Joint Opaque booking systems for online travel agencies
Malgorzata Ogonowska, Dominique Torre

To cite this version:
Malgorzata Ogonowska, Dominique Torre. Joint Opaque booking systems for online travel agencies. 2010. hal-00476150

HAL Id: hal-00476150
https://hal.science/hal-00476150
Preprint submitted on 23 Apr 2010

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L’archive ouverte pluridisciplinaire HAL, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d’enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.
Joint Opaque booking systems
for online travel agencies

Malgorzata OGONOWSKA* and Dominique TORRE*

March 2010

Abstract

This paper analyzes the properties of the advanced Opaque booking systems used by the online travel agencies in conjunction with their traditional transparent booking system. In section 2 we present an updated literature review. This review underlines the interest and the specificities of Opaque goods in the Tourism Industry. It also characterizes properties of the Name-Your-Own-Price (NYOP) channel introduced by Priceline and offering probabilistic goods to potential travelers. In the section 3 of the paper we present a theoretical model, in which we wonder what kind of Opaque system can be implemented by a given online monopoly. We compare the Opaque “Hotwire system”, a NYOP system without any possibility of rebidding and the joint implementation of these two systems. We find that the NYOP system and the joint implementation can have challenging properties if consumer’s information is complete. Then, in section 4, we analyze the case of incomplete information. We develop an appropriate setting to integrate the lack of complete information of potential passengers on their relative propensity to pay. We analyze three cases corresponding to different levels of uncertainty and number of tickets available. We find that in some relevant cases (average number of tickets, moderate uncertainty), the joint implementation of 2 different Opaque booking systems is advantageous for the Online travel Agencies (OTAs) and airlines. This result casts doubt on the current OTAs’ strategies.

JEL Classification: D49, L93
Keywords: Opaque Selling, Name-Your-Own-Price, Economics of Tourism, Online Travel Agencies, Probabilistic Goods.

1 Introduction

In this paper we analyze the advanced Opaque booking systems used by online travel agencies in conjunction with their traditional transparent booking system. This type of pricing system has been introduced since many years by online travel agencies in the

*University of Nice Sophia-Antipolis - GREDEG (DEMOS) - CNRS, 250 rue Albert Einstein, 06560 Valbonne, France. E-mail: malgorzata.ogonowska@gredeg.cnrs.fr, dominique.torre@gredeg.cnrs.fr
USA. They offer the consumers a price advantage, as a counterpart of opacity and uncertainty. Conversely, they provide the companies and intermediaries a good way to manage dynamically the fluctuations of demand function when supply is rigid in the short term. These new systems have already interested few specialists. On the one hand some papers focus on pricing system’s nature and their properties, and on the other hand - other ones interested in the reactions of demand to this pricing innovation. What are the advantages of using such pricing system for suppliers (carriers, airlines and tourism services providers), intermediaries (GDS, travel agencies) and consumers? What sort of competition or segmentation do they involve on the tourism services markets? Does the nature of tourism products change because of these new pricing systems reducing the available information? These points have been raised for each variant of these new Opaque systems. We present briefly the results of these works and the methodology that they have adopted. Then, we consider an important issue still not elucidated: since many variants of Opaque systems exist, is there an advantage to use simultaneously more than one distribution channel?

The literature review is presented in section 2. It underlines Opaque product specificities and advantages for Tourism Industry. It also characterizes properties of the NYOP channel introduced by Priceline and offering probabilistic goods to potential travelers. In section 3, we present a theoretical model, in which we wonder what kind of Opaque system can be implemented by a given online monopoly. We compare an Opaque posted-price “Hotwire system”, a NYOP system without any possibility of rebidding and the joint implementation of these two systems. We find that when information is imperfect (the travelers do not know the nature and the number of tickets available) but complete (the Opaque system’s potential clients know their number and their respective propensity to pay), it is equivalent to implement the most efficient system (the NYOP system) or both of the systems in parallel. We introduce in section 4 an assumption of incomplete information (the travelers know their number but not their relative propensity to pay). In this case, we find that, under moderate uncertainty, the joint implementation of two booking systems dominates the implementation of the NYOP channel only.

2 NYOP and Opaque products: a literature review

In the last years, the emergence of the Internet has deeply changed the industry of tourism, the organization of markets and the pricing mechanisms developed by firms. Tourism is by far the most developed and innovative online business, fostered by the creation of online travel agencies (OTAs) of different kinds and sophisticated pricing and segmentation strategies. Dominant global OTAs have emerged, Expedia, Travelocity, Orbitz, Opodo, which dominate the distribution of travel and tourism services, but the extensive uses of the Internet have given rise to niche players. Most of these players have specialized in specific segment of the market, in terms of destination or services, but some others have been more inventive, experimenting innovative pricing models. Hotwire.com (acquired by Expedia in 2003) and Priceline.com are the two most important companies having successfully developed this strategy on the US market, to account for 6.7% of worldwide online hotel bookings in 2006 for instance. They have developed
online pricing mechanisms such as Name-Your-Own-Price in which instead of posting a price, the seller waits for an offer of the potential buyer that he can then either accept or reject, or such as Opaque offers, in which the characteristics of the services are hidden (hotel or airlines brands, travel schedule). These empirical developments open many different questions. Why would hotels and airline companies be willing to sell their products through Priceline/Hotwire and lose the advantage (and profit) that product differentiation gives them (Shapiro and Shi, 2008)? Why firms would deviate from the standard practices of posting a take-it-or-leave-it offer? Certainly firms should find these strategies more profitable. But as pointed out by Pinker et al. (2003), and underlined in Wilson and Zhang (2008), ‘though on-line auctions are a multi-billion dollar annual activity, with a growing variety of sophisticated trading mechanisms, scientific research on them is at an early stage’. Nevertheless, some interesting advances can be traced in the recent literature, related to the innovative strategies implemented by Priceline or Hotwire. This short review focuses on the two main types of related literature which have been developed. The first one analyses Name-Your-Own-Price selling mechanism, while the second focuses on the Opaque selling with posted prices.

2.1 Name-Your-Own-Price selling mechanism

Wilson and Zhang (2008) present a model of a NYOP intermediary, who sells economy car rentals on a specific date. The retailer capacity is in excess. He provides consumers with a function that describes the chance of a bid to get accepted. Intermediary clients are limited to a single bid. The retailer objective is to force the consumers to bid maximum that will maximize his profits, while consumers intend to maximize their own surpluses. Consequently, the intermediary will choose an appropriate function of bid’s probability of success so as his clients will bid maximum. Since he provides this function, which is the same for all consumers, each of them is treated fairly, even if the prices they pay are different. Using different method (experimental economics), Shapiro and Zillante (2007) analyze seller profits maximization. They emphasize that their importance is a result of a trade-off between the number of bids accepted and their amounts, which depends on the threshold price and on the presence of opacity. They outline that concealing some of good information is detrimental for consumers and does not change anything for the seller, unless the threshold price is too low. In such case his profits will decrease. On the opposite, Wang, Gal-Or and Chatterjee (2005) show that a moderate opacity level can be profitable for the retailer. Indeed, it helps to segment the demand and though to price discriminate the consumers. It will attract some supplementary clients without creating cannibalisation effects of the posted-price channel. The paper considers a monopoly service provider distributing a fixed capacity through its own web site using posted-price mechanism and through a NYOP intermediary during a two stage game. He faces an uncertain and heterogeneous demand. He perceives a signal of the state of the demand after the first stage of the game. On one hand, if the signal is perfectly or highly informative, the uncertainty almost disappears. Then, the market segmentation is only feasible and profitable with sufficiently low or high capacity. On the other hand, if the signal provides no information about the demand and if capacity is high, the service provider will use only the posted-price channel; otherwise, he will use only the NYOP channel. The optimal precision of demand signal is an intermediate value, what means
that some uncertainty remains.

Hann and Terwiesch (2003) identify a double source of profits for the NYOP retailer: intermediary margin, which is defined as a difference between the price paid for the product to a service provider and the threshold price, and the informational margin, which corresponds to the difference between the price paid by a consumer and the threshold price. The customers are heterogeneous in their experience and though in the level of frictional and transaction costs that they afford. This heterogeneity leads to a market segmentation that allows the retailer to price discriminate his clients and improve its profits. Fay (2008) also demonstrates that frictional costs have an influence on market segmentation and on seller profits. Because of market segmentation, price competition on the market is reduced. The duopoly model developed in the paper emphasizes that implementation of a NYOP channel will reduce the competition and improve overall profits in comparison to the situation when both retailers choose posted-price market format. The model stresses that if one of competitors chooses NYOP format, while his rival selects posted-price market, he should restrict its customers to only one bid. Because repeated bidding increases consumer's interest for NYOP selling, it causes a loss of posted-price seller profits, who, in order to attract some consumers, decreases his price, what derives the bid's amounts down. In contrast, Spann, Skiera and Schäfers (2004) show that allowing consumers to repeat their bids may improve seller profits, because the possibility of rebidding leads to higher amounts of maximum bids.

Another type of consumer's heterogeneity is presented by Fay (2004). Some consumers, called "sophisticated" manage to bypass the restriction of a single bid and the others - do not. This creates a new segmentation of demands. The paper compares three situations: single bid, repeat bidding and partial-repeat bidding. The model demonstrates that intermediary profits are exactly the same if the restriction of single bid is kept up or if it is not imposed. On the opposite, partial-repeat bidding deteriorates retailer profits, but this relation is not monotonic. On one hand, if the number of sophisticated consumers is very low, firm’s profits will reduce as their number increases. On the other hand, if their percentage is very important, profits will increase with their number. Therefore, this paper gives the guidelines how to well implement a NYOP strategy in order to better segment the demand and thus reduce price competition. Terwiesch, Savin and Hann (2005) present demand segmentation based on differences in haggling costs occurred by consumers. Retailer can price discriminate his clients, because of this differential. They provide a model of an online haggling process at a NYOP seller web site with no opacity, constant threshold price and possibility of rebidding. Retailer can manipulate consumer’s haggling cost by complicating his website interface or by modifying the time delay with which the customer is notified that his bid was rejected, in order to diminish cannibalisation effect of haggling. Thus, seller profits increase when consumers are sufficiently heterogeneous and if there is a positive correlation between their valuations and haggling costs.

Fay and Laran (2008) add an original idea to the literature on NYOP mechanism.

\footnote{It is the only existing paper considering the case of competition and NYOP selling.}
They analyze the situation, where the threshold price varies under repeat bidding. Every consumer’s bid rejection provides him with new information. If he expects that the threshold price is constant, his bidding pattern is monotonically increasing. However, if he suppose that the threshold price will vary, his bidding behavior will depend on the degree of expected variability and on his patience. The paper’s main implication is that changing threshold price may attract and retain more customers.

Spann, Banhardt, Häubl and Skiera (2005) compare the NYOP format with Select-Your-Price (SYP) mechanism, where consumers are influenced by the range of possible candidate bids. Providing a list of possible bids may be perceived as a format giving more information about the seller threshold price and thus decreases customer’s uncertainty about product’s value. The median and mean bids are the lowest in the NYOP format, so it is dominated by the SYP one. As the threshold price increases, seller profits raise monotonically. They are at their maximum, when the threshold price is equal to variable cost. However, when the candidate bids are high, the profits depend on the tradeoff between the increase in bid’s amounts and the reduction in the number of placed bids.

2.2 Opaque Products

The second type of literature analyses another type of Opaque selling, where prices are posted. These papers focus on the fact that some of the product’s attributes or characteristics are concealed from the consumers. In the traditional channels it was already not always beneficial to fully inform consumers about market prices, because of the risk of increase of their price sensitivity and then of creation of downward price pressures. In that case it is beneficial for service providers to implement multichannel distribution across the mechanisms with different levels of market transparency. Grandos, Gupta and Kauffman (2008) present a model of a supplier, who distributes his product over two online channels, differentiated by the levels of market transparency, characterized by the same marginal distribution costs. They provide mechanisms for a supplier to set optimally the prices and to influence transparency in order to successfully price discriminate his clients. First, they estimate a demand function of the product, then identify the differences in the demand functions across the two online selling mechanisms and finally, set the optimal prices based on those differences. Empirical analysis presented in the paper confirms the model’s results and provides an additional outcome, which states that a supplier in order to increase its revenues can increase the price differential across the selling mechanisms.

Y. Jiang (2007) models a monopoly who distributes tourism devices (airline tickets, hotel rooms) on two types of markets: full-informational and Opaque, and wonders are the consequences of the use of the Opaque channel on firm’s profits and the global welfare. He defines also the conditions of successful implementation of price discrimination. The firm’s profits as well as the overall welfare are greater while serving only the full-informational market. Things get more complicated when the firm decides to serve both of the markets. The firm’s strategy will depend on the degree of homogeneity of demand. If the demand is too homogeneous or too heterogeneous, the monopoly will choose to serve only the full-informational market, because of the risk of cannibalisation.
effect. When the demand is heterogeneous enough, the two types of market will co-exist. The dual-market strategy will improve firm’s profits, by reducing the unsold inventory and social welfare, by serving some extremely price sensitive consumers, who would not travel otherwise. This result is confirmed by Shi and Shapiro (2008), who wonder why the service providers decide to sell their products through Opaque sites and though lose the advantages given by product differentiation; for the consumers Opaque products are indistinguishable and become perfect substitutes. First of all, selling through the Opaque channel helps the service providers to respond to changes in demand without the need to change current branding and pricing policies. In the model the Opaque travel agency act as a "collusion device" which facilitates price discrimination between different types of consumers and increases overall profits, even if the total market demand is perfectly inelastic. The model is a variation of Hotelling’s (1929) and Salop’s (1979) models. The paper’s main result is that for a certain range of parameters values distribution through an Opaque agency enables hotels to discriminate their customers. Without the Opaque agency the hotels would compete for both high and low type consumers on the non-Opaque channel. The presence of low-type consumers intensifies the competition and drives down the equilibrium price and profits. When the Opaque channel is introduced a new equilibrium arises, derived from the Bertrand competition model. In the new equilibrium, hotel’s competition for the low-type consumers increases, but decreases for the high-type ones. It still remains a Hotelling competition, but the hotels do no longer compete for the low-type segment. If there are enough high-type consumers, the overall profits will increase. The intensified competition for the low-type consumers enables hotels to decrease the competition for the high type ones. Another paper, analyzing the case of competition, is the Fay’s (2007) one. He models a duopoly competition with multiple service providers who use a common intermediary. The paper introduces brand loyalty. If there is little brand loyalty in the market, the introduction of Opaque sales will raise price competition and lower industry profits. If there is sufficient brand loyalty, the Opaque sales will reduce price competition and raise the industry profits. The degree of price competition will depend on the number of units allocated to the Opaque channel. Service providers have an incentive to contract with the Opaque intermediary, if there is enough brand-loyalty in the industry. One of the model’s hypotheses is that the firms have no constraints, so the Opaque channel will lead to market expansion.

Opaque products can be seen as probabilistic goods, as emphasized by Fay and Xie (2007). They define a probabilistic good as a gamble involving a probability of getting any one of a set of multiple distinct items. Accordingly, we speak about probabilistic selling, when a seller creates a probabilistic good using existing distinct products or services (called component goods), which he offers as additional purchase possibility. Consequently, implementation of probabilistic selling helps the retailer to segment the market by creating a new different type of consumer’s uncertainty. Thus, the retailer implements price discrimination that can considerably increase his profits if marginal costs are sufficiently low. When there is an advantage from introducing a probabilistic good, it is generally optimal to assign an equal probability to each component products, even if the demand is asymmetric. On the opposite cannibalisation effects may appear. Moreover,

\[2\] Idea developed also in Fay(2008) concerning the NYOP channel.
probabilistic selling is most advantageous when the component goods have moderate
differences. An important advantage of probabilistic selling concerns seller own demand
uncertainty. It provides a buffer against its negative effects and its profit advantages
are even greater with demand uncertainty. Indeed, introduction of a probabilistic good
reduces or even eliminates the dependence of pricing decisions on the identity of the more
popular product. The most optimal results were obtained with sufficiently high demand
uncertainty and mid-range capacity level.

3 The diversification of Opaque Channels with complete information

In this paper we tend to answer one main question: is it suitable and efficient for a given
online agency to use simultaneously more than one alternative Opaque channels?

The answer is complex and depends on many circumstances, and mainly on the com-
petitive environment. If we consider a competitive game, in which every competitor
chooses one single channel, the equilibrium could be an asymmetric equilibrium where
each intermediary specializes and distributes on a specific channel. If Agencies A, B and
C compete at the alternative channels: posted - price Opaque Channel, Last Minute
channel and NYOP channel, each one should specialize in a different type of selling. If
there is only one Agency in a situation of monopoly and that the point is to find the
best allocation of potential travelers on alternative channels, the best solution would be,
theoretically, the first degree price discrimination. As in many other cases, this strategy is
probably not fully implementable due to its complexity. The NYOP Opaque solution (the
“Priceline system”) seems however to be the closest one. Supposing that the population
of travelers is risk neutral, fully informed about the characteristics of remaining tickets on
alternative channels (company, hour of departure . . . ) and knows also perfectly the distri-
bution of the propensities to pay of the others potential travelers, each consumer will be
able to bid (or not) a price corresponding to his reservation price, given the uncertainty
on the number of seats and on their attributes. However, this result validity depends on
the level of incompleteness of potential traveler informations. Suppose for instance that
potential travelers with high propensity to pay are also less informed on ticket’s distribu-
tion and on the propensity to pay of other agents: they will probably over-estimate the
utility they can derive from the NYOP channel and bid lower price than they would, if
the information was complete. In this case, implementation of a “Last minute” channel
would be a better strategy. However, this solution has an inconvenient: it does not re-
sist to time depreciation. If travelers want to book hotel rooms, to rent a car . . . , more
generally if the airline ticket is an element among others of a packaged product providing
them an overall utility, the ”Last minute” solution will sharply decrease their utility. In
this paper we consider ”Last minute” selling as a complementary channel to the Opaque
one.

It is quite complicated to decide if two or more forms of Opaque channels can coexist.
Consider for instance the NYOP Priceline channel and the Opaque Hotwire one. Both of
these channels are Opaque, i.e. do not provide precisely the travelers with the certainty
on the quality of the travel. Once more, if all passengers had complete information on
the flight’s frequency and the other ticket’s attributes and if they knew precisely the
distribution of the other’s consumer’s propensities to pay, all of them would be able to
choose to use the NYOP system only, and leave the Opaque channel, which could be
redundant. In the following subsection we will try to confirm this intuition.

3.1 The model

We suppose that everyday there are 2 flights from city 1 to city 2: the first leaves city 1 at
7:00 am and the other at 6:00 pm. These flights booking level on the traditional channel
is ordinarily estimated with a small error only few days before the date of departure.
This short slot makes the “Last Minute” solution inappropriate for “impatient” low price
tavelers. Indeed, it actually concerns a distinct “patient” sub-population of travelers,
which we are not going to consider in this paper. Subsequently the agency decides to
implement an adapted Opaque channel and to offer to the “impatient” low rate travelers
an adapted booking system. The agency knows the distribution of the states of the world,
which are defined by table 1.

<table>
<thead>
<tr>
<th>States of the world</th>
<th>Number and type of available seats</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>m at 7:00 am</td>
<td>1/4</td>
</tr>
<tr>
<td>2</td>
<td>m at 6:00 pm</td>
<td>1/4</td>
</tr>
<tr>
<td>3</td>
<td>2m at 7:00 am</td>
<td>1/8</td>
</tr>
<tr>
<td>4</td>
<td>m at 7:00 am m at 6:00 pm</td>
<td>1/4</td>
</tr>
<tr>
<td>5</td>
<td>2m at 6:00 pm</td>
<td>1/8</td>
</tr>
</tbody>
</table>

Table 1: Available seats for the flights from city 1 to city 2 on a given date

The agency can implement either:
(i) an Opaque “Hotwire style” posted-price system;
(ii) a NYOP “Priceline style” system;
(iii) both of the systems.
The sequence of the actions is as follows:

- At stage 1, the online travel agency (OTA) chooses between (i), (ii) and (iii). If (i) or
(ii) has been selected, the agency fixes the price of the Opaque channel. If (ii) or (iii)
have been chosen, the travel agency launches a single bid process for the tickets.

- At stage 2, if the OTA has initially chosen (i), the potential travelers decide to buy
or not a ticket on the Opaque channel. If the OTA has chosen (ii), they choose to post
or not a single bid. If the OTA has chosen (iii), they chose to buy a ticket on the Opaque
channel, or to post a bid on the NYOP channel or to reserve.

- At stage 3, the OTA knows the number and the nature of the available seats on each flight. If (i) or (iii) have been chosen at stage 1, the OTA distributes the tickets to the buyers on the Opaque channel. If (ii) or (iii) have been chosen, the agency decides the threshold price for the NYOP channel and sells the tickets to those whose bids exceed this price. Each successful bidder pays the posted rate.

The relevant equilibrium concept is a Stackelberg equilibrium, where the travel agency is leader. The game is solved by backward induction. At stage 3, the travel agency chooses the best action (i.e. fixes the lower limit price of the NYOP channel if the devices (ii) and (iii) have been selected), given the action previously taken by the travelers at stage 2. At stage 2 the potential travelers choose their own best actions, given the travel agency’s decisions at time 1 (the implemented system and the Opaque channel price if the (i) or (iii) schemes have been implemented), their expectations of the travel agency’s decisions at stage 3 and the level of information on the chance that their bids get accepted, if the information on the characteristics of the auction process, when the devices (ii) or (iii) are implemented, is imperfect. At stage 1, the travel agency chooses the appropriate device and the rates of the Opaque channel if the devices (i) or (iii) are implemented.

We suppose that information is imperfect but complete (travelers know the states of the world and their respective probability).

3.2 The optimal choices of the agency

Let’s consider successively the three kinds of solutions for the travel agency.

(i) If the Opaque channel is implemented alone, the agency fixes at stage 1 the price $p^O$ such that $p^O$ maximizes the joint profit of the airline and the travel agency $\pi(p^O) = mp^O$. The quantity of available seats for the Opaque channel is $m$, because it is the higher level of seats available at stage 3 in all states of the world. The level of $p^O$ is then such that the agency extracts the whole surplus of the last traveler choosing the Opaque channel. Whatever the rate of the Opaque channel fixed at stage 1 would be, the potential travelers whose net utility is greater or equal to zero at this rate will choose to buy a ticket on this channel. The best solution for the travel agency is then to charge a rate that exhausts the last potential Opaque channel traveler’s surplus. These travelers will be located on their respective segment on points $a^*_i$ such that $(a - a^*_i)/a = m/2n$, i.e. at $a^*_i = a(2n - m)/2n$. The resulting value of $p^O$ which vanishes the net utility of the agents located on $a^*_i$ is then such that $a^*_i(u + \bar{u}/2) - p^O = 0$ since the states of the world and the distribution of agents on the segments $[0, a]$ is common knowledge. Then we obtain $p^O = a(2n - m)(u + \bar{u}/2)/2n$ and

$$\pi^O = mp^O = (2nm - m^2)a(u + \bar{u}/2)/2n$$ (1)

(ii) If the NYOP channel is the only to be implemented, at stage 3 and in each state of the world, the travel agency will choose the higher threshold value such that all the
potential travelers whose bids are greater or equal that the threshold will exhaust the market. As the OTA determines this value after observing the state of the world, there are two possibilities. If only \( m \) tickets are available, the price \( p^N_H \) will be high: it will correspond to the reservation price of the last of the \( m \) high propensity to pay agents that integrate in their expected utility the possibility to pay less if \( 2m \) seats are available. If the number of available ticket is \( 2m \), the price \( p^N_H \) will be lower as it corresponds to the propensity to pay of the last of the \( 2m \) travelers who integrate in their expected utility the uncertainty. At stage 2, the bidders will be able to integrate the optimal choices of the agency in their own decision and, among other, to consider their bids getting accepted. From usual deductions relative to the optimal bidders behavior, we deduce that, given the resulting expected value of their choices, the bidders will not bid lower price than their reservation price. If they are able to understand correctly the NYOP system, they will calculate the price that they will actually pay as the reservation price of the last successful bid in each state of the world. In fact, there exist two possible bidding prices: bidding prices greater or equal than \( p^N_H \) that guarantee the travel and bidding prices greater or equal to \( p^N_L \) but smaller than \( p^N_H \) that make the travel uncertain. Whatever the level of their bids, if they are greater than \( p^N_H \) or between \( p^N_L \) and \( p^N_H \), the passengers will only pay \( p^N_L \) or \( p^N_H \), their net expected utility is then defined by \( a_i(u + \bar{u}/2) - p^N \) if they decide to bid at price \( p^N_L \) and \( [a_i(u + \bar{u}/2) - p^N_L]/2 \) if they decide to bid at rate \( p^N_H \). From elementary calculus, we deduce the threshold values \( a^1_{2*} \) and \( a^2_{2**} \) separating respectively on each segment \([0, a]\) the potential travelers choosing to reserve and the potential travelers choosing to bid \( p^N_L \), and the potential travelers choosing to bid \( p^N_H \) and \( p^N_L \). These values are \( a^1_{2*} = a(n - m)/n \) and \( a^2_{2**} = a(2n - m)/2n \). Then we deduce the equilibrium prices \( p^N_L = a(n - m)(u + \bar{u}/2)/n \) and \( p^N_H = a(2n - m)(u + \bar{u}/2)/2n \), the joint profit of the airline and of the travel agency \( p^N = mP^N_H + mP^N_L \) or:

\[
\pi^{O/N} = mp^O + mp^N/2 = (3nm - 2m^2)a(u + \bar{u}/2)/2n
\]

(iii) If the two channels are jointly implemented, the OTA allocates the first set of \( m \) seats to the Opaque channel, where it targets the high propensity to pay customers. The second set of \( m \) seats is allocated to the NYOP channel - to the travelers with a lower propensity to pay. At stage 1, the agency chooses the price for the Opaque channel and offers to the travelers the possibility to bid in the NOYP channel. As in case (i), the price of the Opaque channel is \( p^O = a(2n - m)(u + \bar{u}/2)/2n \). The NYOP channel targets the next \( m \) passengers and is activated at price \( p^N = a(n - m)(u + \bar{u}/2)/n \). The joint profit of the airline and the travel agency is then \( \pi^{O/N} = mp^O + mp^N/2 \) or:

\[
\pi^N = (2mp^N_H + mp^N_L)/2 = (3nm - 2m^2)a(u + \bar{u}/2)/2n
\]

Subsequently we deduce the following proposition:

**Proposition 1.** If potential low rate travelers are completely informed on the random number and distribution of available seats and on the propensity to pay of every agent, it is equivalent for the airline and the agency to implement a NYOP channel alone and to realize the joint implementation of an Opaque and a NYOP channel.

**Proof:** Expressions (1), (2) and (3) represent the amounts of the joint profits of the airline and the travel agency at Stackelberg equilibriums associated respectively to the
implementation of an Opaque channel, a NYOP channel and jointly an Opaque channel and a NYOP channel. The comparison of (1), (2) and (3) proves that, whatever the values of the parameters $u, \bar{u}, a, n,$ and $m$ are, $\pi^{O/N} = \pi^N > \pi^O \text{ } \blacksquare$

In accordance with intuition, the Opaque “Hotwire style” channel is not an optimal solution for potential travelers if it is implemented alone: the travelers with high propensity to pay are indifferent between this selling mechanism and its joint implementation with the NYOP channel, while the travelers with low propensity to pay prefer the two other selling technologies. Another observation is that, if travelers are risk neutral (as we have supposed them to be), it is equivalent for high propensity travelers to pay $p^O$ for the Opaque channel or to use the NYOP channel which theoretically provides them with a random price. Note however that when we introduce even a little risk aversion, the agents with high propensity to pay will prefer to pay $p^O$: this observation can provide the background for the joint implementation of an Opaque and a NYOP channels which then could be more efficient than the NYOP channel considered alone.

4 Joint Opaque Channels with incomplete information

A first type of incompleteness is linked with the bad knowledge from travelers of the stochastic distribution of the demand of tickets from the traditional channels. The seasonal, daily and hourly evolution of traditional demand follows complex laws which are not easily understood by travelers. The statistical distribution of demand variations during the period could involve information incompleteness for travelers or informational asymmetries between the OTA and the travelers on the one hand, and on the other hand - between the travelers. It is however advantageous for the airlines to adapt partly their supply to these variations. Consequently, it is advantageous for airlines and OTA to diffuse appropriate statistics on seats distribution for each destination and for every sub-period of time. Then we suppose that this cause of bad information is not the major motive of uncertainty and concentrate on a second type of incompleteness. Indeed, bidders lack relevant information on the other consumers propensities to pay. The number of the potential travelers from which the sample of bidders for a given destination is extracted makes for each bidder very difficult to perceive its relative propensity to pay or the level of its own propensity to pay compared with the propensities to pay of the other bidders. This lack of information has dramatrical consequences: with information completeness, our example provides only two bidding prices when the NYOP is implemented or when the Opaque system and the NYOP are jointly implemented: as we verified analytically, whatever the propensity to pay of the traveler is, it will never be interesting for him to bid at a price different from $p^N_L$ or $p^N_H$. Once travelers cannot calculate $p^N_L$ or $p^N_H$ or calculate the same level from these threshold prices, it could be rational for each of them to offer different prices when the selling system is the NYOP mechanism or another booking system.

11
4.1 The general setting

Let's consider the segment where all potential travelers preferring the 7:00 am (resp. the 6:00 pm) flight to the other one and assume that passengers do not know precisely their position on this segment. This uncertainty implies that their estimations of the other passengers distribution on the segment and especially the distance \([a_i, a]\) between their own location and the location of the agent with the highest propensity to pay are imprecise. Then, we suppose that the agent located at \(a_i\) estimates \(a\) as \(\hat{a}\):

\[
\hat{a} - a_i = q(a - a_i) + (1 - q)a_i, \quad q \in [0, 1]
\]

When \(q = 0\), there is full uncertainty on the position of \(a\) and the traveler locates himself on the middle of the segment \([0, a]\). When \(q = 1\), the information on his position is perfect. When \(q\) is comprised strictly between 0 and 1, the uncertainty on the agent's location is more or less moderate. We suppose that the travel agency knows this imprecision of the agents on their relative propensity to pay. Now, let's consider the three available possibilities of implementation of alternative selling mechanisms.

(i) If the Opaque channel is implemented alone with price \(p^O\), the travelers have all the information on prices while taking their decision at time 2. Their behavior is then unchanged. They will buy a ticket if \(a_i(u + \bar{u}/2) - p^O \geq 0\) and do nothing if \(a_i(u + \bar{u}/2) - p^O < 0\). The result is the same as in case of complete information, i.e., \(p^O = a(2n - m)(u + \bar{u}/2)/2n\) and

\[
\pi^O = mp^O = (2nm - m^2)a(u + \bar{u}/2)/2n
\]

(ii) If the NYOP channel is implemented alone, at time 2 the bidders estimate the probability of success of their bid. Given (4), they still compare \(a_i(u + \bar{u}/2) - p^N_H\) (their estimated net utility if they choose to bid at price \(p^N_H\) and expect being able to travel in all states of the world) and \([a_i(u + \bar{u}/2) - p^N_L]/2\) (their estimated net utility if they choose to bid a price higher then \(p^N_L\), but lower then \(p^N_H\)) and 0 (their utility if they decide to reserve). In this case, they are compelled to use their individual estimations of \(a_i^2\) and \(a_i^{2*}\) to evaluate \(p^N_L\) and \(p^N_H\). Given (4), they calculate \(a_i^{2*} = (aq - 2a_iq + 2a_i)(n - m)/n\) and \(a_i^{2**} = (aq - 2a_iq + 2a_i)(2n - m)/2n\) and then deduce \(p^N_L = (aq - 2a_iq + 2a_i)(n - m)(u + \bar{u}/2)/n\) and \(p^N_H = (aq - 2a_iq + 2a_i)(u + \bar{u}/2)(2n - m)/2n\) as threshold prices (depending on their location \(a_i\) when the total number of seats is respectively \(m\) and \(2m\)). The higher is the propensity to pay of the traveler located at \(a_i\), the greater his expected prices for the NYOP channel at low and high rates are. The potentials travelers located at \(a_i\) on one of the segments \([0, a]\) consider themselves as marginal agents between the passengers choosing reservation and the agents bidding at low rate if \(a_i = a_i^{2*} = (aq - 2a_iq + 2a_i)(n - m)/n\), i.e. \(a_i = aq(n - m)/[2nq - 2mq - n + 2m]\). Agents located on the same segment at \(a_i = a_i^{2**} = (aq - 2a_iq + 2a_i)(2n - m)/2n\), i.e. \(a_i = aq(2n - m)/(4nq - 2mq - 2n + 2m)\) consider similarly themselves as the limit agents between the low rate bidders and high price ones. Note that these thresholds depend on \(q\), i.e. on the level of passenger uncertainty on their relative position on \([0, a]\). Then at stage 2, potential passengers bids depend first on their position on \([0, a]\) and on the level of uncertainty. At stage 3, three cases are possible according to parameters values and
the level of uncertainty:

- **Case 1:** \( a_2^i < a_2^{2p_i} < a_2^{2**} < a_2^{2**p_i} \)
- **Case 2:** \( a_2^i < a_2^{2**} < a_2^{3p_i} < a_2^{3**p_i} \)
- **Case 3:** \( a_2^{3p_i} < a_2^i < a_2^{3**} < a_2^{3**p_i} \)

These cases present different properties and though need to be analyzed separately.

### 4.2 Average number of tickets, relatively strong uncertainty

Let's begin by considering the 2nd case, which is illustrated by Figure 1. In order to define it, it is sufficient to compare \( a_2^{2**} \) and \( a_2^{3p_i} \). Despite the number of parameters and the difficulty to determine exhaustively the ranges of variation of \( n, m \) and \( q \) corresponding to this case, it appears that average number of tickets (\( m \) slightly smaller than \( n/2 \)) and a quite high level of uncertainty (\( q \) values smaller than \( 1/2 \)) can generate such a ranking between \( a_2^{2**} \) and \( a_2^{3p_i} \).

![Figure 1: Case 2, 2m available tickets.](image)

From a direct observation of figure (1), we deduce Proposition 2:

**Proposition 2.** In case of incomplete information, when the threshold reservation utilities are \( a_2^{2i} < a_2^{2**} < a_2^{3p_i} < a_2^{3**p_i} \), the joint implementation of the two systems will never strongly dominate the single implementation of one of the systems.

**Proof:** Suppose that the NYOP channel is dominated by the joint implementation of the NYOP and the Opaque systems. In this case, since the agents located between \( a_2^{2i} \) and \( a_2^{2p_i} \) do not bid when the NYOP system is implemented alone or jointly, only the agents located between \( a_2^{2**} \) and \( a_2^{3**p_i} \) are interested in this joint implementation and will all choose the Opaque channel: consequently, the Opaque system implemented alone is equivalent to the joint implementation of the two systems. With the opposite assumption, the NYOP system alone dominates the joint implementation of the two systems, if we consider OTA’s profits.

In case 2, the level of information incompleteness is such that the Opaque systems are all inefficient to clear the market. In this case, the OTA and the airlines should
develop a non Opaque last minute systems with traditional dynamic pricing, adapted to the last minute demand. Another possibility for the airlines and the OTA is to develop information systems in order to increase $q$ and though make the Opaque channel more efficient.

4.3 Average number of tickets, moderate uncertainty

In case 1 we suppose that the number of tickets available is smaller than half of the number of potential travelers (we must remain that a potential traveler is someone interested in traveling at a positive rate: it is realistic to suppose that there are always potential travelers, especially during holiday periods, that demand the tickets if the rate decreases sufficiently). By moderate uncertainty we mean that $a_i^{2**}$ is less than $a_i^{2p**}$.

![Figure 2: Case 1, 2m available tickets.](image)

In this case, when the NYOP system is applied, as illustrated in figure (2), the threshold between the bid of those who expect to travel in all states of the world and those who expect to travel only when there is a large number of available tickets is higher than in the case of perfect information. The number of the travelers able to travel in all conditions is consequently under-evaluated by themselves. Therefore, the travelers located between $a_i^{2p**}$ and $a$ pay higher (and different) rates than in the case of complete information to travel in all states of the world, while the travelers located between $a_i^{2**}$ and $a_i^{2p**}$ pay smaller (and also different) rates to travel with the same level of certainty. At the same time, the travelers located between $a_i^{2p**}$ and $a_i^{2**}$ pay a relative high (and different) rate to travel only when there are $2m$ available tickets, while those located between $a_i^{2**}$ and $a_i^{2p**}$ do not bid. When the number of available tickets is $2m$, the consequence is again an extra-profit for the OTA and the airlines on the subset of travelers located between $a_i^{2p**}$ and $a_i^{2**}$ and a remainder of unsold tickets corresponding to the potential travelers between $a_i^{2**}$ and $a_i^{2p**}$.

When the Opaque channel is implemented alone, only the travelers located between $a_i^{2**}$ and $a$ choose to travel at the uniform posted rate $a_i^{2**}$.

When Opaque and NYOP systems are implemented jointly, the travelers located between $a_i^{2**}$ and $a$ still choose the Opaque system while the travelers located between $a_i^{2p**}$ and $a_i^{2**}$ still choose to bid relatively high (and different) prices to travel only when there
are 2m available tickets. The Opaque channel is then still dominated by the joint implementation of the two systems. The relevant comparison is then between the NYOP alone and the Opaque and NYOP systems applied jointly and particularly, from the OTA’s point of view, the profits generated by the travelers located between $a_i^{2**}$ and $a$ with the NYOP and with the joint implementation.

If the NYOP channel is implemented alone, the OTA profits are expressed by (6)

$$\pi^N = 1/2 \times \pi^N(m) + 1/2 \times \pi^N(2m)$$

with

$$\pi^N(m) = 2 \sum_{k=1}^{m^P_H/2} P_{H_i}^N [a_i^{2p**} + (k - 1)(a - a_i^{2p**})/((m^P_H/2) - 1)] + 2 \sum_{k=1}^{(m-m^P_H)/2} P_{L_i}^N [a_i^{2**} + (k - 1)(a_i^{2p**} - a_i^{2**})/((m - m^P_H)/2 - 1)]$$

and

$$\pi^N(2m) = 2 \times \sum_{k=1}^{m^P_H/2} P_{H_i}^N [a_i^{2p**} + (k - 1)(a - a_i^{2p**})/(m^P_H/2 - 1)] + 2 \sum_{k=1}^{m^P_D/2} P_{L_i}^N [a_i^{2p**} + (k - 1)((a_i^{2p**} - a_i^{2*})/(m^D_H/2 - 1)]$$

where $m^P_H = 2n(a^2_{2p**} - a^2_i)$ and $m^P_D = 2n(a^2_{2p**} - a^2_{2p**})$, given the level of uncertainty corresponding to case (1), are the number of tickets obtained by travelers able to bid high prices in order to acquire one ticket respectively in all states of the world and the number of tickets obtained by travelers able to bid high prices in order to acquire one ticket if 2m tickets are available.

If the OTA decides to implement jointly both of the channels, its profits are given by equation (7)

$$\pi^{O/N} = (2nm - m^2)a(u + \bar{u}/2)/2n + \sum_{k=1}^{(m^P_H/2 + m^P_D/2 - m/2)} P_i^N [a_i^{2p**} + (k - 1)(a_i^{2p**} - a_i^{2*})/((m^P_H/2 + m^P_D/2 - m/2) - 1)] (7)$$

Let’s begin by an illustration of the smallest case where $n = 4$ and $m = 2$. Then, when there are only 2 tickets available, only 1 potential passenger from each subset (or segment) can travel whereas 2 from each subset can travel when 4 tickets are available. Given that $q < 1$, the passengers able to fly when the number of available seats is $m = 2$ over-evaluate the reservation price $a_i^{2**}$ necessary to fly in such conditions and choose to bid a low rate, while the passengers able to fly only when the available tickets are $2m = 4$ overestimate the reservation price $a_i^{2*}$ necessary to fly at a low rate and choose not to bid. The consequence is that there is only two bidders for the NYOP system, both bidding lower than $a_i^{2**}$. The receipt profits of the OTA are higher if the Opaque system is implemented jointly, since in this case the two bidders of the NYOP system
choose the Opaque system and pay each one $a_{i}^{2**}$. Due to the minimal dimension of $n$ and $m$ (the smallest possible), this example is however a limit case where the joint implementation and the implementation of the Opaque system alone provide the same profit to the OTA. We provide in Appendix 1 another numerical example with slightly larger values for $m$ and $n$ where the joint implementation strongly dominates the single implementation of the two systems taken singularly. We then deduce the following proposition:

**Proposition 3.** When $n$, $m$ and $q$ are such that $a_{i}^{2*} < a_{i}^{2**} < a_{i}^{2**} < a_{i}^{2*p*}$, the joint implementation of the Opaque and the NYOP is always the best solution for the OTA.

**Proof:** see Appendix 2.

This result indicates that with an average number of potential passengers and a level of uncertainty rather moderate, is is advantageous for the OTA to propose jointly two (or more . . . ) Opaque booking systems.

4.4 Large number of tickets

The case 3 is represented in figure (3). It corresponds to large number of available tickets (quite all of potential travelers can travel when $2m$ tickets are available). This is not an unrealistic case. We observe in some periods very low prices on the “last minute” or even the traditional channel that indicate that few number of potential passengers face a large supply of seats. The only doubt on the relevance of this case concerns the level of unpredictability of available tickets number. In low season, low costs companies tend to offer low prices on the traditional channel without increasing indefinitely the proportion of Opaque supply: this is probably the best answer to a rather predictable shortage of demand.

![Figure 3: Case 3, 2m available tickets.](image)

When the case 3 is relevant, it can be considered as very close to case 1. As in case 1, when the OTA considers the option of offering the two systems jointly, it evaluates the trade-off between the high rate passengers (located between $a_{i}^{2*p*}$ and $a_{i}^{2**}$) who tend to bid at low rate. As in case 1, the potential passengers located
between $a_i^{2p*}$ and $< a_i^{2**}$ have the same choices when the NYOP system is implemented alone or jointly with the Opaque “Hotwire” system. The only difference with case 1 is that there exist in these two cases an excess demand: indeed one part of low rate bidders choose to bid a price lower than $a_i^{2*}$ and a more or less large part of them cannot travel, even when $2m$ available tickets remain.

**Proposition 4.** When $n$, $m$ and $q$ are such that $a_i^{2p*} < a_i^{2*} < a_i^{2**} < a_i^{2p**}$, the joint implementation of the Opaque and the NYOP is always the best solution for the OTA.

**Proof:** see Appendix 3.

5 Comments and conclusions

After the literature review analyzing the properties of the Opaque booking systems used by the online travel agencies, this paper considers the possibility of a joint implementation of two different Opaque systems by the same travel agency. We call the Opaque system, the one developed by Hotwire.com and the Name-Your-Own-Price system, the one implemented by Priceline.com. We build a 3 stage game model describing the optimal choices of a travel agency facing a population of potential travelers with differentiated reservation prices. We first develop the game with imperfect but complete information of potential passengers (they do not know how many seats will be available but know the reservation prices of the other passengers). In this case (which is still the only considered by the literature) the joint implementation of the NYOP and Opaque system has no advantages over the single implementation of the NYOP system. We then extend the model to the case of incomplete information (each potential passenger ignore the reservation utility of the others). We decompose this case in 3 sub-cases and prove that in 2 of them, joint implementation dominates the other strategies.

An extension of our analysis will consider the welfare issues associated to the incomplete information case. Another extension could be to develop the case of duopoly as an example of competition. Indeed, in the e-tourism markets, a great number of OTAs compete and co-exist, implementing different distribution strategies. More precisely, two OTAs compete in the Opaque segment, displaying different selling approaches. We could at last evaluate the possibility of threshold price variability according to the number of tickets available and consumer’s arrivals on the market. In fact, it is difficult to pretend that Priceline fixes the threshold price only once at the beginning of the selling period and maintains it unchanged until the date of departure, despite the evolution of number of potential travelers and amount of tickets available. This is a formal limit of our model (and more generally the current limit of models analyzing Opaque channels).
References


Appendix

Appendix 1: illustration of small size of Proposition 3

We present an example of small size in which the joint implementation of the Opaque and NYOP systems strongly dominates the single implementation of the NYOP or the Opaque system. We choose the case where n = 9 and m = 4. In this case, each subset of n agents is located on the segment [0, a].

In case of complete information, travelers 8 and 9 are able to fly in all states of the world and travelers 6 and 7 only when there are 8 available tickets (remind that there are 2n travelers located on
two segments). We normalize \( a = 1 \) and determine the threshold values \( a_i^{2*} = 11/18 \) and \( a_i^{2**} = 15/18 \) corresponding respectively to the reservation prices of travelers 6 and 8. We choose \( q = 35/36 \) which corresponds to a very moderate level of uncertainty (with \( q = 1 \), the potential travelers have a complete information on the reservation price of the NYOP bidders). Since in this case \( a_i^{2**} \) is between \( a_i^{2*} \) and \( a \), when the NYOP system is implemented alone, only the traveler 9 chooses to bid at a high price (what makes his flight certain), while travelers 7 and 8 choose to bid at a low price (with the probability \( p = 1/2 \) to travel) and agent 6 do not bid. When there are \( m = 4 \) available tickets, only agents 8 and 9 travel and at very (in this case) different rates. Given the values on the parameters, we obtain \( a_i^{2**} = 0.859 \) while \( a_8^{2**} = 0.613 \). Their sum 1.472 is the OTA profits obtained by distributing to high rate population when the NYOP system is implemented alone. When the Opaque system is jointly implemented, agents 8 and 9 choose this Opaque system and pay each one the reservation price \( a_8^{2**} = 0.833 \) of agent 8. The resulting profits are then \( i.e. \) 1.666 for the OTA. Then we compare these profits with those obtained if the NYOP system is implemented alone 1.472. Since agent 7 still bid the same amount \( a_7^{2**} \) with or without the Opaque system’s implementation and agent 6 still do not bid, the joint implementation of the two systems then provides higher profits to the OTA.

**Appendix 2: Proof of Proposition 3**

Given that potential travelers located between \( a_i^{2*} \) and \( a_i^{2**} \) choose the same action when the NYOP is implemented alone or jointly with the Opaque system, we consider only the optimal actions of the potential travelers located between \( a_i^{2**} \) and \( a \). According to the relative values of \( n \), \( m \) and \( q \), every agent \( j \) belonging to this subset chooses to bid at price \( P_{H,j}^N \) or \( P_{L,j}^N \) according to his own position related to \( a_i^{2**} \). If the agent is located between \( a_i^{2**} \) and \( a_i^{2*} \), he chooses to bid low price \( P_{L,j}^N \). If he is located between \( a_i^{2**} \) and \( a \), he bids high price \( P_{H,j}^N \). When the two systems are jointly implemented, all the potential travelers located between \( a_i^{2*} \) and \( a \) pay \( P^O \). Equations (6) and (7) can be respectively expressed as (8) and (9) with:

\[
\pi^N = \frac{1}{(8n^2(n + m(-1 + q) - 2nq)^2)a(n^3(m(-2 + q) - 2n(-1 + q))q + 2(m - n)^2(-1 + q)(m + n(-2 + q) - nq)(n + m(-1 + q) - 2nq)) + n^3q((m - 2n)(m(-2 + q) - 2n(-1 + q)) + (2m(-n)mq(n + m(-1 + q) - 2nq)) / (n + 2m(-1 + q) - 2nq)^2)(2u + \bar{u})}{2n^2(n + m(-1 + q) - 2nq)^2}
\]

Figure 4: Comparison between NYOP and joint implementation in case 1
and

\[ \pi_{O/N} = 1/4a(m(2 - m/n) - ((m - n)(mn(5 - 4q) + 2m^2(-1 + q) + 2n^2(-1 + q))q) / (n + 2m(-1 + q) - 2nq)^2) \]

The conditions on parameters \( a, u, \bar{u}, 1/2 < q < 1 \) and the condition \( m \leq n/2 \) are sufficient to make in all cases \( \pi^N \) smaller than \( \pi_{O/N} \) (see Figure 4) \( \blacksquare \)

Appendix 3: Proof of Proposition 4

In the case 3, if the NYOP channel is implemented alone, the OTA profits are expressed by (10):

\[ \pi^N = 1/2\pi(m) + 1/2\pi(2m) \] (10)

Figure 5: Comparison between NYOP and joint implementation in case 3

with

\[ \pi^N(m) = 2 \sum_{k=1}^{mD/2} P_i^N \left[a_i^{2p**} + (k - 1)(a_i - a_i^{2p**})/(mD/2 - 1)\right] + 2 \sum_{k=1}^{(m-mD)/2} P_i^N \left[a_i^{2*} + (k - 1)(a_i^{2p**} - a_i^{2*})/(m-mD/2 - 1)\right] \]

and

\[ \pi^N(2m) = 2 \sum_{k=1}^{mD/2} P_i^N(a_i^{2p**} + (k - 1)(a_i - a_i^{2p**})/(mD/2 - 1)) + 2 \sum_{k=1}^{m-mD/2} P_i^N(a_i^{2*} + (k - 1)(a_i^{2p**} - a_i^{2*})/(m-mD/2 - 1)) \]

When the two systems are jointly implemented, the OTA profits are given by (11):

\[ \pi_{O/N} = (2nm - m^2)a(u + \bar{u}/2)/2a + \sum_{k=1}^{m/2} P_i^N \left[a_i^{2*} + (k - 1)(a_i^{2p**} - a_i^{2*})/(m/2 - 1)\right] \] (11)

Equations (10) and (11) can be respectively expressed as (12) and (13) with:

20
\[ \pi^N = \frac{-1}{4n^2(n + m(-1 + q) - 2nq)^2}a \\
(5m^5(-1 + q)^3 - 8n^5(-1 + q)^3 - m^4n(-1 + q)^2(-27 + 31q) \\
+ m^3n^2(-1 + q)(59 + q(-133 + 72q)) + m^2n^3(65 + q(-215 + 230q - 78q^2)) \\
+ mn^4(-36 + q(117 + 4q(-31 + 10q)))(2u + \bar{u})) \] \tag{12}

and

\[ \pi^O/N = \frac{-am(n(n(5 - 3q) + n^2(-4 - q) + 2m^2(-1 + q))(2u + \bar{u}))/4n^2}{(4n^2)} \] \tag{13}

The conditions on parameters \( a, u, \bar{u}, 1/2 < q < 1 \) and the condition \( m \geq n/2 \) are sufficient to make in all cases \( \pi^N \) smaller than \( \pi^O/N \) (see Figure 5) \[ \blacksquare \]