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# QUATERNIONIC WAVELETS FOR TEXTURE CLASSIFICATION

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## ABSTRACT

This paper proposes a new texture classifier based on the Quaternionic Wavelet Transform (QWT). This recent transform separates the informations contained in the image better than a classical wavelet transform (DWT), and provides a multiscale image analysis which coefficients are 2D analytic, with one near-shift invariant magnitude and a phase, that is made of three angles. The interpretation and use of the QWT coefficients, especially the phase, are discussed, and we present a texture classifier using both the QWT magnitude and the QWT phase of images. Our classifier performs a better recognition rate than a standard wavelet based classifier.

**Index Terms**— Wavelet transforms, 2D Phase, Quaternionic Wavelet Transform, Image texture analysis, Image classification

## 1. INTRODUCTION

### 1.1. Texture Classification

Texture classification is the process which, given any textural image, find the *class* this image most probably belongs to. Texture has still no universal definition, but may be presented by classical cases like ‘tar’, ‘water’, ‘sand’, as macroscopic examples, or ‘town’, ‘ocean’, ‘forest’, as satellite view examples, and characterized by a sort of uniformity and periodicity. Then a *class* is a kind of texture, according to an arbitrary classification we humans make instinctively. In the wide field of Image Processing, texture recognition have been largely studied [1], we present here an approach based on the recent Quaternionic Wavelet Transform (QWT).

We do not aim at carrying out an efficient classifier, our work is rather an innovating first step in applying a new transform. The QWT, which we focus on, has promising theoretical properties, and texture analysis is a famous application of wavelets. Hence we propose to study QWT in comparison with standard wavelets, in a texture analysis context, without emphasis on state of the art techniques. With a new texture classifier, this work gives an QWT application not did yet to our knowledge, and furthers the practical use of QWT coefficients.

### 1.2. Quaternionic Wavelet Transform (QWT)

A standard wavelet transform (DWT) provides a scale-space analysis of an image, yielding a matrix in which each coefficient is related to a ‘subband’ (localisation in the 2D Fourier domain) and to a position in the image. A ‘subband’ means both an oscillation scale (i.e. a 1D frequency band) and a spatial orientation (i.e. rather vertical, horizontal or diagonal). These are coded by an atomic 2-D function called a ‘wavelet’, that is a sort of oscillating, elongated, oriented and well localized ‘fat point’. Each coefficient is calculated by a scalar product between a shifted wavelet and the image, and so represents

the ‘presence’ of a feature in the image, at one position, for one subband.

The QWT [2] is an improvement of the DWT, providing a richer scale-space analysis for 2-D signals. Contrary to DWT, it is near-shift invariant and provides a magnitude-phase local analysis of images. It is based on the ‘Quaternionic Fourier Transform’ (QFT) and the ‘Quaternionic Analytic Signal’ [3], which extend the well known signal theory concepts to 2D, by an embedding into the quaternion algebra  $\mathbb{H}$ , more adapted than  $\mathbb{C}$  to describe 2D signals.

A quaternion is a generalization of a complex number, related to 3 imaginary units  $i, j, k$ , written  $q = a + bi + cj + dk$ , or  $q = |q|e^{i\phi}e^{j\theta}e^{k\psi}$  in its polar form. It is thus defined by one modulus, and three angles that we call *phase*.

The (quaternionic) analytic signal associated with a 2D function is defined by means of its partial ( $\mathcal{H}_1, \mathcal{H}_2$ ) and total ( $\mathcal{H}_T$ ) Hilbert transforms (HT) :

$$f_A(x, y) = f(x, y) + i\mathcal{H}_1 f(x, y) + j\mathcal{H}_2 f(x, y) + k\mathcal{H}_T f(x, y)$$

The mother wavelet is a quaternionic 2D analytic filter, and yields coefficients that are ‘analytic’. Thus, it inherits the ‘local magnitude’ and ‘local phase’ concepts from the 1D analytic signal, very useful in signal analysis.

Note that the usual interpretation of the magnitude remains analogous to 1D, as it indicates the relative ‘presence’ of a *feature*, whereas the local phase is now represented by 3 angles that make a complete description of this 2D feature.

From a practical point of view, if the mother wavelet is separable i.e.  $\psi(x, y) = \psi_h(x)\psi_h(y)$ , the 2D HT’s are equivalent to 1D HT’s along rows and/or columns. Then considering the 1D Hilbert pair of wavelets ( $\psi_h, \psi_g = \mathcal{H}\psi_h$ ) and scaling functions ( $\phi_h, \phi_g = \mathcal{H}\phi_h$ ), the analytic 2D wavelets are written in terms of separable products.

$$\begin{aligned}\psi^D &= \psi_h(x)\psi_h(y) + i\psi_g(x)\psi_h(y) + j\psi_h(x)\psi_g(y) + k\psi_g(x)\psi_g(y) \\ \psi^V &= \phi_h(x)\psi_h(y) + i\phi_g(x)\psi_h(y) + j\phi_h(x)\psi_g(y) + k\phi_g(x)\psi_g(y) \\ \psi^H &= \psi_h(x)\phi_h(y) + i\psi_g(x)\phi_h(y) + j\psi_h(x)\phi_g(y) + k\psi_g(x)\phi_g(y) \\ \phi &= \phi_h(x)\phi_h(y) + i\phi_g(x)\phi_h(y) + j\phi_h(x)\phi_g(y) + k\phi_g(x)\phi_g(y)\end{aligned}$$

This means the decomposition is heavily dependent on the position of the image with respect to  $x$  and  $y$  axis (rotation-variance), and the wavelet is not isotropic, but the advantage is an easy computation with separable filter banks.

Each subband of the QWT can be seen as the analytic signal associated with a narrowband<sup>1</sup> part of the image. The QWT magnitude  $|q|$ , shift-invariant, represents features at any spacial position in each frequency subband, and the 3 phases ( $\phi, \theta, \psi$ ) describe the ‘structure’ of these features. We discuss below the interpretation of these phases.

<sup>1</sup>The 1D analytic signal provides a time analysis considering the entire frequency spectrum. So in practice, the extracted local (instantaneous) characteristics are only meaningful when the signal itself is narrowband.

The QWT uses the Dual-Tree algorithm [4], a filter bank implementation that uses a Hilbert pair as a complex 1D wavelet, allowing shift invariance and analytic coefficients, while circumventing the undecimated filter bank<sup>2</sup>. Two complementary 1D filter sets, odd and even, lead to four 2D filter banks, slightly shifted each other, providing a sub-pixel accuracy and then the near-shift invariance, for a redundancy of only 4:1. Originally combined by Kingsbury to compute two directional complex analytic wavelets, the 4 outputs of the Dual-Tree here constitute one 4-valued quaternionic wavelet decomposition, embedding the structural informations into a local phase concept, rather than an oriented separation. As the Dual-Tree makes an approximation, the QWT coefficients are approximately analytic, so the extraction of 2-D local amplitude and phase, as well as their interpretation, are actually approximative.

### 1.3. Wavelet-based texture classification

Feature extraction via the standard wavelet representation of images (DWT) has been widely used as a *signal processing approach* to texture analysis [1]. Accordingly, the multiscale analysis provided by the DWT is well adapted to textural images. From each subband, one may calculate a mean, a standard deviation, an energy or a mean power. Those features, well combined, can yield a powerful texture descriptor.

Recently, Celik and Tjahjadi [5] used the *Dual-Tree Complex Wavelet Transform* (CWT), a complex extension of the DWT, motivated by the (near) shift invariance of its magnitude, and the oriented aspect of these wavelets, and obtained better results with CWT than with DWT. The invariance of the magnitude to shifts makes the extracted feature independent of the precise location of the textural patterns, and so allows a better characterization.

We here propose to extend the analysis using the QWT phase. We use the QWT algorithm from [2], and for comparison, a DWT with the well known CDF 9/7 wavelets<sup>3</sup>. The 3-level decompositions provide 9 subbands for analysis, and an unused low-frequency subband.

### 1.4. The proposed classifier

We propose a simple *k nearest neighbors* classifier. From each image of a *training base* a feature vector describing the texture is extracted, and labelled with its class number. When a *test* image is given, its feature vector is calculated and compared with those in the training base, by using simple Euclidean distances. According to a parameter *k*, the *k* nearest vectors are kept to find the most represented class.

By testing several unknown images for each class, we can calculate a general recognition rate, to evaluate the quality of the feature vector.

This paper first presents a classical magnitude based feature extraction and exposes some results in terms of recognition rate with both the DWT and the QWT. Then a phase based approach is developed, and the combination of the magnitude with the phase is discussed, so the final classifier is carried out.

<sup>2</sup>The undecimated DWT is shift invariant but is not a tight frame, and have too high redundancy.

<sup>3</sup>It's usual to consider that a good separation of the frequency components is well adapted to texture analysis, and the CDF 9/7 wavelets are known to offer that property. Note that the analysis filters are the same size (9-tap) for both the DWT and the QWT

	<i>Energy</i>	<i>Standard Deviation</i>
DWT magnitude	59%	68%
QWT magnitude	66%	68%

**Table 1.** Magnitude based classification recognition rates.

## 2. MAGNITUDE BASED ANALYSIS

### 2.1. Feature extraction

First we calculate the magnitude  $M_{ij} = |q_{ij}|$  of the Wavelet Transform of the given image (*i* and *j* are the discrete coordinate of a pixel). Then from each subband, we consider this two different measures (*i* and *j* span a subband) :

- *Energy* :  $m = \frac{1}{E} \sum_{i,j} M_{ij}^2$  where E is the energy of the whole image minus the energy of the low-frequency subband. This normalisation makes *m* the relative amount of energy with respect to the other relevant subbands.
- *Standard deviation* :  $m = \sqrt{\frac{1}{N} \sum_{i,j} (M_{ij} - \mu)^2}$  where *N* is the number of pixels in the subband, and  $\mu$  is the mean value.

Note that we don't use the low-frequency subband that is not relevant for a texture analysis, especially the low-frequency *energy* that could trivially discriminate images with their intensity, rather than with their textural content.

### 2.2. Test procedure

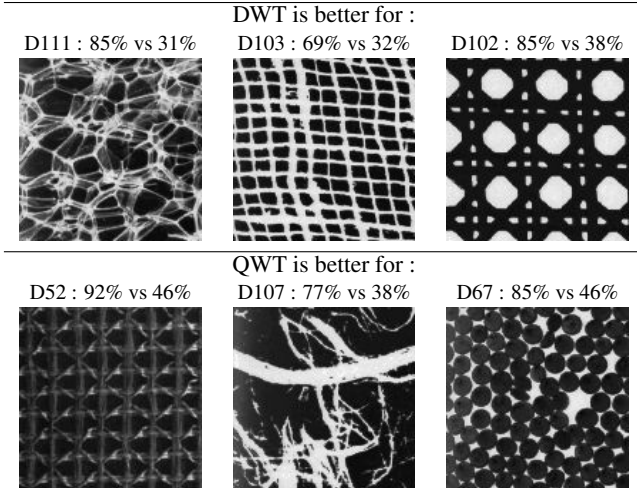
Classes of textures are created from the Brodatz album [6]. Each of the 111 Brodatz textures<sup>4</sup> are cut into 25 square little images (128 × 128 pixels), and separated in a chessboard way to create a *test* ensemble (13 examples the program doesn't know) and a *training* ensemble (12 examples the program knows). So we have a *training* ensemble of 12 × 111 = 1332 little images from which we extract the *training* feature vectors ensemble prior to classification. Then we give each image of the *test* ensemble to the program, and count times the classifier decides the right class, that provides the recognition rate.

### 2.3. Results

We obtained many results depending on the considered decomposition levels (1, 2 and/or 3), the used feature extraction, and the value *k*. Better recognition rates are performed by using the 3 levels of decomposition, and *k* = 3. Since our training base is quite small, we cannot use a high *k* value. On the other hand a too small *k* value makes a poor density estimation, so *k* = 3 is a good compromise, and gives the best global results experimentally. Note that globally for any measure, the recognition rate decreases near monotonically with *k* > 3. We present here (Table 1), and in the sequel, the results with 3 levels and *k* = 3. Note that our results are quite good, considering the heterogeneity of the album (some images are irrelevant in our context of 'rather uniform' texture). In the following, we always use the *standard deviation* measure for the magnitude.

Let us observe particular textures Fig. 1. The D67 texture is composed of randomly placed identical elements, that must be quite

<sup>4</sup>Contains 111 texture images 512×512 B&W, available on the internet at <http://www.ux.uis.no/~tranden/brodatz.html>.



**Fig. 1.** Particular textures (The first  $128 \times 128$  part) better recognized by either the QWT-based or the DWT-based method (highest difference of recognition rate examples)

differently encoded in the DWT, according to the various shifts. In this case the invariance of the QWT magnitude gives us a more robust description of this texture, that explains the better result. To go further, we may note that the D102 and D52 textures are the most *periodic* examples (sampled Fourier spectrum), and the main difference between them is that the D102 contains much more low frequency energy. So maybe the QWT is less efficient for low subbands, this problem is open.

As a first conclusion, the QWT magnitude offers a quite similar performance than the DWT, little better for some measures (*energy*), certainly due to the near shift-invariance. But the work of Bülöw [3] and Chan *et al.* [2] shows that the QWT phase should provide powerful image analysis, so the QWT is obviously not fully exploited here. We present below a review of the interpretation of those phases, and propose a new QWT-based feature extraction.

### 3. PHASE BASED ANALYSIS

#### 3.1. The QWT phase

In his thesis [3], Bülöw demonstrates the importance of the phase in image analysis, defines a Quaternionic Fourier Transform (QFT), a Quaternionic analytic 2D signal, and analytic quaternionic 2D Gabor filters. In a Gabor based texture segmentation, the filtered images are analytic, and form a scale-space analysis of the image, from which Bülöw extracts local magnitudes and phases at each point, to characterize the texture.

First, due to the *QFT shift theorem* [3], it comes that the two first phases  $\phi$  and  $\theta$  indicate a small shift of the encoded feature, around the position of the quaternionic coefficient. This information is analogous to the classical 1D local phase, encoding the shift of an impulse.

Note that in the 1D case, this shift is enough to fully characterize the structure of the feature. Actually, it is the same information (See [3]), since a phase around 0 or  $\pi$  simply means an ‘impulse’ (positive or negative), and a phase around  $\pm \frac{\pi}{2}$  means a ‘step’ (rising or falling), being actually an edge of a shifted impulse. In 2D, this shift is not sufficient to characterize all structures, in particular ‘intrinsically 2D’ structures (e.g. corners, T-junctions), that are more

complex than edges and ridges.

The third phase  $\psi$  completes the structure analysis, and is considered to be a texture feature. Bülöw found that  $\psi$  is seemingly near proportional to a certain  $\lambda$  in a mixture of two plane waves defined as :  $f_\lambda(x, y) = (1 - \lambda) \cos(\omega_1 x + \omega_2 y) + \lambda \cos(\omega_1 x - \omega_2 y)$ . In his application of texture segmentation, he obtains very good results using only  $|\psi|$  and  $\psi$ .

With the QWT, Chan *et al.* [2] use  $\phi$  and  $\theta$  in a disparity estimation algorithm. They consider that since the QWT performs a local QFT analysis, the QFT shift theorem holds approximately for the QWT.

In an other application (‘wedgelet’ representation estimation),  $\phi$  and  $\theta$  are used to calculate the position of edges and  $\psi$  is to calculate their orientation.

#### 3.2. Feature extraction

Now, how can we use the QWT phase to describe textures? Actually  $\phi$  and  $\theta$  are irrelevant because they inform about the position of the features, whereas we are interested in their structure, so we focus on  $\psi$ .

However we must assume that what is described by a  $\psi$ -coefficient is a *local* feature, in that sense it’s a ‘complicated point’, thus not as complex as a whole texture pattern. A pattern would be represented by a set of  $\psi$ -coefficients, so a simple mean of them within a subband would be irrelevant.

We do not aim to carry out high level process such as spatial measures (extrema search, connexity ...) and this work is centered on the local phase interpretation, so we would rather extract a single value for a whole subband, as a global feature, but we found nothing in the literature about the extraction of a simple global phase<sup>5</sup>.

It seems that the simple calculus of the standard deviation (st. dev.) within a subband would be adapted, because it describes a part of the behavior of  $\psi$ . Moreover, since  $\psi \in [-\frac{\pi}{4}, \frac{\pi}{4}]$ , we avoid the usual problems about circular data ( $\pm\pi$  discontinuity), and there is no ambiguity to calculate angle differences or means.

An other idea is to weight the  $\psi$ -deviation by the QWT magnitude. A high magnitude means an important presence of the feature while a low value means ‘no feature’. So it should be interesting not to consider the structure of low magnitude features, and would make the measure more representative. The weight function  $W$  is the magnitude of the QWT coefficients normalized so the sum within the subband is 1, and is integrated in the standard deviation formula as defined below. Here are the two phase measures we use for the feature extraction :

- *St. dev.* :  $m = \sqrt{\frac{1}{N} \sum_{i,j} (\psi_{ij} - \mu)^2}$  where  $\mu = \frac{1}{N} \sum_{i,j} \psi_{ij}$  and  $N$  is the number of pixels in the subband, and  $(i, j)$  span a subband.
- *Weighted st. dev.* :  $m = \sqrt{\sum_{i,j} W_{ij} (\psi_{ij} - \mu)^2}$

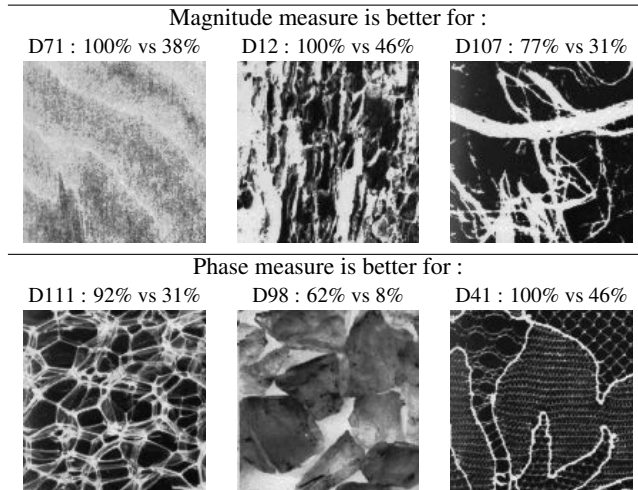
#### 3.3. Results

We used the same procedure as for the magnitude, the simple *st. dev.* measure performs 62% recognition and the *weighted st. dev.* 66% (See table 2). Those results are quite good, and we can see that the weight provides a real improvement.

<sup>5</sup>The *Global Phase Coherence* from Blanchet *et al.* (2008) gives a measure of image sharpness, using the local *phase congruency*, but seems to us not relevant enough in this paper.

	<i>St. dev.</i>	<i>Weighted st. dev.</i>
QWT phase	62%	66%

**Table 2.** Phase based classification recognition rates.



**Fig. 2.** Particular textures (The first  $128 \times 128$  part) better recognized by either the QWT-magnitude or the QWT-phase

By observing the recognition rates within classes, it comes that many textures are substantially better recognized by one of the measures, see Fig. 2 the most significative ones.

It is clear that the images D111, D98 and D41, containing sharp singularities, have many features well described by the  $\psi$ -phase, while noise in D71 and blur in D12 may make the phase less significant.

In that way, the extracted phase information seems complementary to the magnitude information, adding a good sharp feature characterization. Therefore, it may be interesting to combine both to carry out an improved feature vector.

#### 4. COMBINING MAGNITUDE AND PHASE

We now consider the best measures we found for both QWT magnitude and phase. So the *standard deviation* is used for the magnitude, and the *weighted standard deviation* is used for the phase, using the 3 levels so we have 18 measures. We here present some improvement ideas, and the recognition results we obtained for the whole Brodatz album.

First, we simply put the two feature vectors together and obtained 79% recognition, a quite good result. Note that in this case, the recognition rate for  $k = 1$  is better, 80%, so we may use this value which allows a fast calculation of the  $k$  nearest neighbors, that reduces to a *minimum distance search* algorithm.

Note that the two measures are not of the same type, as the first is homogeneous to a magnitude ( $\in \mathbb{R}^+$ ) and the other to an angle ( $\in [0, \frac{\pi}{2}]$ ). This causes a lack of coherence because every measure is viewed the same way by the Euclidean distance. A metric often allows to avoid this inequality, but it is irrelevant here, because the difference is qualitative. Moreover, the metrics we tried did not improve our process.

## 5. CONCLUSION

We proposed a new wavelet based texture classifier using the QWT which offers a magnitude and phase analysis.

To summarize the results, we have a good magnitude measure, the *standard deviation*, giving the same performance with a DWT, and a good  $\psi$ -phase measure, the *weighted standard deviation*, completing the QWT magnitude based analysis, which makes the QWT a better tool for texture classification. By simply concatenating the two measures, yielding 18 features, we obtain 69% recognition rate with the DWT and 79% with the QWT, over the whole Brodatz album.

According to us, the cases where the DWT is *little* superior just correspond to textures for which the shift invariance is not very necessary, and which contain no sharp contours. These cases may rather be interpreted as a similar performance to QWT, as well as those where the QWT is *little* superior.

In contrast, there are some textures substantially better recognized by the QWT, in particular with circular patterns, that demonstrates the importance of its properties.

Actually, a common visual interpretation is hard to find. But there is not any texture in the Brodatz album that makes the DWT really superior to QWT, so not only the QWT performs better results, but this new wavelet transform also keeps the good analysis properties of DWT, making it a real improvement for a texture analysis purpose.

Our results are quite good considering the heterogeneity of the Brodatz album, although we didn't take in account the rotation variance of the QWT, as every texture of a same class has the same orientation, but the QWT phase interpretation is still in its early stages. . .

The monogenic phase of Felsberg and Sommer seems to provide a better local description of 2D signals (rotation invariant). However for now, to our knowledge there exists no monogenic filterbank.

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