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Summary : When studying the oscillations of reed instruments different approaches and simplifications can be used to understand the behaviour of such instruments. Two theoretical approaches were successfully used by previous authors. The first one consists in analysing the small oscillations near the threshold. The second one, applied until now only on cylindrical resonators, analyses the squared oscillations with a lossless model of the resonator. The aim of this paper is on one hand to apply the lossless analysis to conical like instruments and on the other hand to try to understand how the small oscillations model matches with the lossless model. In this study the cone is replaced by resonators which are known to be approximately equivalent and for which the input impedances allow analytical developments. Differences in behaviour between cylinders and cones are demonstrated. Contrary to cylindrical instruments, for conical instruments there is a subcritical threshold and no small amplitude oscillations. Moreover, there is more than one regime at the fundamental frequency in conical instruments. Two of these, similar to the Helmholtz motions in bowed string instruments are studied in this paper. Theoretical results are qualitatively compared with experiments using a blowing machine.

1. Introduction

The sound in reed instruments is generated by the self-sustained oscillations of a mechanical oscillator, the reed. It acts as a valve which modulates the flow into the instrument. The steady-state periodic regimes characterized by their fundamental frequency and harmonic spectrum depend on both the instrument (the bore, the resonator) and the excitator (the reed and the coupling flow supplied by the player).

Many papers deal with the physical modelling of reed instruments and most of them study the clarinet which is, because of its cylindrical bore, easier to model (for a first overview, see for example [1,2,3,4]). Mainly two different approaches have been used. Firstly linear stability of the equilibrium position and the small-oscillations behaviour have been investigated, the static reed being destabilized by one of the acoustical modes of the instrument [5,6,7]. Secondly the use of a model ignoring losses allows the study of the saturated regime (almost squared oscillations). This has been applied to the cylindrical case [8,9,3]. The aim of the present study is on one hand to study how the small oscillations results match with the saturated regime and, on the other hand to study the specific case of a conical resonator. In this paper, for mathematical convenience, a conical resonator will be approximated by a lattice of cylinders having harmonic resonance frequencies [10]. These resonators allow a simple and effective analysis of the oscillations which appear to be equivalent to the Helmholtz motions wellknown in bowed string instruments [11,12].

The present paper considers only the internal acoustic pressure (i.e. inside the mouthpiece) of reed instruments, the external can be approximately deduced through a simple transfer function calculation [13,14,15,16]. Our study is based on a simplified model of the instrument which is divided into two parts : the resonator which is assumed to be linear and the source in which the flow entering into the instrument is assumed to be determined by the pressure drop according to the Bernoulli law. The reed is assumed to be a pure stiffness, its position being assumed to be determined by the pressure difference between its in and outside area. After this introduction, the simple model is detailed and discussed in section 2. Models of the resonators are presented in section 2.1 and a model of the source in section 2.2. An analysis of the model near the threshold, based on previous work, is summarized in section 3.1. Lossless oscillations for a

cylindrical resonator and for a conical like resonator are studied in section 3.2. In section 3.3 we try to understand how the two different approaches can match. For this study the analytical and numerical harmonic balance technique is used.

The proposed model leaves out some secondary aspects: reed damping, reed resonance, anharmonicity of tube resonances, radiation losses, etc. These effects are actually not negligible but our aim here is to try to find a general though schematized description for reed instruments which may not be so much affected by the omissions. To validate the newly found results, experiments using an artificial blowing machine with a real clarinet or saxophone mouthpiece have been carried out and are presented in section 4. These experiments consist in measuring the pressure signal in the mouthpiece versus the blowing pressure in the mouth. The experiments have been made for different resonators.

2. Elementary physical model

2.1. Resonators

Most resonators used in reed instruments are close to cylindrical or conical tubes which are known for having harmonically related frequencies. Cylinders are used for clarinets and cones are used for saxophone, basson and oboe. In practice the cone is truncated because a reed has to take place at the top. This may introduce trouble when the cone is short because the harmonicity of the resonance frequencies becomes poor. It has been shown [17,4,18,19] that, in order to achieve a better harmonicity of the resonance frequencies, the cone must be terminated by a mouthpiece of a volume V_e close to the volume of the missing part of the cone: $V_e = S_e \ell / 3$ where S_e is the input section of the cone and ℓ the distance from the input to the apex of the cone (see Figure 1.a). The impedance of a cone including an equivalent volume $V_e = S_e \ell / 3$ of the mouthpiece is , ignoring losses, given by:

$$Z = \frac{jZ_c}{1/k\ell - k\ell/3 + \cot kL} \quad \text{with} \quad Z_c = \frac{\rho c}{S_e} \ (\rho \text{ density of air and } c \text{ speed of sound}) \qquad (1)$$

This compensation leads to a large anharmonicity when $k\ell$ is not small compared to unity. A higher order improvement can be attained by changing the conicity of the top part of the truncated cone [19] or by optimizing the mouthpiece [20].

Cylinders and cones belong to a larger family of resonators having harmonically related resonance frequencies [10]. Resonators of this family consist in a succession of N cylinders, all of the same length, the n^{th} cylinder having a cross sections area S_n, given by the following relation:

$$S_n = \frac{n(n+1)}{2} S_1 \ , \qquad \text{where } S_1 \text{ is the cross section of the first cylinder} \tag{2}$$

N equal to one is corresponding to the case of the single cylinder: the clarinet, N equal to infinity is corresponding to the cone. Resonators with N in the range of 3 to 5 find limited use in folk instruments. To our knowledge the only instruments made with this shape are bamboo saxophones build by the instrument maker Angel Sampedro del Rio (Argentina). The input impedance Z of such resonators is, ignoring losses and additional masses at the discontinuities, given by (for notation see Figure 1.b):

$$Z = \frac{2jZ_c}{\cot k\ell + \cot kL} \quad \text{with} \quad Z_c = \frac{\rho c}{S_1} \tag{3}$$

where $L=N\ell$ is the total physical length of the instrument and ℓ the length of one cylinder (for a single cylinder $L=\ell$). This impedance, including losses, is plotted on Figure 2.a and 2b for N=2 and N=3 for an input radius of 6 mm and a first resonance frequency of 100 Hz. The impedance appears to be very close, for $k\ell << 1$, to the one for a truncated cone of the same length L, a truncation length ℓ and with a mouthpiece of equivalent volume $V_e = S_e\ell / 3$. Only the term $\cot k\ell$ has to be replaced by $1/k\ell - k\ell/3$. The two terms are very close for low frequencies since $\tan k\ell \cong k\ell + (k\ell)^3/3$ and then $\cot k\ell \cong 1/k\ell - k\ell/3$. However, resonances are strictly harmonic only for the lattice of cylinders. Note that this impedance is the impedance of an open-

open cylindrical tube with a length $L+\ell$, the reed being located not at the input of the cylinder but at a distance ℓ from one open end and L from the other open end [21]. This model shows an analogy between bowed string instruments and reed instruments. If a clarinet can be regarded as equivalent to a string bowed in the middle [8], conical woodwinds can be considered as equivalent to a string bowed at a relative distance ℓ / L from the bridge [21].

For simplicity, the previous discussion ignores losses, radiation, side holes, small bore changes etc. In practice a conical woodwind has only a finite number of resonances. Because of the existence of the cut-off frequency the number of significant peaks varies, for a saxophone or an oboe, from 2 to 8 (somewhat more for a bassoon, see for example [22]). Then the main difference between a conical and a cylindrical instrument is that a conical instrument has at least two resonance frequencies, with the second one being close to the octave of the first one. In this paper we will assume that a set of cylinders is not very different from a truncated cone with an optimised mouthpiece. This is convenient because the impulse response of such a resonator is mathematically simple and allows analytical developments in the time domain. Using the open-open cylindrical tube analogy, such a resonator can be modeled as a double delay line [3, 21].

2.2. Source model

The acoustic source consists in a mechanical oscillator (the reed) acting as a pressure-controlled valve modulating the flow blown into the pipe of the musical instrument (the air resonator). Under some conditions the reed can oscillate. Its oscillation is controlled by the pressure difference on both sides (the mouth pressure and the internal mouthpiece pressure). The elementary model presented below is the same as the one used e.g. by [3].

The flow through the reed slit is controlled by the aperture between the reed and the mouthpiece, and by the velocity of the air depending nonlinearly on the pressure difference (Bernoulli relation). This gives two conditions:

$$\left|\begin{array}{l} u(t) = w(H+h)v \\ \Delta p(t) = \frac{1}{2}\rho\, v(t)^2 \end{array}\right. \qquad (4)$$

where ρ is the air density, u is the instantaneous volume flow, v the velocity assumed to be uniform in the reed channel, H the height of the slit in the absence of flow and w the effective width of the reed channel, h the deviation of the reed from its rest position in the absence of flow (see Figure 3).

To obtain the simplest model we assume the reed to be a linear spring. Then h is directly proportional to the pressure difference. Writing $h = -H\Delta p / P_M$, where P_M is the limit value of the static pressure in the mouth P_m above which the reed remains closed, from equations (4) an instantaneous relationship between flow and pressure is derived:

$$\left|\begin{array}{ll} u(t) = wH(1-\frac{\Delta p}{P_M})\sqrt{\frac{2|\Delta p|}{\rho}}sign(\Delta p) & \text{if} \quad P_m < P_M \quad \text{with} \quad \Delta p = P_m - p\,. \\ u(t) = 0 & \text{if} \quad P_m \geq P_M \end{array}\right. \qquad (5)$$

This function is displayed on Figure 4 for different values of P_m.

When the pressure difference is large the reed is blocked against the mouthpiece, the entering flow is zero. Sufficiently far above the threshold of oscillations, there are large amplitude oscillations where the so called "beating reed" oscillates between two states : an open state and a blocked state. Notice that the transition between the two states implies a discontinuity of the derivative of the velocity at the particular value where the reed begins to beat (P_m=P_M/2 in lossless model of a clarinet). In practice this discontinuity does not exist, since the lay is curved due to which the reed stifness increases.

The non-linear relationship between flow and pressure depends on the parameters of the system : the main ones are the rest height H and the mouth pressure P_m which is the main control parameter of the dynamical system.

This elementary model can be summarized in two equations. The first one is a condition in the frequency domain, it is linear and describes the resonator including the equivalent volume of the reed (the input impedance relation). The second one is in the time domain and is nonlinear (equation (5)). The system is often presented as a closed feedback loop [23] used in free oscillations because the input oscillating variable is zero. In other words the mouth pressure is assumed to be constant. Fortunately for the player this loop is unstable, so it can generate periodic oscillations ! The following section summarizes the theoretical results about these periodic regimes from small to large oscillations.

Many assumptions behind this elementary model require further discussion (for a more sophisticated model and for discussions about the limit of each assumption see [2]). For example the dynamic model of the reed is the simplest one: damping has been ignored even if it is essential to avoid the reed to oscillate in its own resonance frequency (the wellknown squeaking, regime "in tempo" in Bouasse terminology [24]), as shown by [6]. Since this mechanical resonance of the reed is not taken into account this regime does not exist in the model. To take this into account a third variable as the reed tip position would be needed which complicates drastically the study of the equivalent dynamical system (see for exemple [25] who takes into account the effect of the reed resonance in the pressure spectrum). As explained before, the beating-reed model is crude: in reality the reed is coiling up the lay of the mouthpiece which means that the equivalent mechanical parameters are not independent of the reed position. Subtle hydrodynamic effects in the reed channel exist in static regimes [26] but these effects have not been observed in dynamical regimes. The mouth cavity could be taken into account as a second resonator in series with the instrument [27,28]. The model does not consider the volume velocity induced by the motion of the reed. This effect can be included in the input impedance as an equivalent volume at the input of the instrument [29, 18, 19].

3. Theoretical results : periodic oscillations, from small to large oscillations

3.1. Small oscillations near threshold

A trivial solution can be easily found from equation (5). This solution corresponds to a steady-state, non-oscillating regime of the acoustical variables, and then to an equilibrium position of the reed. Another trivial solution is obtained when the static mouth pressure P_m is larger than the particular value P_M, the reed is blocked on the mouthpiece and thus the flow into the resonator is zero. These equilibrium positions are not always stable, allowing other permanent regimes like the periodic one. The question is whether or not the equilibrium is stable (linear or local stability of the trivial solution of the problem). Using the feedback loop analogy, it is known as the free oscillation linear stability problem in a closed loop obtained when the nonlinear component of the loop is linearized around the trivial solution. Then the linear stability of the reed can be studied with respect to each resonance of the impedance curve. Wilson and Beavers [6] have published a nice study illustrated by a set of experimental results obtained by using an artificial mouth with cylindrical tubes of variable length and different single reeds. For each acoustic resonance of the tube, a threshold of instability can be defined by two values : a threshold frequency and a threshold mouth pressure P_{th}. The linear stability of a single reed coupled with an acoustical resonator has been studied for example in [30, 31, 32, 33]. If the resonator has no losses the threshold mouth pressure P_{th} is exactly equal to $P_M/3$ (see e.g. [3]).

The behaviour of this self-sustained oscillator near the threshold has been studied in [7]. The study is based on a Fourier expansion of the periodic internal pressure. The resonator is represented in the frequency domain by an impedance (or admittance) relation applied for each corresponding component of the two Fourier series of the acoustic velocity and pressure; the excitator is represented by a third order Taylor series expansion of the nonlinear relation in the vicinity of the threshold:

$$u = u_{oo} + Ap + Bp^2 + Cp^3 \qquad (6)$$

where u_{oo} is the flow when there is no oscillation and A,B,C are the three coefficients of the series expansion.

The balance between the harmonic components gives an infinite number of equations. To solve this set of equations, some aditional hypotheses are needed. In the small oscillations analysis, it is assumed that the fundamental component of the pressure signal P_1 is not vanishing

and this coefficient is considered as a first order quantity, P_1 is small compared to the mouth pressure P_m, the other Fourier components P_n are at most of the same order as P_1. In addition, it is assumed that the admitance Y_1 corresponding to the fundamental frequency is different from all the other Y_n's (admittance corresponding to the n^{th} harmonic). After careful inspection of the order of magnitude of each term (see Section 3 of [7]) it can be shown that all second and higher order of P_n with n>1 can be neglected. After the suppression of these higher order terms, the equations for the first three components are then found to be :

$$\begin{cases} Y_1 P_1 = AP_1 + 2BP_1P_2 + 3CP_1^3 \\ Y_2 P_2 = AP_2 + BP_1^2 \\ Y_3 P_3 = AP_3 + 2BP_1P_2 + CP_1^3 \end{cases} \qquad (7)$$

where the impedance is assumed to be zero at zero frequency ; P_1, P_2 and P_3 are the first three complex amplitudes of the acoustic pressure defined by its Fourier expansion. Y_1, Y_2, Y_3 are the first three admittances corresponding to the first three resonance frequencies.

From these equations, P_1 is found from:

$$P_1^2 = \frac{(Y_1 - A)(Y_2 - A)}{2B^2 + 3C(Y_2 - A)}. \qquad (8)$$

and the expressions for P_2 and P_3 as functions of P_1 are:

$$P_2 = \frac{BP_1^2}{Y_2 - A} \qquad (9)$$

and

$$P_3 = \frac{P_1^3}{Y_3 - A}\left(C + \frac{B^2}{Y_2 - A}\right). \qquad (10)$$

These results are characteristic for small oscillations when they exist (right member of (8) positive). The n^{th} harmonic increases as the nth power of the amplitude of the first one (equation (9) and (10)), as reported earlier in [5]. P_1 is assumed to be real because the phase of the

fundamental frequency component can be assumed to be zero, there is no other zero phase reference in such a self-sustained oscillator. In order to simplify the discussion, only the non dispersive case is discussed: the first two resonances of the resonator are assumed to be perfectly harmonic: if F_1 is the resonance frequency with Z_1 real, Z_2 at the frequency F_2=2F_1 is real as well.

Assuming, at the frequency F=F_1, the right hand side of equation (8) to be purely real, only its sign needs to be discussed. Assuming A is slightly increasing on both sides of the threshold value A_{th1} (A_{th1} is the particular value of A equal to Y_1 defining the threshold mouth pressure P_{th1}), the criterion for a "direct" bifurcation is (see [7]) : infinitely small oscillations exist for values of the control parameter $A>A_{th1}$ (or $P_m>P_{th1}$; A is an increasing function of P_m) for which the trivial solution is not stable. If, on the opposite, the trivial solution is stable (case $A<A_{th1}$, or $P_m<P_{th1}$), the bifurcation is called "inverse". The criterion defining the bifurcation state is depending on both the characteristics of the resonator, via the value of (Y_2 - Y_1) and the excitator, via the parameters A, B and C of the Taylor expansion, equation (8). The condition for a direct bifurcation is given in [7]:

$$C < \frac{-2B^2}{3(Y_2 - Y_1)}. \qquad (11)$$

The results can be summarized in a plane picture, axes being defined by (Y_2-Y_1) and C, where lines separate the direct and inverse bifurcation behaviour areas (see Figure 5). For the simplified excitator model used here, it can be shown that A is an increasing function of the mouth pressure P_m and that C is always negative. For a cylindrical resonator (clarinet case), Y_2-Y_1>>0 because of the anti-resonance at F_2=2F_1, so the behaviour is this of a direct bifurcation. Notice that when Y_2 and Y_1 are of the same order of magnitude (i.e. for a conical like resonator), results depend on the sign of (Y_2-Y_1). It will be shown in section 3.3 that, in that case, linear analysis near the threshold is not sufficient.

The general properties of the Hopf bifurcation [34,35] imply that small oscillations are stable for the direct bifurcation case. They are instable for the inverse bifurcation case ; this does

not mean that stable oscillations are never possible, but, if they exist, they necessarily occur for larger values (not infinitely small and not quasiharmonic) of the oscillation amplitude: the oscillation may jump discontinuously to a finite amplitude in the vicinity of the instability threshold of the equilibrium reed position, for mouth pressures slightly smaller than $P_{th}1$. In that case the subcritical threshold of oscillation P_{sc} is lower than the threshold of linear instability of the equilibrium position $P_{th}1$. It implies that an "hysteresis phenomenon" is present: the threshold of oscillation may occur for different values of the control parameter depending on whether the mouth pressure is decreasing or increasing.

These results can be applied for the lattice of cylinders defined in section 2.1. for N=1, 2 and 3. When N=1 (a cylindrical resonator, a clarinet), $Y_2 >> Y_1$ and the third coefficient C of the Taylor expansion (equation (6)) is always negative (assuming the source model presented in section 2.1.), the bifurcation is direct (see Figure 5.). When N=2 (a lattice of 2 cylinders), the impedance peak at the first resonance frequency F_1 is higher than the second one at F_2 (see Figure 2.a.) but of the same order of magnitude (Y_2 - Y_1 positive but close to 0), the bifurcation is inverse (see Figure 5.). In that case the solutions given by the small oscillations theory are instable periodic regimes corresponding to values of the mouth pressure P_m smaller than the threshold pressure P_{th1}. To get stable periodic regimes we need another kind of calculation: it will be presented in a following section (§3.3.1.). Notice that we could do the same analysis around the threshold corresponding to the second resonance frequency : the bifurcation corresponding to the octave regime is direct. When $N \geq 3$ (a lattice of 3 or more cylinders), the impedance peak at the first resonance frequency F_1 is lower than the second one at F_2 (see Figure 2.b.), knowing that the parameter C is always negative the bifurcation is direct (see Figure 5.). But $Y_2<Y_1$ implies as well that the linear threshold P_{th2} corresponding to the second resonance is lower than the threshold P_{th1} corresponding to the first one. Moreover, the direct bifurcation originating from P_{th1} corresponds to another solution as for $Y_2>Y_1$ because the sign of P_2 is different. In the following sections, it will be shown that this corresponds to an inverted Helmholtz motion.

3.2. Large amplitude oscillations : the lossless resonator approximation

Results of section 3.1. are valid for any kind of resonator but only for small oscillations. They do not give information about the behaviour at large amplitude. Another approximate approach is to ignore losses in the instrument. Even if that is not very realistic (e.g. the spectrum is independent of the amplitude) this approach can give basic informations on the behaviour of real instruments. Ignoring losses means that the impedance becomes infinite at resonance. Assuming harmonic resonances and a periodic solution of which the fundamental frequency is equal to a resonance frequency of the resonator, all the volume velocity a.c. components are then equal to zero. Thus the flow through the reed aperture is constant. Therefore, the pressure can have only two different values. There are a large number of solutions (see [12]). The Helmholtz motions are the solutions for which the pressure is equal to a value for one part of the period and to another value during the other part. The signal then is a square signal. Assuming that the input impedance at zero frequency is equal to zero, the mean value of the pressure signal is zero too. A consequence is that the ratio between the absolute values of the two pressure values is equal to the ratio of the durations of the two parts of the signal. This ratio is determined by the resonator. For a resonator whith an impedance given by equation (3), a ratio equal to N is a possible solution [36, 12]. For N=1 (clarinet) the signal is symmetrical, for N>1 it is asymmetrical. In that case two solutions are possible namely the longer part of the signal is positive (open reed) or negative (closed reed). The first one is the standard Helmholtz motion and the second one the inverted Helmholtz motion (see Figure 6). The spectrum of these idealised periodic oscillations is fixed for a given value of N, i.e. for every n, P_n/P_1 does not depend on the mouth pressure. The coefficients P_n of the corresponding Fourier series are given by the following formula :

$$P_n = -sign[(-p)^N]\frac{\sin(X_n)}{X_n} \text{ with } X_n = \frac{n}{N+1}\pi . \qquad (12)$$

where p is the value of the longest part of the signal; p is positive for the standard Helmholtz motion and negative for the inverted Helmholtz motion. All harmonic components are real, the first one being defined as real and positive. Coefficients of even harmonics are negative for the standard Helmholtz motion and positive for the inverted Helmholtz motion. In particular

coefficient P_2 is negative for the standard Helmholtz motion and positive for the inverted one. Amplitudes of harmonics N, $2N$, $3N$... are equal to zero. In a cylindrical case, N=1, amplitudes of even harmonics of the internal pressure are equal to zero. As emphasized by Kergomard [3], in the limit case of a beating reed, when $P_m > P_M / (N+1)$, the permanent zero value of the flow entering into the mouthpiece is a paradox : during one part of the period the reed is blocked, during the second part the pressure is the same on both sides of the reed. This is due to the fact that losses are ignored. In fact this model does not give information on the flow entering through the reed in a real instrument.

Starting from the work of McIntyre & al [8], Maganza & al [9] developed an attractive graphical method to find the periodic regimes (squared oscillations). More recently Kergomard [3] proposed to describe the dynamical behavior of such a system characterized with only two dimensionless parameters. The periodic oscillations of the resonators based on a lattice of cylindrical tubes without losses have been studied as well [37]. This motion is analysed in studies dealing with the bowed strings (see for example, [8, 38, 36]).

The solutions for p can be found by solving the equation :

$$u(p) = u(-Np) \qquad (13)$$

where u is given by equation (5). We obtain the following two solutions:

$$p_+ = \frac{(N-1)(2P_M - 3P_m) + \sqrt{[(N-1)P_M]^2 + (P_M - P_m)(3P_m - P_M)}}{2(N^2 - N + 1)} \quad \text{for } P_m < \frac{P_M}{N+1}$$

$$p_+ = P_m \quad \text{for } P_m > \frac{P_M}{N+1} \qquad (14)$$

$$p_- = \frac{(N-1)(2P_M - 3P_m) - \sqrt{[(N-1)P_M]^2 + (P_M - P_m)(3P_m - P_M)}}{2(N^2 - N + 1)} \quad \text{for } P_m < \frac{NP_M}{N+1}$$

$$p_- = -P_m / N \qquad \text{for } P_m > \frac{NP_M}{N+1}$$

The corresponding solutions are plotted for N=1 to 4 in Figure 7. Positive solutions correspond to standard Helmholtz motions and negative solutions correspond to inverse Helmholtz motions. For a non-beating reed, $p(P_m)$ are elliptic curves. For a beating reed, the amplitude of the pressure signal is proportional to the mouth pressure. For the standard Helmholtz motion (positive values of p) the slope is equal to unity whereas it is equal to $1/N$ for the inverted one (negative values of p). Then the level is much lower for the inverted motion : the mouth pressure must be N times higher for the same level. The beating reed regime occurs for $P_m > P_M / (N+1)$ for the standard Helmholtz motion and for $P_m > NP_M / (N+1)$ for the inverted one.

According to this lossless approximation, an inverse bifurcation occurs for $N > 1$ and for the standard Helmholtz motion. In that case, there are no infinitely small stable oscillations and the lower values for the pressure level are 0.192, 0.189, 0.166 P_M i.e 68, 84, 89% of the mouth pressure P_m, for respectively N=2,3 and 4. The non-beating reed regime occurs between the (subcritical) threshold of oscillation and the beginning of the beating reed regime. The threshold of oscillation is obtained by solving the equation $p_+ = p_-$. It is equal to 0.282, 0.226 and 0.186 P_m / P_M for N=2,3 and 4 respectively. The begining of the beating reed regime is given by $P_m / P_M = 1/(N+1)$ i.e 0.333, 0.25, 0.2 P_m / P_M for respectively N=2,3 and 4. It appears that for N>1 the non-beating reed regime range is very small and probably difficult to observe experimentally. Furthermore, it must be pointed out that there is no theoretical limit in this model for the amplitude of the pressure signal, in particular there is no reason why the oscillation should be unstable when the mouth pressure is higher, during a part of the period, than the pressure P_M.

For the inverted Helmholtz motion, the bifurcation is always direct but only from a stability analysis it could be concluded if small oscillations are possible or not.

3.3. From small to large oscillations, approximated calculations

The two extreme situations for a cylindrical resonator case appear to be well understood:

(i) for very small oscillations, in the direct bifurcation case, when the excitation pressure level increases, the amplitude of the harmonics increases as the n^{th} power of the amplitude of the first one [5]. The amplitude is also a function of the value of the input impedance value for a given harmonic.

(ii) For a non-dissipative cylindrical resonator the spectrum is that of a squared signal, independent of the excitation pressure level. The amplitudes of the odd harmonics are proportional to $1/n$ and the amplitudes of the even harmonics are zero.

Strictly speaking these results are not compatible: near the threshold there should exist a range of P_1 for which $P_1 > P_2 > P_3 > 0$. Then even harmonics amplitudes cannot be strictly equal to zero. Actually the range for which $P_2 > P_3$ is very small and probably not measurable. Consequently, the approximation $P_2=0$ remains very good for the spectrum of the internal pressure (see the experimental results in Section 4). This kind of approximation is part of a fruitful one to get realistic results on the internal spectrum of the clarinet for a large range of mouth pressures (see the companion paper [39]). What are the intermediate situations between these extreme cases ? To answer this question one needs to know the relationship between the spectrum and both the excitation level and the dissipation factor. The answer is partially given in [39] and summarized in section 3.3.2. Another question is the understanding of how the two different approaches can match when the number N of cylinders of the lattice is larger than unity: from the small oscillations analysis, there is one branch originating from the linear threshold, but from the lossless analysis, there are two solutions (giving the standard and the inverse Helhmholtz motions). In the following section (3.3.1) is shown how the matching between one branch towards the two Helmholtz motions can be understood ?

3.3.1. Inverse bifurcation for a two-harmonic system

In section 3.1.2. a set of equations based on the harmonic balance technique and on the small oscillations approximation is given (see equations 7). To extend the calculation to finite oscillations, a system with only two resonances is now studied (we could say that it is the simplest nontrivial resonator). Above the second resonance the input impedance is assumed to be negligible. This approximation may correspond to the higher notes of the first register of the saxophone or the oboe for which only two resonance frequencies remain under the cutoff frequency. For simplicity these two resonances are assumed to be harmonic (the second one being exactly the octave of the first one, F_2=2F_1). The amplitudes of harmonics number n are equal to zero for n>2, and the infinite number of equations obtained by the harmonic balance technique leads to only two equations for the two harmonics of non zero amplitude P_1 and P_2:

$$\begin{cases} Y_1 P_1 = AP_1 + 2BP_1 P_2 + 3CP_1 (P_1^2 + 2P_2^2) \\ Y_2 P_2 = AP_2 + BP_1^2 + 3CP_2 (2P_1^2 + P_2^2) \end{cases}, \qquad (15)$$

Because the two resonances are harmonic these components are real, P_1 being positive and P_2 either positive or negative. This set is different from the one of section 3.3.1 obtained for the small oscillations (7) because P_3 is equal to zero and no terms have been ignored in the expansion of the harmonic balance applied to the first two harmonics.

From first equation of (15) P_1^2 is expressed as a function of P_2, and substituted in the second equation of (15) to obtain the following 3rd degree equation in P_2 :

$$P_2^3 + \frac{2B}{3C} P_2^2 + \frac{1}{9C}\left[(Y_2 - A) - 2(Y_1 - A) + \frac{2B^2}{3C} \right] P_2 - \frac{B}{27C^2}(Y_1 - A) = 0 , \qquad (16)$$

P_1 being deduced from the following equation (derived from the second equation of (15)) when its right member is positive:

$$P_1^2 = \frac{1}{3C}\left[(Y_1 - A) - 2BP_2 \right] - 2P_2^2 \qquad (17)$$

Equation (16) has one, two or three real solutions depending on the sign of the discriminant (negative, zero, positive). On figure 8.a are plotted the P_2 solutions for the two-harmonic resonator of which the amplitudes of the peaks are taken equal to those of the first two peaks of

the lattice of two cylindrical tubes presented before (see the corresponding impedance curve in Figure 2). Firstly, the P_2 solutions corresponding to unrealistic solutions are rejected (values of P_2 having an order of magnitude equal to two times the minimum closing reed pressure P_M). Secondly, one of the remaining two solutions is rejected for a particular range of P_m because it leads to negative values of P_1^2 (see first solution for P_1^2 for which $P_{th1} < P_m / P_M < P_{th2}$ in Figure 8.a). This range is limited by the linear threshold of oscillation P_{th1} for the fundamental regime and a value a little bit larger than the linear threshold of oscillation P_{th2} for the octave.

The bifurcation diagram (Figure 8.b) shows two branches :

- First branch is originating from the linear threshold (P_m=P_{th1}) according to an inverse bifurcation as predicted by the small oscillations analysis and corresponding to a standard Helmholtz motion. In this case, the system now has a threshold (subcritical mouth pressure P_{sc} i.e. the minimum value of the mouthpressure for which there is an oscillation) which is actually smaller than the threshold value P_{th1} corresponding to the first resonance (frequency F_1). At this value, the oscillation amplitude may jump discontinuously towards a finite level, there is no infinitely small (quasi-harmonic) stable oscillation. An extensive study should be done to show how the subcritical threshold mouth pressure is depending on the nonlinearity and resonances parameters.

- Second branch does not link up the P_m axis (no infinitely small oscillations) and corresponds to an inverted Helmholtz motion. It can be seen that P_1 tends to 0 when P_m is decreasing along this second branch (see Figures 8.a and 8.b). The branch "seems" to come from the second threshold value P_{th2} (threshold corresponding to the second resonance, frequency F_2). There is no infinitely small oscillation at the fundamental frequency for the inverted Helmholtz motion but strictly speaking, it doesn't correspond to an inverse bifurcation. When the control parameter P_m is decreasing, P_1 is tending towards 0 while P_2 is tending towards a finite value : the branch is matching the octave regime branch coming from the second threshold value P_{th2} and corresponding, here, to a direct bifurcation. This phenomenon reminisces a period doubling bifurcation coming from the octave regime.

The lattice of two cylindrical tubes is know studied extensively by numerical harmonic balance [40], and the bifurcation diagram is displayed in Figure 9. Dissipation in the resonator is taken into account, but not dispersion. There are two linear thresholds corresponding to the first two resonances of the resonator : first one corresponds to the fundamental regime and second one to the octave regime. The first one is lower than the second one because the first resonance peak is higher in amplitude than the second one. The behaviour of the small periodic oscillations in the vicinity of the two linear thresholds are those predicted by the theory : an inverse Hopf bifurcation for the fundamental regime, a direct one for the octave regime. The other fundamental regime corresponds to an inverse Helmholtz motion for the large amplitude oscillations. When the mouth pressure is decreasing, this second fundamental regime tends towards the octave regime for a finite amplitude: there are no infinitely small oscillations corresponding to this second fundamental regime. So there is no contradiction with the linear analysis near the threshold of oscillations. Furthermore, the second harmonics in inverse and standard Helmholtz motions haven't the same sign as it is the case for the lossless model. These results are well matching with the two theoretical well-known cases : the small oscillations case with one unstable branch originating from the linear threshold (inverse Hopf bifurcation), the lossless oscillations approximation with two limit branches, the inverse and standard Helmholz motions, the first one continuing the small oscillations branch, the second one being linked with the small oscillations octave branch without infinitely small amplitude solutions. It was verified that the bifurcation diagram of the lattice of cylinders (N=2) obtained by numerical harmonic balance (figure 9) is qualitatively similar to the one obtained analytically with the "two peaks resonator" (in the last case, the impedance is 0 for frequencies greater than F_2 ; the non-linear relation is approximated by its 3rd order Taylor's expansion ; the beating reed is not considered).

The bifurcation diagrams studied in the previous part of this section correspond to resonators for which $Z_1>Z_2$ as it is the case for a lattice of two cylindrical tubes (N=2). For N greater than 3, $Z_2>Z_1$ (see Figure 2.b) and then the threshold P_{th1} is higher than P_{th2}. But we already know that according to the small oscillations analysis there is a modification of the behaviour near the threshold : the bifurcation becomes again direct when Y_2-Y_1 becomes negative, i.e. when $Z_2>Z_1$ (see Figure 5). Then, the matching of this small oscillations behaviour with the two Helmholtz motions is different from the previous cases. The two peaks resonator

with $Z_2>Z_1$ is studied similarly according to equations (16) and (17). The solutions are displayed Figures 10.a and 10.b in the same way than those displayed Figures 8.a and 8.b.

In accordance with the small oscillations analysis, the branch originating from the threshold associated to the first resonance ($P_m=P_{th1}$) is corresponding to the inverted Helmholtz motions with a direct bifurcation (case where (Y_2-Y_1) and C are negative in Figure 5.). The branch coming from the second threshold ($P_m=P_{th2}<P_{th1}$) corresponding to the octave regime is a direct bifurcation as well (Y_4-Y_2 is positive and infinite because we have assumed that Z_4=0). The second branch corresponding to the standard Helmholtz motion is linked with the small oscillations octave branch. There is no infinitely small amplitude solutions for this branch. Then it is very similar to an inverse bifurcation with a threshold of oscillation P_{sc} smaller than P_{th2}. The nature of this inverse bifurcation is different from the one described before when $Z_2<Z_1$: in the first case ($Z_2<Z_1$), the inverse bifurcation is the one predicted by the small oscillations analysis coming from $P_m=P_{th1}$, in the second case ($Z_2>Z_1$), the inverse bifurcation is originating from the octave threshold ($P_m=P_{th2}$). In that case, the periodic oscillations existing for the lowest values of P_m are far from the linear threshold and then the small oscillations solutions is valid only for the inverted Helmholtz motion. This shows a limit of small oscillations analysis near the threshold of instability of the trivial solution: it misses the standard Helmholtz motion which is certainly the only one played in practice. Experimentally because of the quasi inverted bifurcation, no small oscillation corresponding to the standard Helmholtz motion can be observed for N different from unity and then the difference in the bifurcation diagrams of Figures 9 and 11 may have no consequence in practice. It can be considered that the bifurcation is inverse for every N larger than 2 even if it is a strict inverse bifurcation only for N=2.

3.3.2. Harmonic balance technique applied to a cylindrical resonator

In sections 3.1 and 3.2, theoretical results on small and large oscillations have been given. The first are valid in the vicinity of the threshold, the second are valid for lossless resonators

(squared oscillations for a cylindrical resonator). But there is, up to now, no theoretical result about the spectrum in-between these two extreme situations.

A companion paper [39] discusses with many details this problem, for a cylinder and using a model of the excitation mechanism assuming a non beating reed. Using a method called ″variable truncation method″, this paper proposes approximated formulas for the first harmonics, which are compared to numerical results obtained by the harmonic balance technique. For a resonator with harmonically related frequencies, assuming that even harmonics are very weak in the internal spectrum, an expression for the amplitude of the fundamental is obtained:

$$P_1^2 = \frac{A - Y_1}{3C} \quad , \qquad (18)$$

where A and C are the coefficients of the Taylor expansion (6) and Y_1 the admittance corresponding to the first resonance frequency. This equation is equivalent to equation (8) with Y_2 assumed to be infinite but it is demonstrated in [39] that it remains a good approximation beyond the threshold of oscillation.

With the same hypothesis, the following formula is obtained for the third harmonic:

$$\frac{P_3}{P_1} = -\frac{1}{3}\frac{A - Y_1}{A - Y_1 + Y_3 - Y_1} \quad , \qquad (19)$$

where Y_3 is the admittance corresponding to the third resonance frequency. This equation is compatible with equation (10) when the quantity (A-Y_1) is small, i.e. near the threshold of oscillation and Y_2 assumed to be infinite. But when (A-Y_1) increases and tends to the infinity, the equation (19) is drastically different from equation (10): the denominator does not vanish and the ratio tends to the value corresponding to the square signal. Actually (A-Y_1) tends to a finite value when the mouth pressure increases to the beating threshold and then the ratio P_3/P_1 tends to a value slightly lower than 1/3.

Results for higher odd harmonics are more intricate but the behaviour is qualitatively the same as this for the third harmonic.Equation (19) can be rewritten, using (18), as follows:

$$P_3 = -\frac{CP_1^3}{3CP_1^2 + Y_3 - Y_1} \quad . \qquad (20)$$

This result has been proved to be valid for the full nonlinear model (5) as well as for the Taylor expansion (6) [39]. It exhibits two different slopes for the asymptotic lines: $P_3 \propto P_1^3$ near the threshold and $P_3 \propto P_1$ above. Similar results for the higher order odd components show that $P_n \propto P_1^n$ near the threshold and $P_n \propto P_1$ far above it. In the latter case all the lines corresponding to the odd harmonics are parallel (see for example experimentals results in Figure 12.c).

Concerning the value of the threshold of oscillation, theory based upon the model described in section 2.2 [3, 39] shows that it is significantly higher in practice than predicted with the lossless model. Writting $A=Y_1$ and solving equation (37) in [39], the threshold pressure P_{th} is approximately given by:

$$P_{th} \cong \frac{P_M}{3}\left(1 + \frac{Y_1}{wH}\sqrt{\frac{2\rho P_M}{3}}\right) \quad , \qquad (21)$$

where H the height of the slit in the absence of flow and w the effective width of the reed channel, ρ the density of air and P_M the minimum closing pressure (see section 2.2, Figure 3). Typically this leads to P_{th} =0.4 P_M instead of P_{th} =1/3 P_M with the lossless model.

4. Experimental results

4.1. Experimental set-up

Many authors have developed artificial mouths for blowing single reed instruments (see e.g. [6, 30]). The present one, developped by Gazengel [41], consists in a hermetically closed chamber (volume 1500 cm3) fed by a high-pressure air supply. The mouth pressure in the chamber is controlled. The "embouchure" is formed by an artificial lip, a latex tube filled with water, on a metal backing positioned by a screw (see [41]). The water-filled tube appears to be a noticeable improvement over the foam plastic used in the experiments in the past.

The static pressure in the mouth is measured with a pressure transducer. The internal acoustic pressure (in the mouthpiece) is measured with a microphone (an acceleration-compensated piezo-electrical gauge, type PCB M116B). For the same resonator a lot of different regimes have been observed and results are very dependent on the "embouchure" i.e. on the lip shape, position and pressure. Some of these regimes are not periodic (multiphonics), even for the fundamental frequency different regimes can be seen which can not always be ascribed to a Helmholtz motion. We have done measurements of the input pressure in a clarinet mouthpiece with various resonators (lattice of cylinders with N=2, 3, 4 as defined in section 2.1). Results are shown in Figure 11. In the pure Helmholtz motion a rectangular (square for N=1) signal should be observed. The ratio between the two parts of the period should be equal to N if N is an integer [36]. Measured signals looked a little bit "rounded" [21], like those obtained for a violin [42,43], but the ratio seems to be preserved. When calculated with the method given by Schelleng [42] these are given by 2.01,3.02 and 4.10 for N=2,3 and 4, respectively (Figure 11).

For a given "embouchure", the periodic permanent regimes are analyzed with a spectrum analyser. These measurements are carried out from the oscillation threshold up to large amplitude oscillations.

4.2. Cylindrical resonator

The experiment for a cylindrical resonator has been carried out with a clarinet mouthpiece on a PVC tube of inner diameter 15mm and length 30cm. The dimensions of this are close to those of a clarinet playing with about half its holes closed. For a given "embouchure" the experiment consists in varying the mouth pressure and analysing the pressure signal in the mouthpiece. The fundamental frequency obtained varies approximately from 190 Hz near the threshold to 205 Hz for the highest pressure.

The shape of the signal, when increasing the blowing pressure, varies from a sinusoidal one to a more or less square one. For a certain pressure value P_{M+} the signal vanishes and the reed closes. When the mouth pressure is slowly decreased the reed remains closed until the

pressure reaches a certain value for which the reed starts oscillating again. This pressure is the minimum pressure for which the closed state is stable and is assumed to be the minimum closing pressure P_M.

The RMS input pressure is plotted against mouth pressure P_m in Figure 12.a. The threshold pressure is less than a third of the minimum closing pressure P_M. The amplitude of the first harmonic is proportional to the mouth pressure for the main part of the curve, corresponding to the beating reed regime. This beating reed regime seems to start at a value which is approximately one half of the minimum closing pressure P_M. Considering the model presented in section 3.3.2 all the qualitative behaviour is correctly predicted, but the precise ratios of the threshold of oscillation to the minimum closing pressure P_M is lower in the experiment than in the model. This discrepancy can be interpreted as a consequence of the bad representation of the beating phenomenon: the model does not take into accout the curvature of the lay. The slope of the curve is approximately 0.7 instead of 1 for an ideal Helmholtz motion. It is possible that this difference is due to losses.

The ratio of the third and fifth harmonic to the first harmonic as presented Figure 12.b indicates that the spectrum tends to a saturated regime for which the spectrum becomes independent of the amplitude as predicted by equation (19). This may correspond more or less to the square signal analysis despite that relative amplitudes of the harmonics are lower than that for a true square signal. An interesting feature is that the saturation of the spectrum of the lowest odd harmonics begins below the beating threshold and is prolonged above this threshold.

Near the threshold the evolution of the signal indicates that the bifurcation is a direct one. The growing of the harmonics 3 and 5 is according to the law stated by Worman (Figure 12.c). Figure 12.c is very similar to the one obtained for the external pressure and also to that obtained with a professional musician by Benade [18]: up to a certain pressure, called change of feel in [18], a constant spectrum is observed.

4.3. Lattices of cylinders and saxophone

The procedure for these resonators is the same as for a cylindrical resonator : the blowing pressure is increased until the reed closes and then decreased to find the minimum closing pressure P_M. For these resonators the oscillations start for a given mouth pressure but persist when decreasing the mouth pressure below this value: a hysteresis is observed (Figure 13.a). Moreover, no harmonic oscillations can be observed and the spectrum does not vary much with the mouthpressure (Figure 13.b). All this indicates that it correspond to an inverse bifurcation. The first regime obtained can be interpreted as the standard Helmholtz motion because the main part of the pressure signal is positive (see Figure 11). Above a given blowing pressure, close to the minimum closing pressure P_M, another signal is obtained which can be interpreted more or less as an inverted Helmholtz motion. Between these two regimes an intermediate regime can be observed (Figure 14). For this intermediate regime the second harmonic tends to become zero and, as expected, change its sign.

In the main part of the curve the amplitude is found to be approximately proportional to the mouth pressure, indicating a beating reed regime. The non-beating range seems to be very small (Figure 13.a). This corresponds to experimental observations on conical reed instruments where the reed closes a short time and remains open the rest of the time [44]. The slopes are, as in the cylindrical case, lower than the theoretical ones: 0.85 instead of unity for the ideal Helmholtz motion for the RMS pressure in the case of N=2 (Figure 13.a). For the inverse Helmholtz motion the proportionality is not so clear. However, the slope can be compared to the theoretical one: 0.4 instead of 0.41 for N=4 (Figure 14). It is rather surprising that in that case the slope is not very different from the theoretical one for square signal. This, as well as the fact that the spectrum is not rigourously the same for the inverted and standard Helmholtz motions, indicates that the theoretical scheme is not perfectly confirmed. As for the cylindrical case the ratio between the upper harmonics and the first one is lower than that for the ideal square signal. It must be noticed too, that results can differ slightly with the embouchure. For some embouchures the octave can be observed between the two Helmholtz motion.

As a verification, experiments have also been carried out with an alto saxophone (Figure 15). The behaviour is very similar to the one for a lattice of 4 cylindrical tubes: the bifurcation

appears to be inverse and the range of the non-beating reed regime is small. In some cases inverted Helmholtz motion have also been observed.

5. Conclusion

The aim of this paper is to show how small and large amplitude analysis with very simple models can be used for the understanding of reed instrument operation. For the clarinet the non saturated regime is reasonably well described by the small oscillation analysis and the saturated regime by the large amplitude analysis. Moreover using a series expansion technique with a new method of approximation, the spectrum can be theoretically derived for a large range of mouthpressures [39]. Our study has shown that large and small analysis match rather well for cylindrical and conical resonators. The set of lattices of cylinders described in [10] is particularly useful to understand the difference between a cylinder and a cone from the oscillation's behaviour point of view (bifurcation diagrams). It has been shown in particular, for conical instruments, how the two Helmholtz motions join the single solution of the small oscillation analysis in the case of a two peaks resonator. One of the Helmholtz motions is originating from the small oscillations solution, the other one can be interpreted as a period doubling bifurcation originating from the octave regime. It is shown that the bifurcation types are well predicted by large amplitude analysis : bifurcation is direct for cylindrical instruments and inverse for conical instruments. On the contrary small oscillations analysis does not give sufficient information to find the bifurcation type of non cylindrical resonator: it analyses only one regime which can be either the standard or the inverted Helmholtz motion. This study confirms a musician’s experience: it is much more difficult to play pianissimo with a saxophone than with a clarinet.

Experiments have been made for different resonators and it was found that most of the theoretical results are qualitatively correct under normal playing condition showing that our simplified approach is helpful to understand the basic behaviour of reed instruments from small to large oscillations. It may be noticed that experimental results are depending much on the embouchure and especially on the reed damping including the added damping due to the instrumentalist's lip on the reed. With a very tight embouchure a direct bifurcation can be observed with a saxophone: this may have a relation with the “subtone” playing technique which

allows to play pianissimo on a saxophone. This gives the limit of this kind of study in which different phenomena are ignored. The influence of damping on the spectrum should also be studied as well as the effect of anharmonicity on spectrum and playing frequency. Effect of reed resonance has also been neglected. Wilson & Beavers [6] have shown that reed damping is crucial to avoid the reed regime in a clarinet, a similar study could be done for conical instruments especially for double reed instruments [2]. However, analysis on a simplified model can be very useful. As the model is general, some results can probably be applied to bowed string instruments. As the input pressure signal in the mouthpiece of a lip reed instrument is very similar to that in conical reed instruments, some results can probably also be applied to lip reed instruments. Many aspects of reed oscillation remain to be studied: the stability of different regimes, the modelling of the reed especially when beating, the influence of the damping etc.

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FIGURE CAPTIONS

FIGURE 1a :

Schematic view of a conical resonator with an added volume at the top.

FIGURE 1b :

Schematic view of a lattice of 4 cylindrical tubes having harmonically related eigenfrequencies.

FIGURE 2. :

Calculated input impedance (magnitude in dB) versus frequency of a lattice of N cylindrical tubes having harmonically related eigenfrequencies. Input radius is r=6 mm and $L + \ell = 1.65\text{m}$, with viscothermal losses.

a) N=2 b) N=3

FIGURE 3. :

Schematized view of a single reed instrument mouthpiece.

FIGURE 4. :

Non linear curves $u(t)$ as a function of $p(t)$ for three differents values of P_m.

from the right to the left P_m/P_M= 1/6; 1/3; 1/2.

FIGURE 5. :

Diagram showing the regions where the bifurcation is direct as well as those regions where it is inverse (equation (11)). The horizontal axis shows the values of the coefficient C (cubic term in the Taylor expansion of the non linear function between u and p (6)), the vertical axis the difference Y_2-Y_1 between the admittance at F_2=2F_1 and the admittance at F_1.

The hatched region is for a direct bifurcation and the unhatched region for an inverse bifurcation.

Three typical cases are pointed out:

N=1: single cylinder (case of the clarinet)

N=2: lattice of two cylinders with harmonically related resonance frequencies. In that case the analysis correspond to a standard Helmholtz motion (standard HM)

$N \geq 3$: lattice of three or more cylinders with harmonically related resonance frequencies. In that case the analysis correspond to an inverted Helmholtz motion (inverted HM).

FIGURE 6. :

Schematic plot of the input pressure signal p, as a function of time, corresponding to an ideal standard (p>0) and inverted (p<0) Helmholtz motion, both for the same mouthpressure P_m.

a) Standard Helmholtz motion b) Inverted Nelmholtz motion.

FIGURE 7. :

Pressure of the longest episode p as a function of the mouth pressure P_m for, from the right to the left N=1 to 4 (equation (14)). The pressures are divided by the minimum closing pressure P_M.

* beating reed limit. o linear threshold $P_{th}=P_M/3$ and minimum closing pressure P_M .

FIGURE 8.a :

Bifurcation diagrams corresponding to the two peaks resonator where $Z_1>Z_2$.

Second harmonic P_2 solution of equation (17) as a function of the control parameter P_m

and first harmonic squared P_1^2 solution of equation (16) as a function of the control parameter P_m.

All quantities are divided by the minimum closing pressure P_M .

o o o o first solution for P_2, + + + + second solution for P_2,

– – – – first solution for P_1^2, --------- second solution for P_1^2.

FIGURE 8b. :

Bifurcation diagrams corresponding to the two peaks resonator where $Z_1>Z_2$.

The RMS acoustical pressure is displayed as function of the mouth pressure P_m (results deduced from (17)).

All quantities are divided by the minimum closing pressure P_M .

oooo first solution ++++ second solution _____ octave.

FIGURE 9. :

Bifurcation diagrams corresponding to the lattice of two cylindrical tubes.

The RMS acoustical pressure is displayed as function of the mouth pressure P_m (results obtained by numerical harmonic balance technique).

All quantities are divided by the minimum closing pressure P_M .

oooo first solution ++++ second solution _____ octave.

FIGURE 10a :

Bifurcation diagrams corresponding to the two-harmonics resonator where $Z_1<Z_2$.

Second harmonic P2 solution of equation (17) as a function of the control parameter P_m and first harmonic squared P_1^2 solution of equation (16) as a function of the control parameter P_m.

All quantities are divided by the minimum closing pressure P_M .

o o o o first solution for P_2, + + + + second solution for P_2,

– – – – first solution for P_1^2, --------- second solution for P_1^2.

FIGURE 10b :

Bifurcation diagrams corresponding to the two peaks resonator where $Z_1<Z_2$.

The RMS acoustical pressure is displayed as function of the mouth pressure P_m (results deduced from (17)).

All quantities are divided by the minimum closing pressure P_M .

oooo first solution ++++ second solution _____ octave.

FIGURE 11: Typical experimental input signals for N=1, 2, 3, 4 and a soprano saxophone.

dotted lines: corresponding idealized Helmholtz motion.

FIGURE 12. : Cylindrical resonator (clarinet case) blown with an artificial mouth, internal acoustical pressure experimental results.

(a) RMS input pressure (in Pa) against mouth pressure (in Pa) ;

(b) relative amplitude of the third and fifth harmonic to the first harmonic against the mouth pressure (in Pa) ;
(c) second, third, four and fifth harmonic as a function of the first harmonic (dB scales).

FIGURE 13 : Lattice of cylindrical tubes with N=2 blown with an artificial mouth, internal acoustical pressure experimental results.
(a) RMS input pressure (in Pa) against mouth pressure (in Pa) ;
(b) second, third and fourth harmonic as a function of the first harmonic (dB scales).

FIGURE 14 : Lattice of cylindrical tubes with N=4 blown with an artificial mouth, internal acoustical pressure, experimental results.
Amplitude of the first and second harmonics (in Pa) against mouth pressure (in Pa).
* first harmonic, o second harmonic.
Straight lines correspond to the case in which the amplitude of the first harmonic is proportional to the mouthpressure. Vertical dashed line separate the standard Helmholtz motion range and the inverted Helmholtz motion range.

FIGURE 15. : Conical resonator (saxophone) blown with an artificial mouth, internal acoustical pressure experimental results.
Amplitude of the first and second harmonics (in Pa) against mouth pressure (in Pa).
* first harmonic, o second harmonic.
Straight lines correspond to the case in which the amplitude of the first harmonic is proportional to the mouthpressure. A vertical dashed line separate the standard Helmholtz motion range and the inverted Helmholtz motion range.

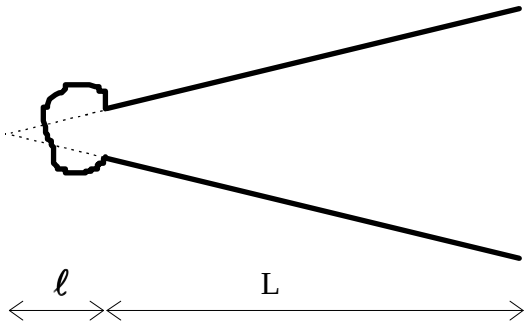

Figure 1a

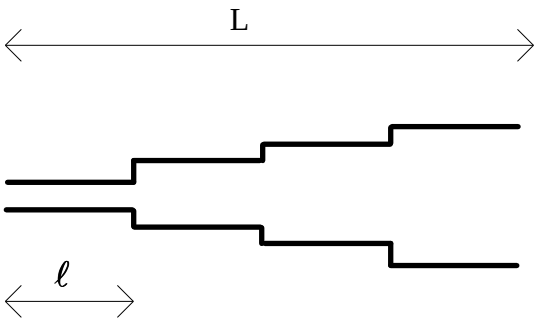

Figure 1b

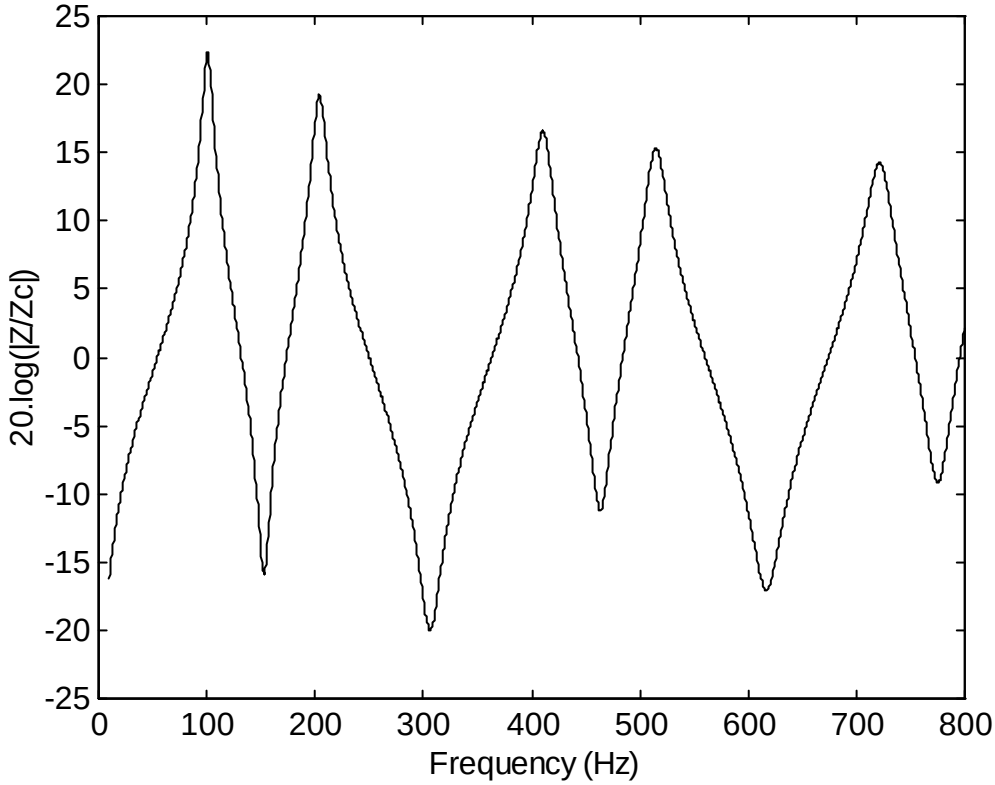
25
20
15
10
5
0
-5
-10
-15
-20
-25
20.log(|Z/Zc|)
0
100
200
300
400
500
600
700
800
Frequency (Hz)

Figure 2a

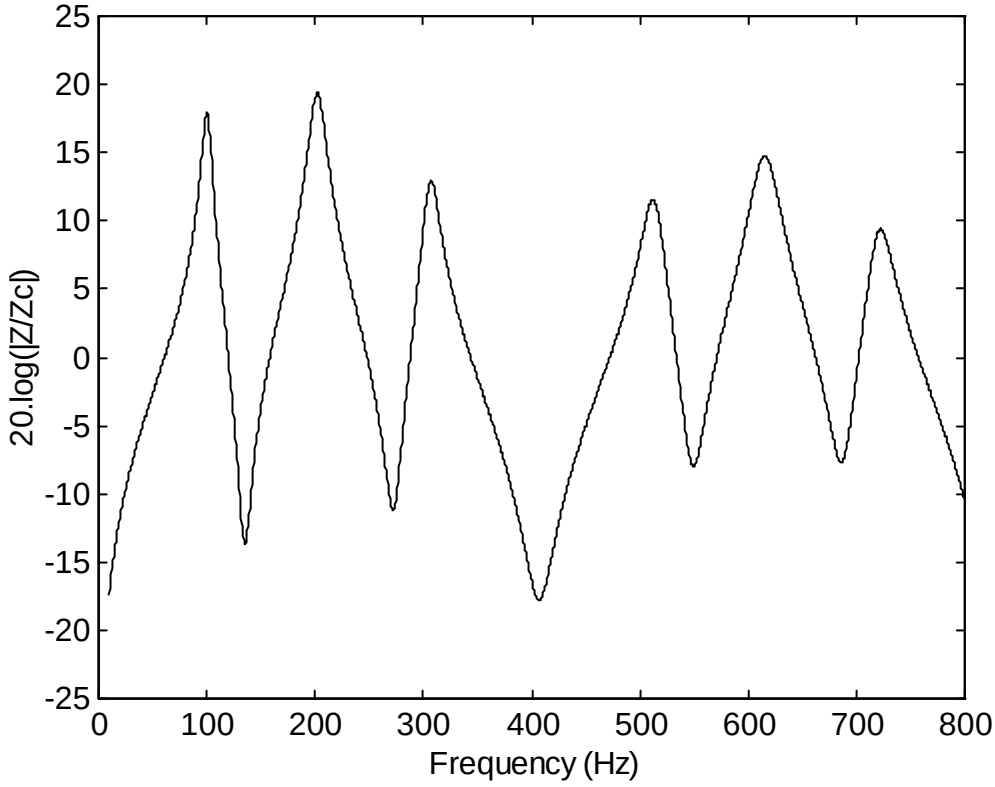
25
20
15
10
5
0
-5
-10
-15
-20
-25
20.log(|Z/Zc|)
0
100
200
300
400
500
600
700
800
Frequency (Hz)

Figure 2b

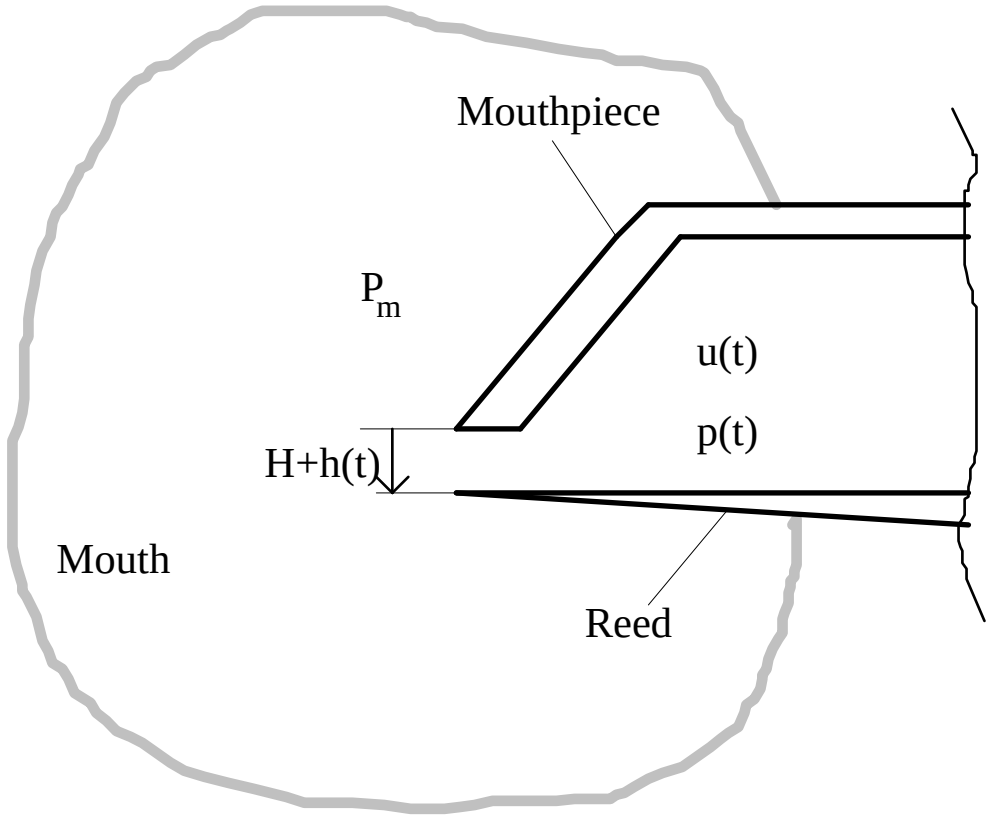

Figure 3

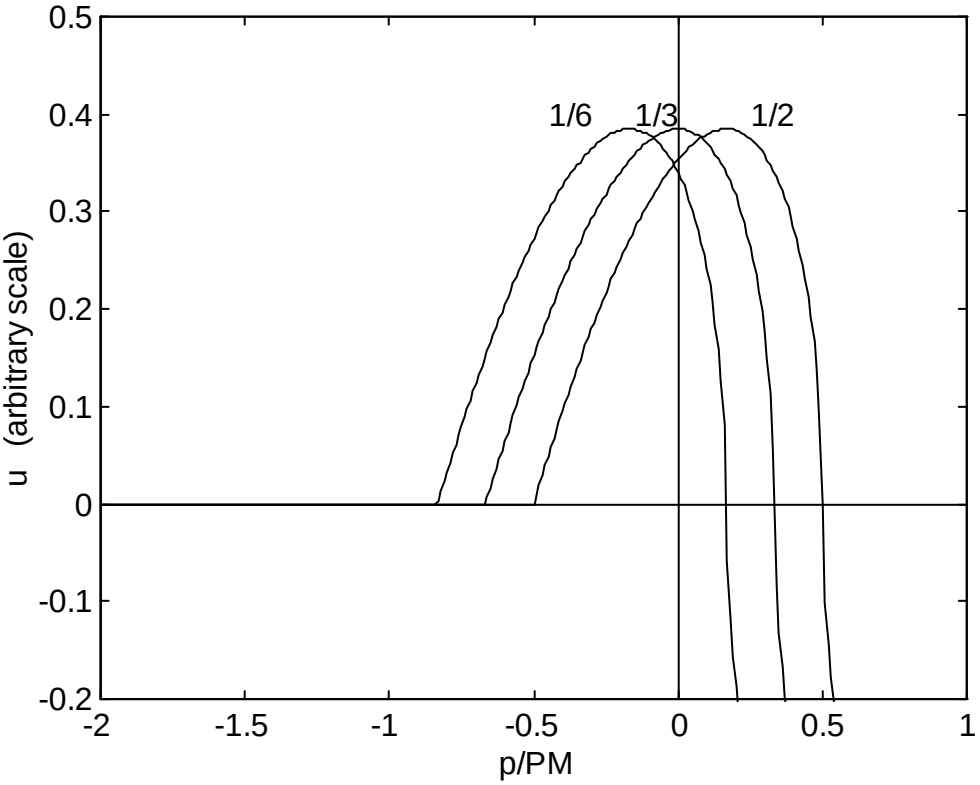

Figure 4.

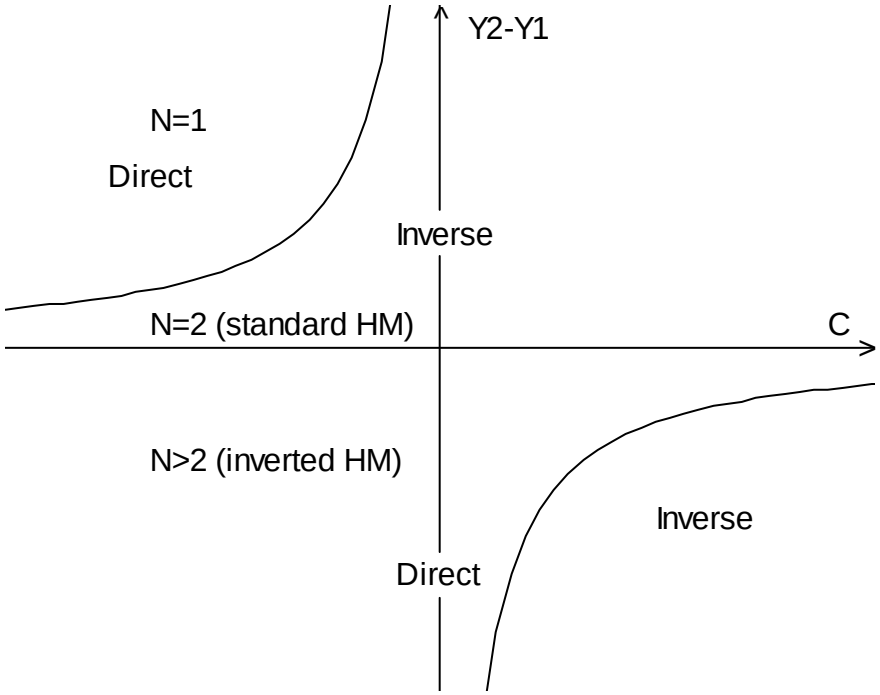
Y2-Y1
N=1
Direct
Inverse
N=2 (standard HM)
C
N>2 (inverted HM)
Inverse
Direct

Figure 5.

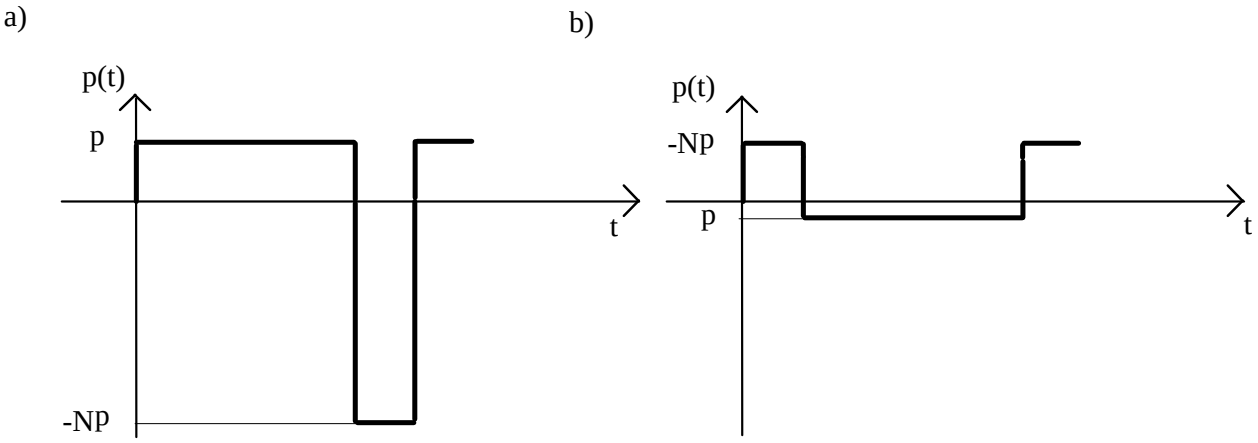

figure 6

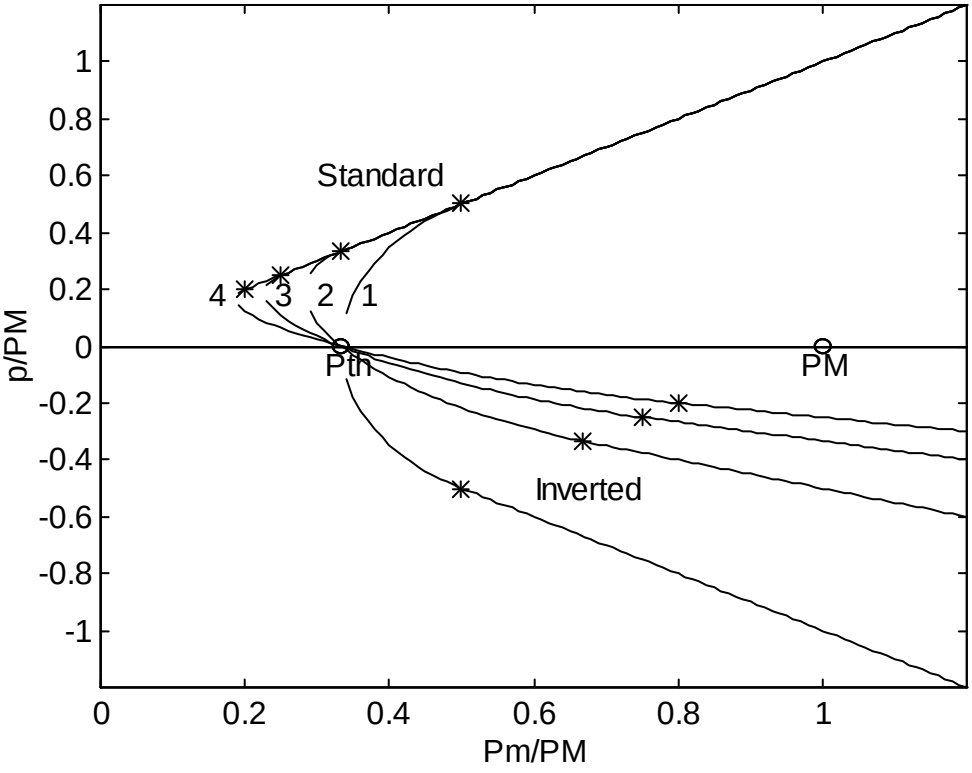
Standard
Inverted
4
3
2
1
Pth
PM
p/PM
Pm/PM
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
0
0.2
0.4
0.6
0.8
1

figure 7.

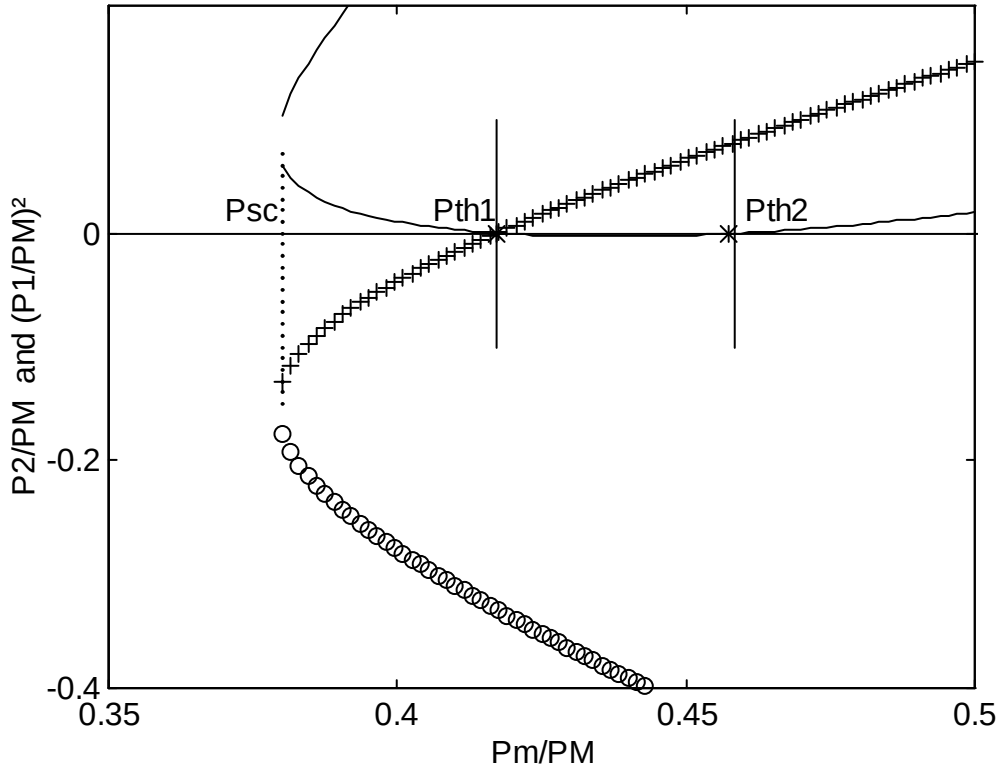

Figure 8a

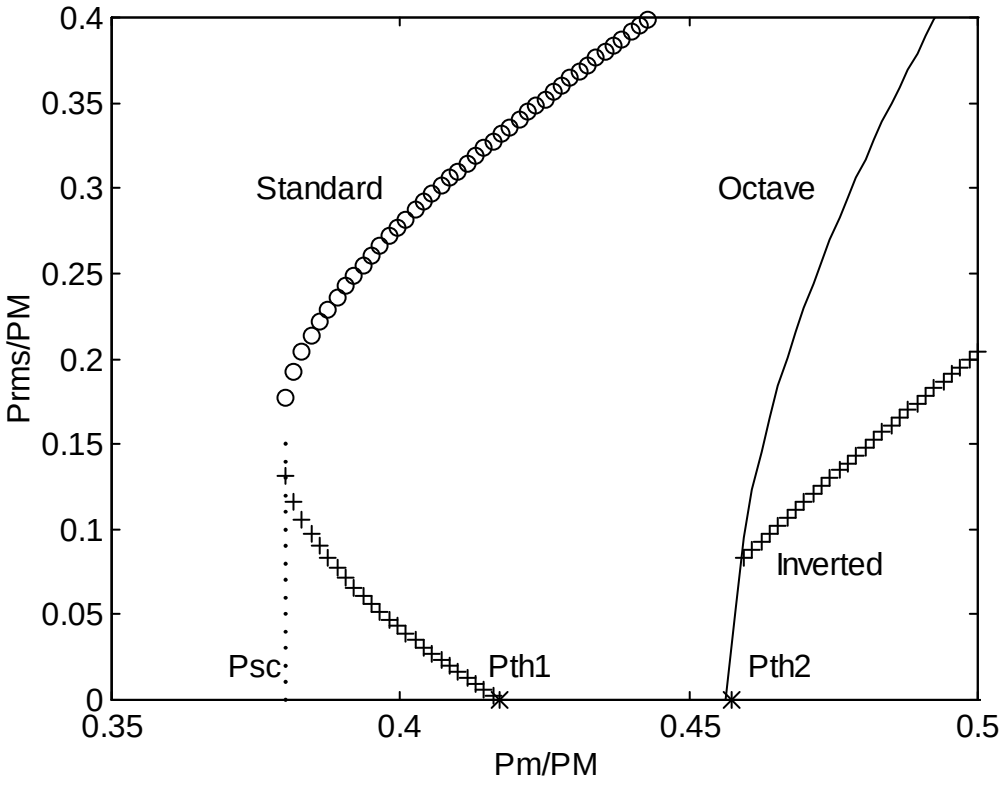
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
Prms/PM
0.35
0.4
0.45
0.5
Pm/PM
Standard
Octave
Inverted
Psc
Pth1
Pth2

Figure 8b

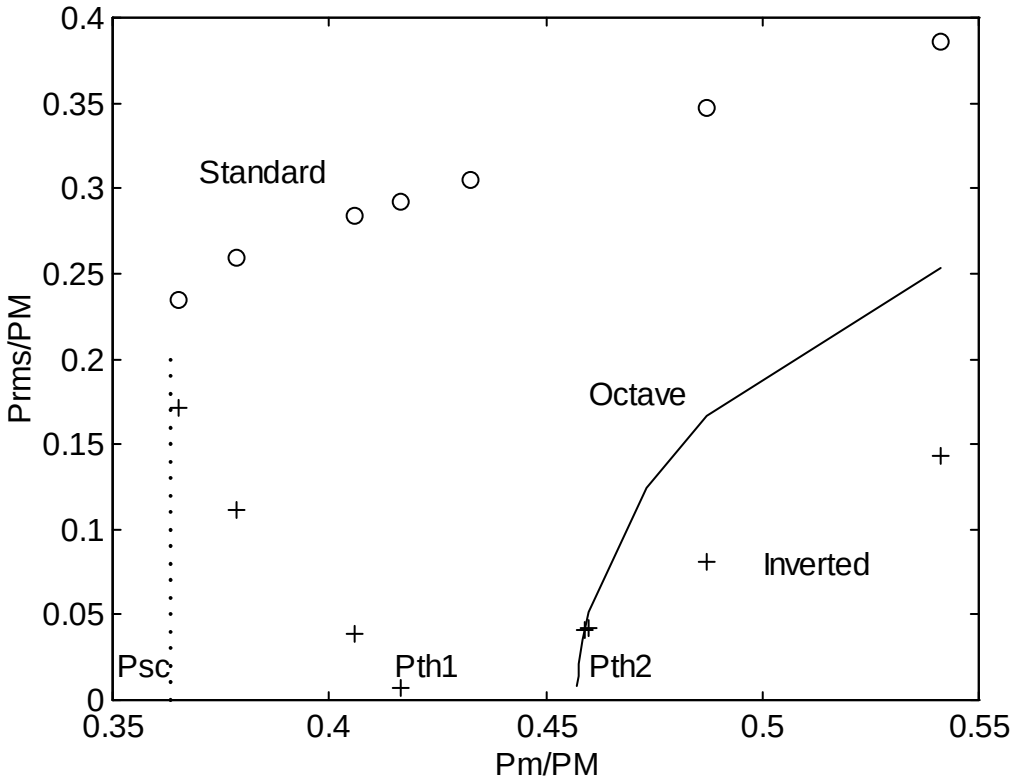
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
Prms/PM
0.35
0.4
0.45
0.5
0.55
Pm/PM
Standard
Octave
Inverted
Psc
Pth1
Pth2

figure 9

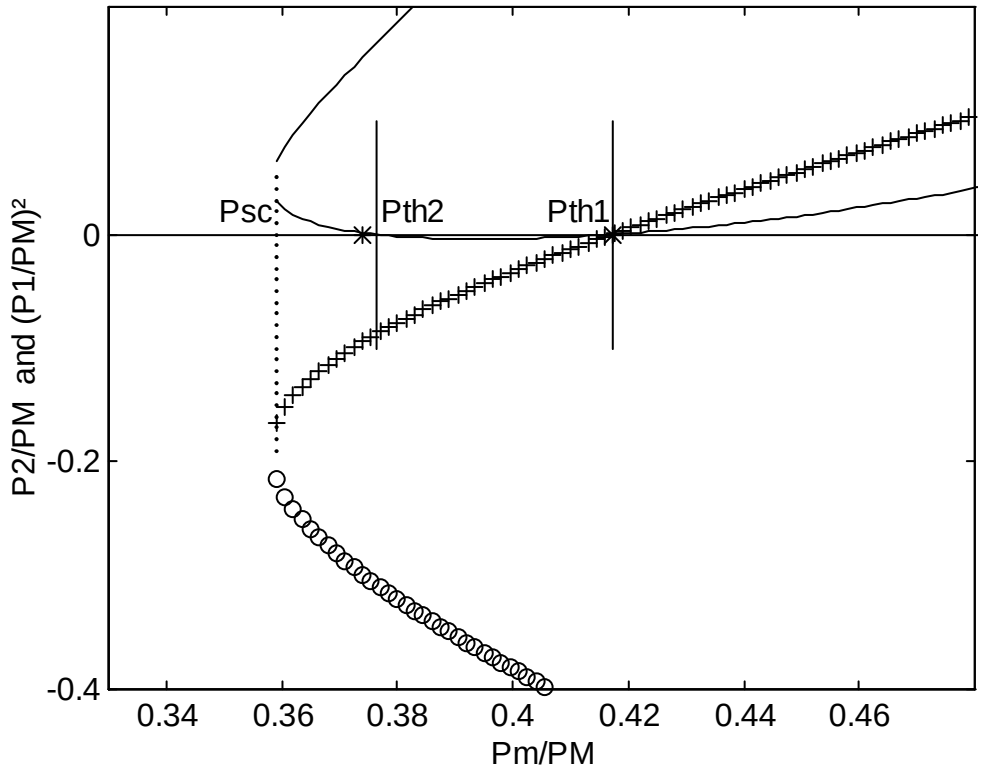
Psc
Pth2
Pth1
0
-0.2
-0.4
P2/PM and (P1/PM)²
0.34
0.36
0.38
0.4
0.42
0.44
0.46
Pm/PM

figure 10a

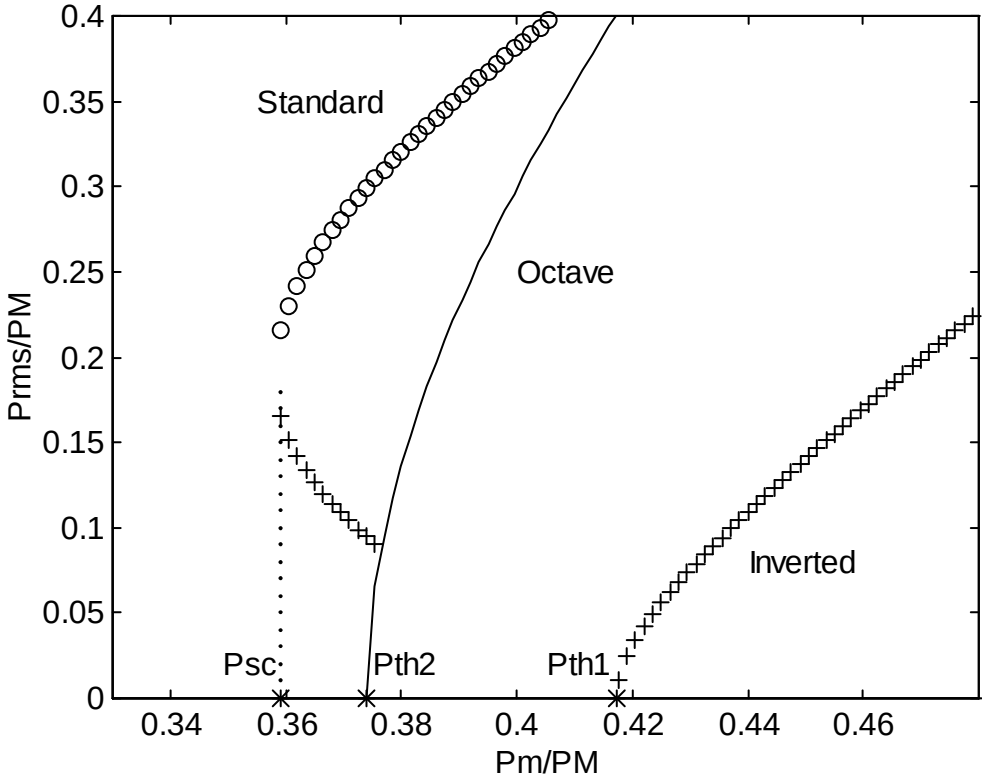
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
Prms/PM
Standard
Octave
Inverted
Psc
Pth2
Pth1
0.34
0.36
0.38
0.4
0.42
0.44
0.46
Pm/PM

figure 10b

Figure 11

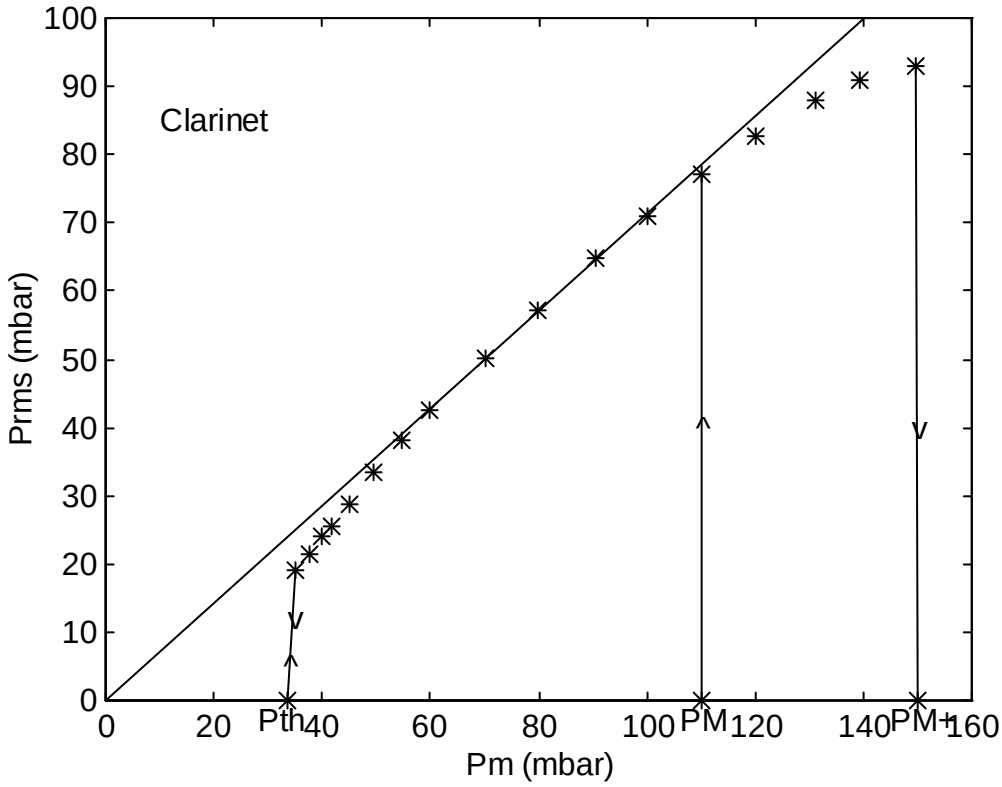
Clarinet
Prms (mbar)
Pm (mbar)
Pth
PM
PM+
0
10
20
30
40
50
60
70
80
90
100
120
140
160

Figure 12 a

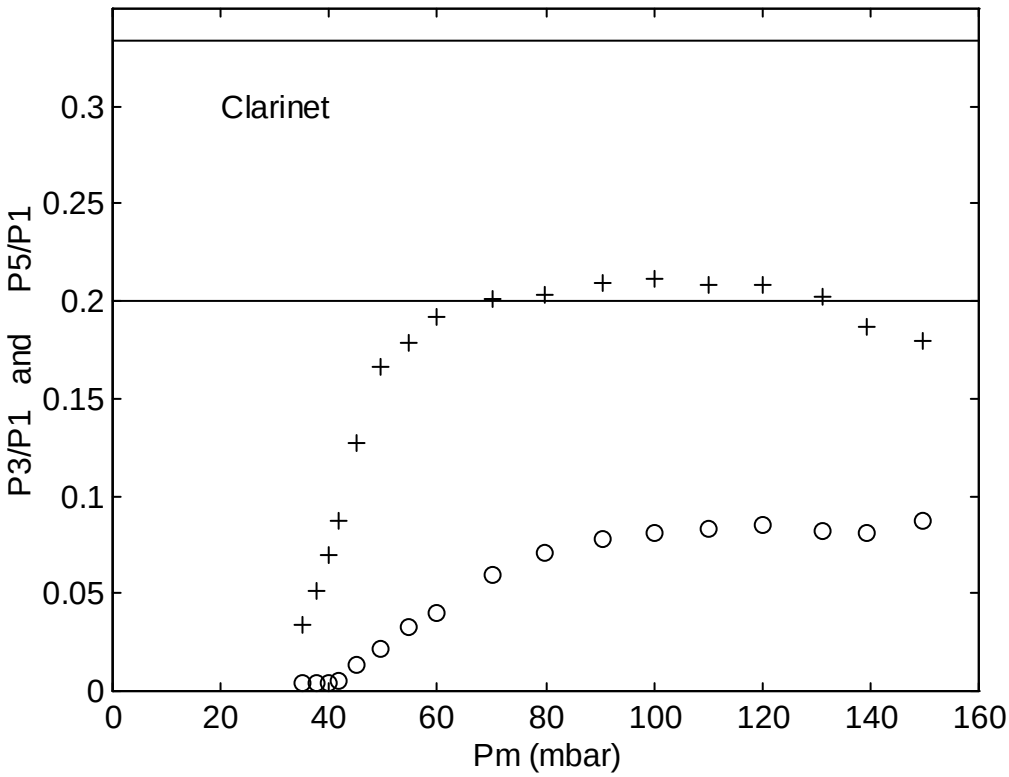
Clarinet
P3/P1 and P5/P1
Pm (mbar)
0
0.05
0.1
0.15
0.2
0.25
0.3
0
20
40
60
80
100
120
140
160

Figure 12 b

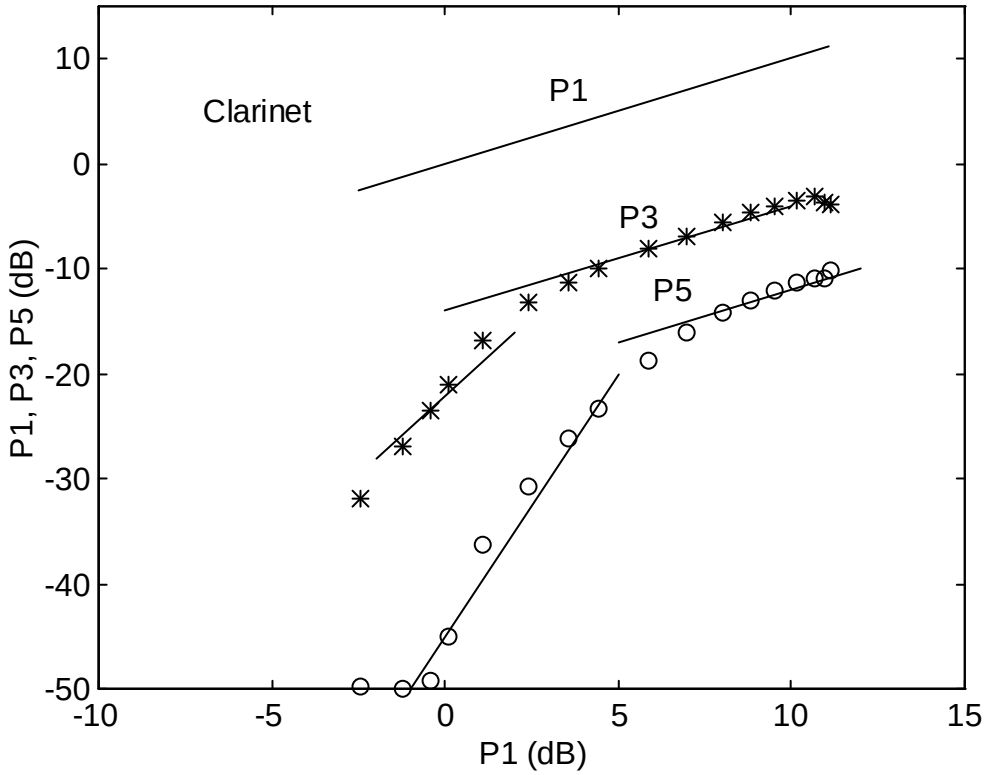

Figure 12 c

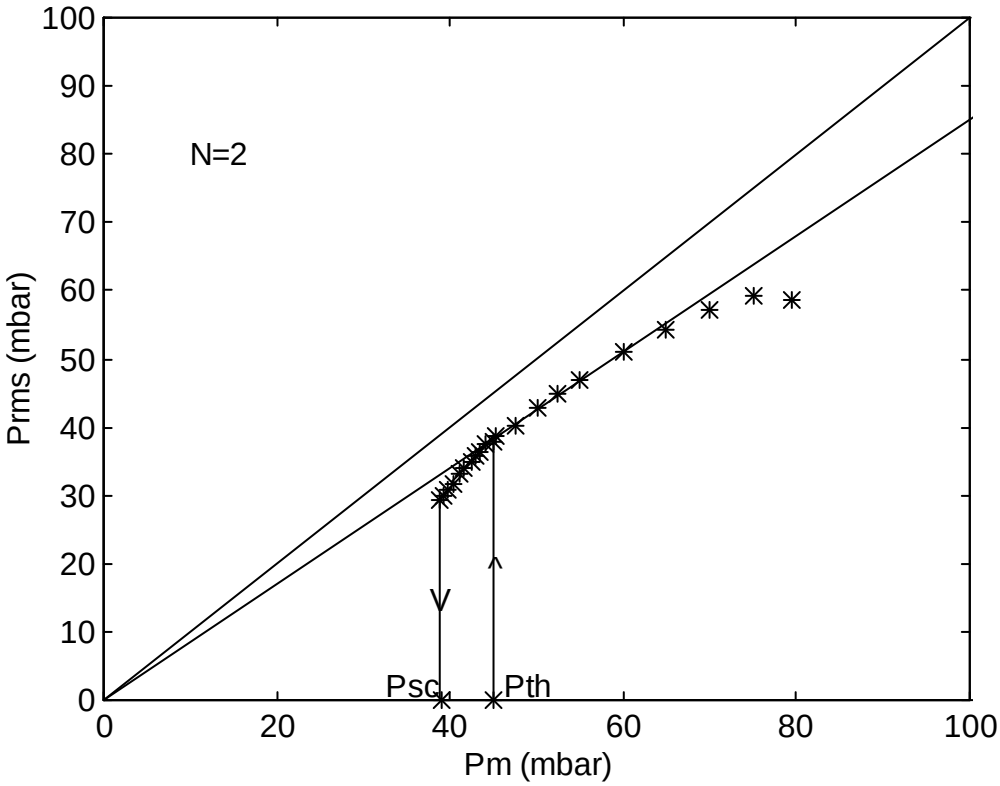
N=2
Prms (mbar)
Pm (mbar)
Psc
Pth
0
10
20
30
40
50
60
70
80
90
100
0
20
40
60
80
100

Figure 13a

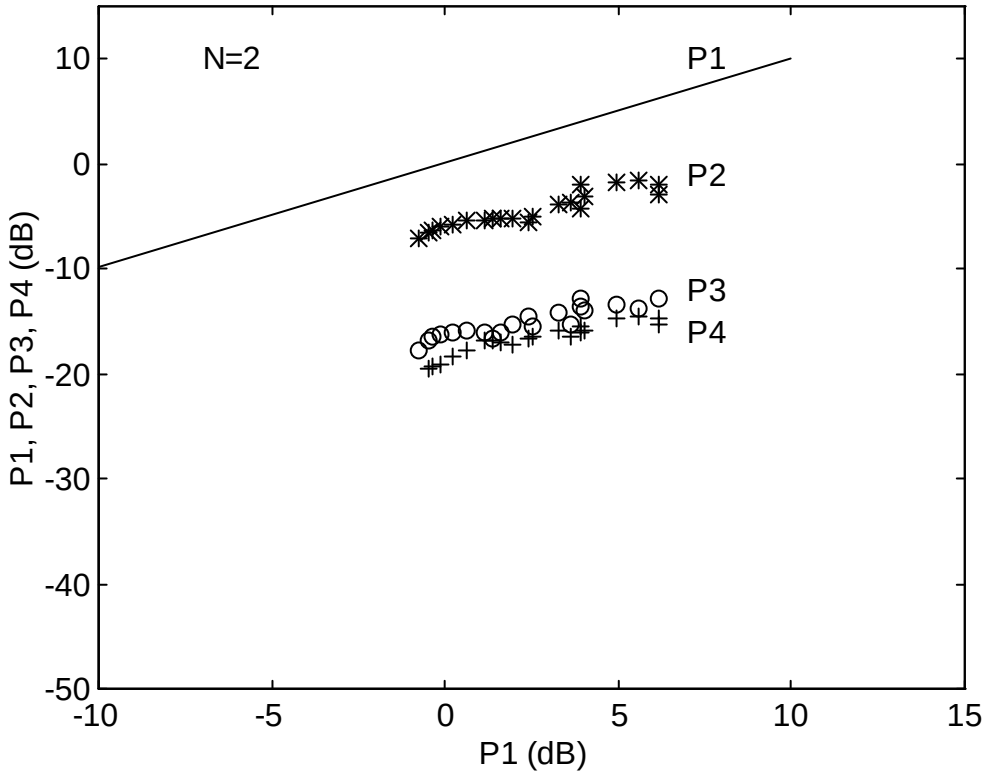

Figure 13b

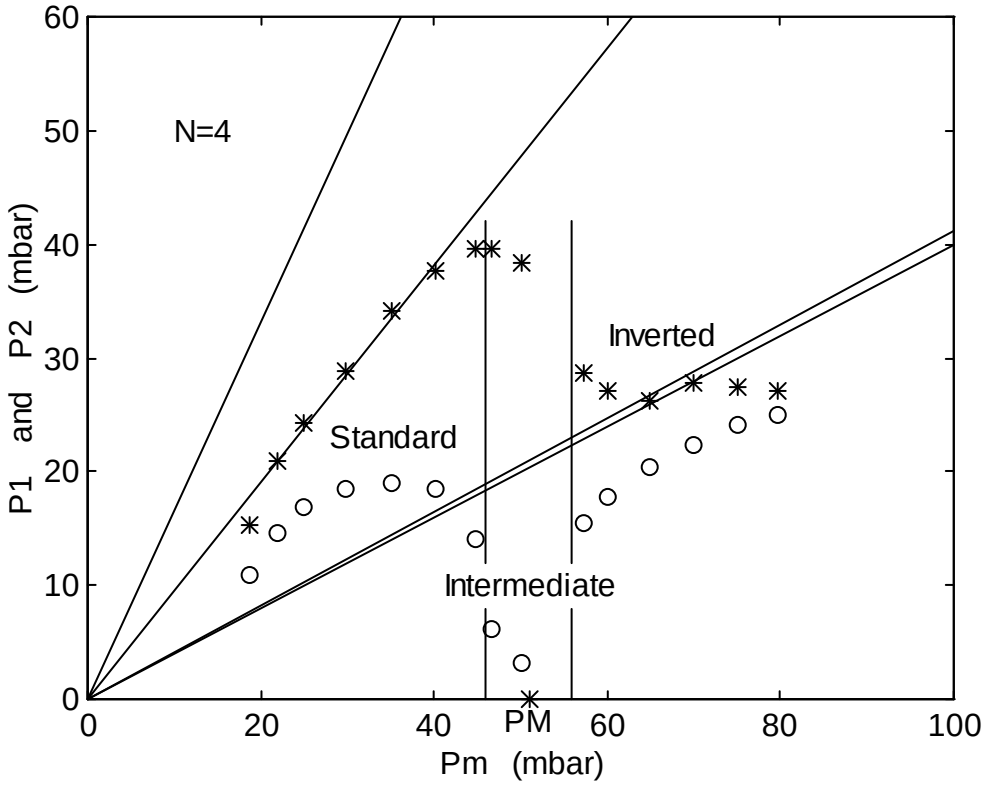
N=4
Standard
Inverted
Intermediate
PM
P1 and P2 (mbar)
Pm (mbar)
0
10
20
30
40
50
60
0
20
40
60
80
100

Figure 14

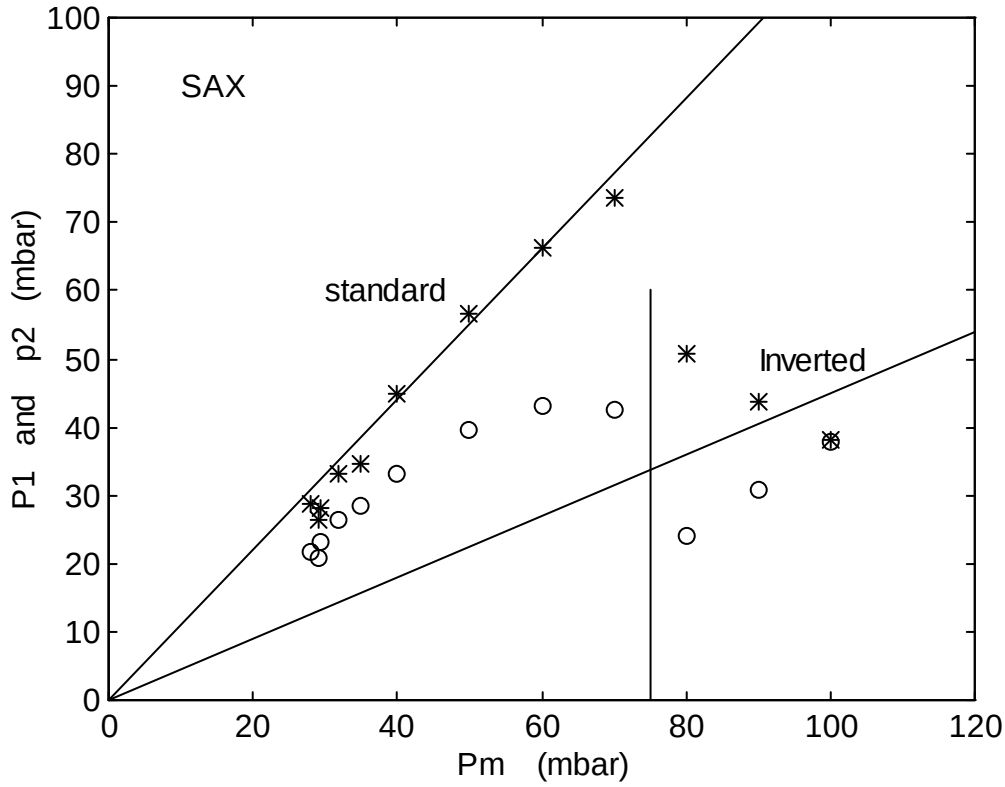
SAX
standard
Inverted
P1 and p2 (mbar)
Pm (mbar)
0
10
20
30
40
50
60
70
80
90
100
0
20
40
60
80
100
120

Figure 15