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Interfacial properties of immiscible Co-Cu alloys

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Abstract

Using electromagnetic levitation under microgravity conditions, the interfacial properties of an $Cu₇₅Co₂₅$ alloy have been investigated in the liquid phase. This alloy exhibits a metastable liquid miscibility gap and can be prepared and levitated in a configuration consisting of a liquid cobalt-rich core surrounded by a liquid copper-rich shell. Exciting drop oscillations and analysing the frequency spectrum, both surface and (liquid-liquid) interfacial tension can be derived from the observed oscillation frequencies. This paper briefly reviews the theoretical background and reports on a recent experiment carried out on board the TEXUS 44 sounding rocket.

Keywords

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miscibility gap, interfacial tension, surface tension, levitation, oscillating drop, microgravity

1 Introduction

Alloys with a metastable miscibility gap are fascinating systems due to the interplay between phase separation and solidification. In contrast to systems with a stable miscibility gap, the demixed microstructure can be frozen in by rapid solidification from the undercooled melt. Electromagnetic levitation offers the possibility to study compound drops consisting of a liquid core, encapsulated by a second liquid phase. The oscillation spectrum of such a compound drop contains information about both,

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the surface and the interface tension. The binary monotectic alloy CuCo is an ideal model system for such investigations. Its phase diagram is shown in Figure 1.

In order to study this system, including potential industrial applications, the European Space Agency ESA funded a European project, COOLCOP [1]. In the past years, this team devoted a lot of effort to understand the behaviour of such systems, starting from phase diagram calculations [2], drop dynamics [3], modelling of interfacial properties [4], and extending to solidification theories and experiments [5]. The investigations laid the ground for microgravity experiments. First results for a Co25Cu75 alloy onboard a sounding rocket are reported here.

As the temperature of a homogeneous melt of the alloy is lowered below the binodal temperature, demixing sets in and small droplets of one liquid, L1, in the matrix of the other liquid, L2, are formed. These two immiscible liquids do not consist of the pure components, but have concentrations according to the phase boundary of the miscibility gap; therefore, L1 is rich in component 1, while L2 is rich in component 2. Initially, depending on the nucleation kinetics, a large amount of liquid droplets is created. This initial phase is energetically very unfavourable, due to the high interface area created between the different drops. In the next stage, Ostwald ripening sets in [6]. This diffusive mechanism leads to the growth of large drops at the expense of the small ones, thereby coarsening the structure of the dispersion and finally leading to two separated liquid phases. For a levitated drop, without contact to a substrate, the liquid with the lower surface tension – in the present case the copper-rich liquid – encapsulates the liquid (cobalt-rich) core.

Figure 1. Phase diagram of Cu-Co showing the metastable miscibility gap. Symbols indicate experimentally determined liquidus and binodal temperatures.

Terrestrial levitation experiments suffer from the detrimental side effects of the levitation field, in particular by electromagnetic stirring effects which destroy the separated two-phase configuration. Therefore, it was decided to perform such an experiment under microgravity conditions on board a TEXUS sounding rocket. As will be discussed below, the drawback of this carrier is the short available experiment time of about 160 s. Due to a specific preparation of the sample, it was nevertheless possible to conduct three melting cycles during this short time.

2 Drop Dynamics

Generally speaking, the interfacial tension between two liquids is difficult to measure, and only few data exist [7]. The oscillating drop technique [8] is a non contact measurement technique for surface tension measurements of levitated liquid drops. In its original form, it assumes a homogeneous non-viscous drop, free of external forces. In this ideal case, the frequency of surface oscillations is simply related to the surface tension σ_0 by Rayleigh's formula [9] :

$$
\omega_0^2 = \frac{8\sigma_0}{\rho_0 R_0^3} \tag{1}
$$

where ρ_0 is the density of the drop and R₀ its radius. By substituting ρ_0 R₀³ = 3M/4 π , the apparent density dependence of the frequency disappears which makes this equation particularly easy to use.

The oscillating drop technique can be extended to the measurement of the interfacial tension between two immiscible liquids [10]. Based on the theory of Saffren et al. [11], the theory was worked out for force-free, concentric spherical drops.

Figure 2. Cross section of a spherical, concentric compound drop consisting of two immiscible liquids with densities ρ_i and ρ_0 , radii R₀ and R_i, surface tension of the outer liquid σ_0 , and interfacial tension σ_{12} .

The geometry considered is summarized in Figure 2. Due to the presence of the interface between liquid L1 and L2, this sytem possesses two fundamental frequencies, driven by the surface tension σ_0 and the interfacial tension σ_{12} .

Adopting the nomenclature of ref [11], the normal mode frequencies ω of a concentric, force-free, inviscid compound drop read:

$$
\omega_{\pm}^2 = K_{\pm} \frac{W}{J} \tag{2}
$$

K and *J* are dimensionless, while *W* is a frequency squared. *W/J* is given by:

$$
\frac{W}{J} = \frac{\omega_0^2 \tau^8}{\sigma} \frac{1}{(1 + \Delta \rho_i) \tau^{10} + 2/3 \Delta \rho_i}
$$
(3)

Here, a number of symbols has been introduced which are defined as follows: ω_0 is the unperturbed Rayleigh frequency (eqn (1)) of a simple drop with density ρ_0 , Radius R_0 and surface tension σ_0 (see also Figure 2 for the definition of the symbols).

$$
\tau = \sqrt{\frac{R_0}{R_i}}\tag{4}
$$

is the square root of the ratio between outer and inner radius,

$$
\sigma = \sqrt{\frac{\sigma_0}{\sigma_{12}}} \tag{5}
$$

is the square root of the ratio of the surface tension and the interface tension, and

$$
\Delta \rho_i = \frac{3}{5} \frac{\rho_i - \rho_0}{\rho_0} \tag{6}
$$

is the weighted relative density difference between liquids *L1* and liquid *L2*. It remains to write down the expression for *K*. It is given by:

$$
K_{\pm} = \frac{1}{2} \left(\frac{\sigma m_i}{\tau^3} + \frac{m_0 \tau^3}{\sigma} \right) \pm \sqrt{\frac{1}{4} \left(\frac{\sigma m_i}{\tau^3} - \frac{m_0 \tau^3}{\sigma} \right)^2 + 1}
$$
 (7)

where two additional symbols have been introduced, namely:

$$
m_0 = \frac{3}{5} \tau^5 + \frac{2}{5} \tau^{-5}
$$
 (8)

and

$$
m_i = (1 + \Delta \rho_i) \tau^5 - \Delta \rho_i \tau^{-5}
$$
\n(9)

For large σ and small $\Delta \rho_i$, approximate equations can be derived for the two frequencies ω_+ and ω_- :

$$
\omega_{+}^{2} = \omega_{0}^{2} \left(I + \frac{I}{\sigma^{2} \tau^{4}} \right)
$$
 (10)

$$
\omega_{-}^{2} = \omega_{0}^{2} \frac{\tau^{6}}{\sigma^{2}} \left(1 - \frac{5}{3} \tau^{-10} \right)
$$
 (11)

From an experimental point of view it is interesting to discuss the frequencies as a function of the initial, homogeneous composition of the drop. To this end we introduce the relative mass fraction of component 2, i.e. the component with the lower surface tension which will eventually constitute the outer liquid shell. it is given by:

$$
m_{rel} = \frac{m_{shell}}{m} = \left(1 + \left(\frac{R_i}{R_0 - R_i}\right)^3 \frac{\rho_i}{\rho_0}\right)^{-1}
$$
(12)

In Figure 3, the frequency spectrum is shown as a function of m_{rel} for parameters corresponding to the Cu-Co system.

Figure 3. The normalized normal mode frequencies ω_1/ω_0 as a function of the relative mass fraction m_{rel} . For the figure, following parameters were chosen: σ_0 =1.3 N/m, σ_{12} =0.5 N/m, ρ_0 =7.75 g/cm³, ρ _i=7.86 g/cm 3 .

Although the oscillations of the inner radius, R_i , cannot be observed optically for nontransparent liquid metals, both eigenfrequencies can be determined from the oscillations of the outer radius, R_0 , alone. This is due to the coupling of the two oscillators via the common velocity field in the melt. The relative amplitudes of the oscillations of the outer and inner surface are shown in Figure 4 for both oscillatory modes. The larger the value of $|\delta R_0/\delta R_i|$, the better the detectability. Consequently, the optimal choice to detect both modes, lies between $0.7 < m_{rel} < 0.8$.

Figure 4. Relative amplitudes of the oscillations of inner and outer surface as a function of mass fraction for both modes.

3 Results

The experiments were carried out using the TEXUS-EML module during the TEXUS 44 campaign. Two experiments, one on demixing of CuCo, described here, and one on calorimetry and undercooling of an Al-Ni alloy were accommodated. The allotted time span of microgravity for the present experiment was 160 s.

As this time is much too short for undercooling and complete phase separation, it was decided to perform the experiment on a sample which was prepared *ex-situ* as a two-phase compound drop using a DTA furnace and a melt flux technique which allows deep undercooling and subsequent phase separation of the Cu-Co sample [12]. Of course, such a system is not in equilibrium when it is remelted, but it takes some time to destroy the interface between the two liquids L1 and L2, and this time is sufficient to excite and observe oscillations of the (unstable) interface.

The experiment consisted of three heating cycles:

- one cycle with a completely phase-separated sample
- one cycle with a homogenised sample
- one cycle for maximum undercooling

The experiment was conducted with one sample of the composition $Cu₇₅Co₂₅$ with a pre-separated microstructure. Careful heating should melt the Cu shell first and then, at higher temperature, the Co core. The aim of the experiment was to observe the oscillations of this separated microstructure in the liquid and to experimentally determine the interface energy of Cu-Co. In a second heating cycle, the microstructure was homogenized and two pulses were applied to observe oscillations of a homogeneous sample for comparison. A third heating cycle was used to investigate growth of a droplet dispersion starting after undercooling at the binodal.

The most important parameter in the preparation of the experiment was the choice of the maximum temperature in the first heating cycle. It had to be chosen such that both, the outer copper shell, and the inner cobalt core are fully molten, but not

intermixed. Microstructure analysis of the samples from previous parabolic flights showed that a maximum temperature of 1800°C of the heating cycle was too high, as the pre-separated microstructure has been destroyed. On the other hand, a minimum temperature of about 1500 °C is required to melt the cobalt-rich core. Consequently, a maximum temperature of 1600°C has been chosen for the TEXUS experiment.

The experiment was successful and three heating cycles could be conducted. The second cycle led probably to homogenisation of the liquid sample ($T_{max} \approx 1850^{\circ}C$). Two heating pulses for excitation of the homogeneous droplet oscillations have been applied in the high temperature region. The third cycle led to an undercooling of the melt and a recalescence due to release of latent heat, which is indicated by an arrow in the temperature-time profile in Figure 5. The sample was saved and the experiment with Cu-Co was finished.

4 Discussion

4.1 Temperature Calibration

The emissivity of the sample changes depending on whether or not it is phase separated. The pyrometer data were calibrated for ε =0.1, corresponding to a demixed sample. It is assumed that the second heating homogenizes the sample, leading to an emissivity of ε =0.13. Therefore, the pyrometer signal had to be corrected according to:

$$
\frac{1}{T_1} - \frac{1}{T_2} = \frac{\lambda_0}{c_2} \ln \frac{\varepsilon_1}{\varepsilon_2}
$$
\n(13)

where λ_0 is the operating wavelength of the pyrometer, and c₂ = 1.44 10⁴ µm K.

The pyrometer operates in a band of $1.45 - 1.8$ µm. Assuming an effective wavelength of λ_0 = 1.5 um results in a correction of -2.733 10⁻⁵ K⁻¹.

Taking this correction into account, the pyrometer signal was recalibrated and is shown in Figure 5. Also shown (dashed line) is the heater control voltage, controlling the heating power in the coil system of the EML module. The sample is molten within 30 s, between 45260 and 45290 s. During cooling, short heater pulses are applied to excite oscillations of the liquid drop. Due to the time resolution of the data acquisition, not all such pulses are shown in Figure 5.

The temperature signal is rather noisy, especially during heating. This is due to sample movement and sample rotation. As explained above, the sample was prepared in a melt flux, and part of this flux was still attached to the sample surface. This flux has a much higher emissivity than the metallic sample. Whenever such a clod of glass entered the measuring spot of the pyrometer, its signal went up, resulting in spikes. In fact, the temporal distance of these spikes is a quantitative measure of the sample's rotation frequency.

The solidus temperature is, according to the phase diagram, $T_s = 1080$ °C and is visible in the signal around 45280 s. The liquidus temperature, T_L , of the phaseseparated sample is around 1437 °C, also visible at 45290 s. The liquidus

temperature of the homogeneous, single-phase sample is determined as $T_L = 1357$ °C. The binodal temperature is located at T_b = 1248 °C and can be recognised around 45350 s. After the final sequence the sample undercooled and solidified at 45410 s, displaying a recalescence peak. Undercooling relative to the corresponding liquidus temperature was about $\Delta T = 200$ K.

Figure 5. Temperature-time profile of the Cu-Co TEXUS 44 experiment. Dotted lines show the heater activity, not all pulses are shown due to the time resolution of the display. The arrow indicates final solidification.

4.2 Oscillation Spectra

For the analysis of the spectra, a number of sample parameters need to be known. First of all, the mass was determined before and after the flight. The sample mass before the flight was M_0 = 1.31 g and M_∞ = 1.30 g after the flight, resulting in a small weight loss due to evaporation of $\delta M = 0.01$ g, which can be assumed to be mainly copper.

The initial masses of copper and cobalt were $M_{Cu} = 1.00566$ g, $M_{Co} = 0.3064$ g, resulting in 76.65 wt% copper. Due to evaporation, the copper content decreased to 76.38 wt% after flight. Therefore, the concentration changed only by 0.27 wt%, which is acceptable.

For the evaluation of the oscillation frequencies, the radius in the liquid phase is required. This cannot be measured directly, and we estimate it from the sample mass according to

$$
R_{\text{eff}} = \sqrt[3]{\frac{3M}{4\pi\rho}}
$$
 (14)

The densities of liquid copper and cobalt were measured by Saito and coworkers [13, 14]. At the melting point, the quoted values are: $\rho_{Cu}(T_m)=7.86$ g/cm³, $\rho_{Co}(T_m)=7.75$ g/cm³. The temperature dependent densities are as follows:

$$
\rho_{\text{Co}}(T) = 9.71 - 1.11 \, 10^{-3} \, T \, \text{g/cm}^3
$$
\n
$$
\rho_{\text{Cu}}(T) = 8.75 - 0.675 \, 10^{-3} \, T \, \text{g/cm}^3
$$

At T = 2000 K, we obtain p_{Co} (2000 K)=7.49 g/cm³ and p_{Cu} (2000 K)=7.44 g/cm³. As these two densities are very close, we have decided to neglect the density difference and to assume $p_{Co} = p_{Cu} = 0.765 p_{Cu}(2000 \text{ K}) + 0.235 p_{Co}(2000 \text{ K}) = 7.45 \text{ g/cm}^3$ throughout the analysis. Inserting this value into above equation, we obtain

$R_{\text{eff}} = R_0 = 3.475$ mm

We still need to determine m_{rel} and R_i . For these two quantities we need to know the compositions of the two separated liquids L1 and L2. This of course depends on the solidification path and is not known a priori. From EDX analysis of samples prepared identically to the flight sample we estimate that the L2 liquid consists of app. 90 wt% copper and 10 wt% cobalt, while L1 is composed of 16 wt% copper and 84 wt% cobalt. We therefore estimate

$$
m_{rel} = (1.31)^{-1}
$$
 and obtain $R_i = R_0 \sqrt[3]{1 - m_{rel}} = 2.149$ mm.

In order to get a feeling for the oscillation frequencies, we need estimates for the surface and interfacial tensions. The surface tensions of the Cu-Co system have been measured by Eichel *et al.* [15]. For the composition Cu₇₀Co₃₀ which is very close to our sample, their result is:

$$
\sigma_0(T) = 1.22 - 0.29 \cdot 10^{-3} (T - 1365^{\circ}C) \frac{N}{m}
$$

For T = 1665 °C this yields σ_0 =1.13 N/m. Inserting this into the Rayleigh equation, eqn (1), we obtain a Rayleigh frequency

$$
v_0 = \omega_0/(2\pi) = 27.06
$$
 Hz.

For the interfacial tension, we assume complete wetting, yielding $\sigma_1 = \sigma_1 - \sigma_2$.

From ref 15 we have:

$$
\sigma(Cu_{15}Co_{85}) = 1.4 - 0.2 \cdot 10^{-3} (T - 1440^{\circ}C) \frac{N}{m}
$$

$$
\sigma(Cu_{90}Co_{10}) = 1.22 - 0.29 \cdot 10^{-3} (T - 1365^{\circ}C) \frac{N}{m}
$$

At 1665 °C we obtain:

$$
\sigma_{12}=0.225\,\frac{N}{m}
$$

Inserting these values into eqn (2), we obtain

$$
v_{+} = 28.75 \; Hz, \quad v_{-} = 17.26 \; Hz
$$

for a separated two-phase drop.

4.3 Surface Tension

As pointed out before, sample oscillations were excited by short current pulses through the heating coil, which led to a compression and subsequent damped oscillations of the sample. The sample shape was recorded by a video camera, looking along the symmetry axis of the sample (top view) operating at 196 Hz.

The obtained images were analysed off-line by image processing with respect to a number of geometrical parameters; the most important ones are: area of the visible cross section and radii in two orthogonal directions. From the latter, two more parameters can be constructed, namely the sum and the difference of these two radii. In case of non-spherical samples, the latter should have slightly different peaks in their oscillation spectra [16], while the Fourier spectrum of the area signal should contain all peaks. Although there were no big differences between the signals, the area signal was used for further analysis. The time signals of these oscillations are shown in Figure 6 for the first melting cycle. In the first cycle, three oscillations are clearly visible, but the first one is somewhat disturbed. The second cycle also shows three oscillations; they are not shown here.

Figure 6. Oscillations of the visible cross section during the first melting cycle.

In order to obtain the oscillation frequencies, each oscillation was analysed separately by performing a Fourier transformation. The result is shown in Figure 7 for all pulses analysed.

Figure 7. Fourier transforms of the area signal for all evaluated pulses. The spectra are shifted vertically for clarity. From bottom to top: cycle1/pulse 1, cycle 1/pulse 2, cycle 1/pulse 3, cycle 2/pulse 1, cycle 2/pulse 2.

Except for the first pulse of the first cycle, all spectra display a single peak around 28 Hz. The first pulse of the first cycle displays two peaks at 28 Hz, and a small peak around 15 Hz. Positions and corresponding temperatures of the main peaks are shown in Table 1. Assuming that, after the first pulse, the sample is single-phase, these frequencies correspond to the Rayleigh frequency, eqn(1). We then obtain the surface tension as a function of temperature, as shown in Figure 8. Linear fit to the data yields

$$
\sigma(T) = 1.29 - 2.77 \, 10^{-4} \, (T - 1357^{\circ} \text{C}) \quad N/m \tag{15}
$$

This is in excellent agreement with the data measured by Eichel [15].

As is evident from Figure 6 and Figure 7, the oscillations during the first pulse of the first cycle are more complex than for the other pulses. We have therefore analysed this pulse in greater detail, as shown Figure 9. Regardless of the parameter analysed, two peaks around 29 Hz and 28 Hz and a small peak at 15 Hz are clearly visible. Therefore, we conclude that the liquid drop was initially phase separated, giving rise to two peaks around 15 Hz and 29 Hz, and homogenized in the course of the oscillations yielding the Rayleigh frequency at 28 Hz.

Table 1. Temperatures and peak positions of the pulsed oscillations.

Figure 8. Surface tension of $Cu_{75}Co_{25}$ alloy.

If this is correct, we must be able to fit all three frequencies by two values for the surface tension σ_0 and the interfacial tension σ_{12} . This is shown in Table 2. From the fit we obtain:

 σ_0 = 1.21 N/m, σ_{12} = 0.17 N/m

The value for the surface tension corresponds to 1590 °C and agrees well with the fit obtained from the other pulses, see Figure 8. The value of the interfacial tension is somewhat lower than previously estimated. This may be due to a slight shift in the compositions of the two liquids L_1 and L_2 .

	measured	calculated
V٥	28.1	28.0
v+	29.1	29.1
v.	15.5	15.4

Table 2. Measured and calculated frequencies for the first pulse of the first cycle.

Figure 9. Fourier spectra of the 1st pulse in the 1st cycle. FFT of cross section (top) and radius sum (bottom) is shown. Spectra are shifted vertically for clarity.

5 Summary

Using the EML module on board the TEXUS 44 microgravity mission, a $Co₂₅Cu₇₅$ sample was successfully processed. Following results were obtained:

- surface tension as function of temperature
- interfacial tension at 1590 °C
- size distribution of precipitated Co drops

The interfacial tension could not be measured as a function of temperature because the unstable interface was destroyed during the first pulse. The final and decisive experiment will have to be performed on board the ISS, when time is sufficient to keep the sample in the undercooled phase until complete phase separation is obtained and a metastable interface exists between the two liquid phases.

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