_1K_i \quad (26d)$$

□

Proof:
Let define a positive and symmetric bloc diagonal matrix:

$$Q = \left(\begin{array}{c|c} Q_1 & 0 \\ \hline 0 & Q_2 \end{array}\right)$$

Using the bounded-real lemma [9], the condition (20a) is formulated as follows:

$$\begin{pmatrix} \bar{A}^T_iQ_1 + Q_1\bar{A}_i & 0 & Q_1\bar{B}_i & (MC)^T \\ 0 & A^T_{if}Q_2 + Q_2A_{if} & Q_2B_{if} & -C^T_{if} \\ \bar{B}^T_iQ_1 & B^T_{if}Q_2 & -\gamma_f^2 I & \bar{D}^T_i \\ MC & -C_{if} & \bar{D}_i & -I \end{pmatrix} < 0$$

where:

$$\bar{A}_i = (T_1A + K_iC)$$
$$\bar{B}_i = (T_1(F_i - A_iT_2H) - K_i(CT_2H - H))$$

To resolve these LMIs, for $i = 1, ..., h$, one poses:
$\Omega_i = Q_1K_i$, then the LMIs (25a) are satisfied.
In the same manner, the application of the bounded real lemma to constraint (20b) by using the relation (18b), makes it possible to obtain LMIs (25b). The condition (20c) is ensured by the stability of the matrix Π_i in (25a). ■

To design robust residuals for multi-model descriptor systems, a multi-objective optimization problem needs was solved. This paper uses the method of inequalities to solve this multi-objective optimization problem. All objectives are reformulated into a set of inequality constraints on performance indices. This method is an extension of the FDI methodology proposed in [7], for ordinary multi-model to descriptor multi-model systems.

IV. Illustrative example

In order to illustrate the efficiency of the presented method, let consider the nonlinear descriptor system (1) defined by

$$\begin{cases} \dot{x}_1(t) = -1.5x_1^2(t) + 0.2x_3(t)x_4(t) + d(t) \\ \dot{x}_2(t) = -u_1(t)x_1^2(t) - x_4(t)x_3^2(t) - 0.5x_2(t) \\ 0 = 0.5x_2(t) - x_3(t) + 0.2x_4(t) \\ 0 = -x_2^2(t) + x_3^2(t) - 2x_4(t) + u_2(t) \\ \\ y(t) = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} x(t) \end{cases} \quad (28)$$

where $u_1(t)$ and $u_2(t)$ are constant signal of magnitude 10 and 7 respectively. $d(t)$ is gaussian noise of mean zeros and variance 0.1 applied for $8 \leq t \leq 11$.

A. Multi-Model Representation

The above nonlinear descriptor system is approximated by a multi-model representation as follows:

$$\begin{cases} E\dot{x}(t) = \sum_{i=1}^{3} h_i(x_3(t))(A_ix(t) + B_iu(t) + R_id(t) + \Delta x_i) \\ y(t) = Cx(t) \end{cases} \quad (29)$$

The numerical values of those parameters are as follows:

$$E = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, A_1 = \begin{bmatrix} -0.8775 & 0 & 0.526 & -0.0274 \\ -5.85 & -0.5 & 0.1481 & 0.0026 \\ 0 & 0.5 & -1 & 0.2 \\ 0 & 2.6522 & -0.274 & -2 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} -0.6375 & 0 & 0.6166 & 0.0307 \\ -4.2500 & -0.5 & 0.2176 & -0.0036 \\ 0 & 0.5 & -1. & 0.2 \\ 0 & 1.8526 & 0.3068 & -2 \end{bmatrix}, B_1 = \begin{bmatrix} 0 & 0 \\ -0.0856 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} -0.6357 & 0 & 0.6047 & 0.0226 \\ -4.2380 & -0.5 & 0.1162 & -0.0015 \\ 0 & 0.5 & -1. & 0.2 \\ 0 & 1.966 & 0.2264 & -2 \end{bmatrix}, B_2 = \begin{bmatrix} 0 & 0 \\ -0.0452 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}$$

$$B_3 = \begin{bmatrix} 0 & 0 \\ -0.0449 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}, R_i = R = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, C = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$\Delta x_1 = \begin{bmatrix} 0.2004 \\ 1.6753 \\ 0 \\ 1.7397 \end{bmatrix}, \Delta x_2 = \begin{bmatrix} -0.0268 \\ 0.8083 \\ 0 \\ 0.8346 \end{bmatrix}, \Delta x_3 = \begin{bmatrix} -0.0011 \\ 0.8505 \\ 0 \\ 0.9535 \end{bmatrix}$$

The weighting functions are:

$$h_i(x_3(t)) = \frac{\mu_i(x_3(t))}{\sum\limits_{i=1}^{3} \mu_i(x_3(t))}$$

where $\mu_i(x_3(t))$ are defined by:

$$\begin{array}{c} \mu_1(x_3(t)) = \exp(-1/2(\frac{x_3+5}{2})^2) \\ \mu_2(x_3(t)) = \exp(-1/2(\frac{x_3}{2})^2) \\ \mu_3(x_3(t)) = \exp(-1/2(\frac{x_3-5}{2})^2) \end{array}$$

In order to show the effectiveness of the used modeling method, the nonlinear state and here approximation by the multi-model approach are given in the following figures:

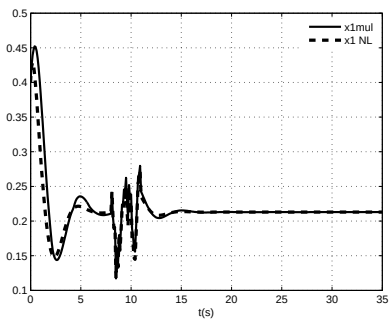

Fig. 1. $x_1(t)$ of nonlinear model and $x_{1mu}(t)$ of the multi-model

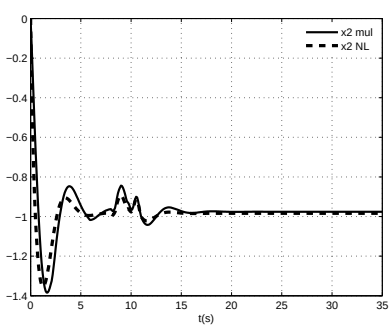

Fig. 2. $x_2(t)$ of nonlinear model and $x_{2mu}(t)$ of the multi-model

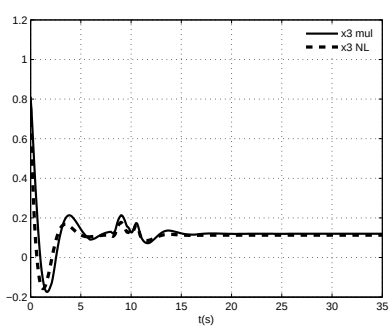

Fig. 3. $x_3(t)$ of nonlinear model and $x_{3mu}(t)$ of the multi-model

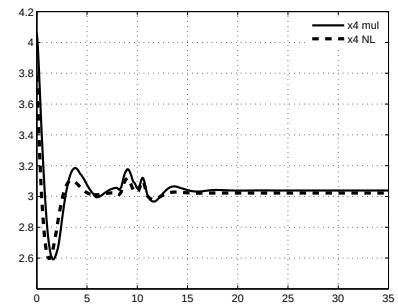

Fig. 4. $x_4(t)$ of nonlinear model and $x_{4mu}(t)$ of the multi-model

These figures illustrate the superposition of the nonlinear states with the states coming from of the multi-model representation. We can see that the Multi-Models well represent the nonlinear dynamic behavior.

B. Residual generation

Firstly, based on (5), the multi-model (29) becames:

$$\begin{cases} E\dot{x}(t) = \sum\limits_{i=1}^{3} h_i(x_3(t))(A_i x(t) + B_i u(t) + R_i d(t) + F f(t) + \Delta x_i) \\ y(t) = Cx(t) + Hf(t) \end{cases} \tag{30}$$

where $F_i = F$ and H are the actuator and sensor faults distribution matrices respectively. The decision variable $x_3(t)$ is considered free of faults. The first component of the vector $f(t)$ is a sensor fault and the second component is an actuator fault on the second input. These failures are given as:

$$f_1(t) = \begin{cases} 1.5 & if \quad 15 \leq t \leq 20s \\ 0 & \text{elsewhere} \end{cases}$$

$$f_2(t) = \begin{cases} 0.3u_2(t) & if \quad 25 \leq t \leq 30s \\ 0 & \text{elsewhere} \end{cases},$$

$$F = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}, H = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \end{bmatrix}$$

Then, to design the multi-model residual generation, for $i = 1, \ldots, h$, the constraints (6) are verified , the necessary assumptions to the diagnosis are thus satisfied. W_f is chosen to be a diagonal of first order low-pass filters as follows.

$$W_f = \left(\begin{array}{c|c} A_f & B_f \\ C_f & D_f \end{array} \right) = 0.17 \left(\begin{array}{cc|cc} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ \hline 0 & 0 & 1 & 1 \end{array} \right)$$

For each fault, a dedicated residual generator is designed with the following parameters which are obtained by resolving the minimization of the multi-objective problem given in Theorem (1). It results that for a chosen parameter $a = 0.9$ with positive reals scalars $\gamma_f = 2.1396$ and $\gamma_d = 1.9071$, the gains matrices K_i given as follows:

$$K_1 = \begin{bmatrix} -0.8473 & -2.3216 & 0.1307 \\ -1.6730 & 0.7307 & 0.0039 \\ 0.2612 & 0.2710 & -2.5090 \\ -1.9139 & -1.1541 & 0.1335 \end{bmatrix}$$

$$K_2 = \begin{bmatrix} -0.7355 & -2.1091 & 0.0345 \\ -1.6764 & 0.8449 & -0.0341 \\ 0.1542 & 0.1850 & -2.4650 \\ -1.7240 & -1.1420 & 0.1522 \end{bmatrix}$$

$$K_3 = \begin{bmatrix} -0.7390 & -2.1054 & 0.0373 \\ -1.6836 & 0.8443 & -0.0283 \\ 0.1563 & 0.1821 & -2.4648 \\ -1.7181 & -1.1435 & 0.1460 \end{bmatrix}$$

and a matrix $M = [-0.1765 \quad 0.4162 \quad 0.0180]$. The obtained residuals under a gaussian noise of variance 1, are displayed on figures (5) and (6). The residual $r_1(t)$ is sensitive only to the first fault (sensor fault) and the second residual $r_2(t)$ is only sensitive to the second fault (actuator fault).

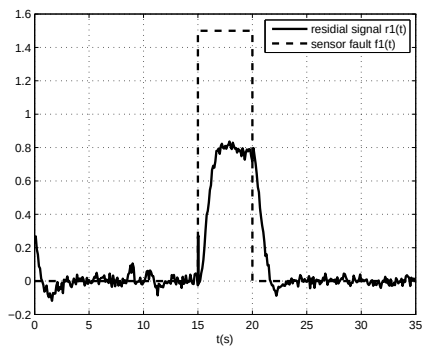

Fig. 5. Sensor fault and corresponding residual signal $r_1(t)$

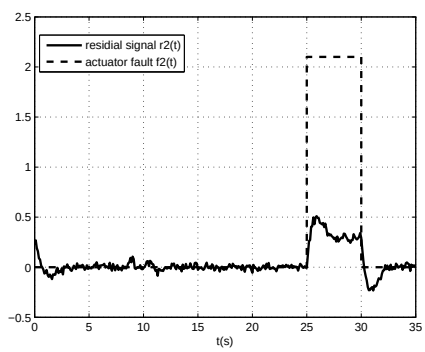

Fig. 6. Actuator fault and corresponding residual signal $r_2(t)$

From figure (5) and (6), it can be seen that the residual is almost zero throughout the time simulation run for fault-free residuals. The residuals of the respective unknown inputs observers increase in magnitude considerably, when actuator fault occur at t= 25s and sensor fault at t=15s. The faults can be easily isolated using the information provided by residuals.

V. Conclusion

The design of a robust multi-model residual generation based on unknown inputs multi-observer for descriptor systems has been studied. The existence condition and an LMI-based computation have been established. The residuals generation are optimally robust since they are designed in order to minimize the sensitivity of the residual. So, a multi-objective optimization problem is resolved. The introduction of weighting function ensures the performance of the detection in a prescribed frequency range. The given example illustrates the effectiveness of this approach.

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