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## Poisson point processes applied to sensors networks

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## Motivation

The development of wireless ad hoc networking capability together with the decreasing costs and sizes of the electrical circuits allow an increasing using of sensor networks in building, utilities, industrial, home, agriculture, defense and many other contexts. The topology of such networks, particularly the connectiveness and coverage area, are important and, sometimes, critical factors. Recently, many works dealing with topology give interpretations and techniques capable to be applied on sensor networks. Besides, it is not possible to control the positions and number of sensors in many of those networks, which lead us to study the topology of random sensor networks by using the Poisson point processes.

## Problem Formulation

The principal idea of the problem is that sensors  $\{S_1, S_2, \dots, S_n\}$  have a power supply allowing them to transmit their ID's and maybe some environmental information (such as temperature, pressure, presence/absence of an element etc). At the same time, the sensors have receivers which can identify the transmitted ID's of other sensors above a threshold power. The sensors, knowing their ID's and the ID's of the close neighbors, create an information network.

## Physical Features of the System

- The sensors lie over a  $d$ -torus  $\mathbb{T}_a^d$  and the dimensions of the sensor are considered too reduced compared to the system, so the position of the sensor  $S_i$  is given by  $x_i \in \mathbb{T}_a^d = (u_{i,1}, \dots, u_{d,i}), u_{d,k} \in [0, a]$ ;
- A sensor receives the ID's from all other sensors closer then a deterministic distance  $\epsilon$ , so if  $\|x_i - x_j\| \leq \epsilon$ , sensors  $S_i$  and  $S_j$  are directly connected;
- We use the maximum norm, i.e.,

$$\|x_i - x_j\| = \max_k (u_{i,k} - u_{j,k})$$

## Random Features of the System

- The number of sensors lying over on  $\mathbb{T}_a^d$ ,  $\Phi(t)$ , is distributed as poisson with mean  $\lambda a^d$ , where  $\lambda$  is a constant in the model. Indeed,  $\lambda$  represent the density of sensors;
- The distribution of the position of each sensor is independent of the other sensors and given by

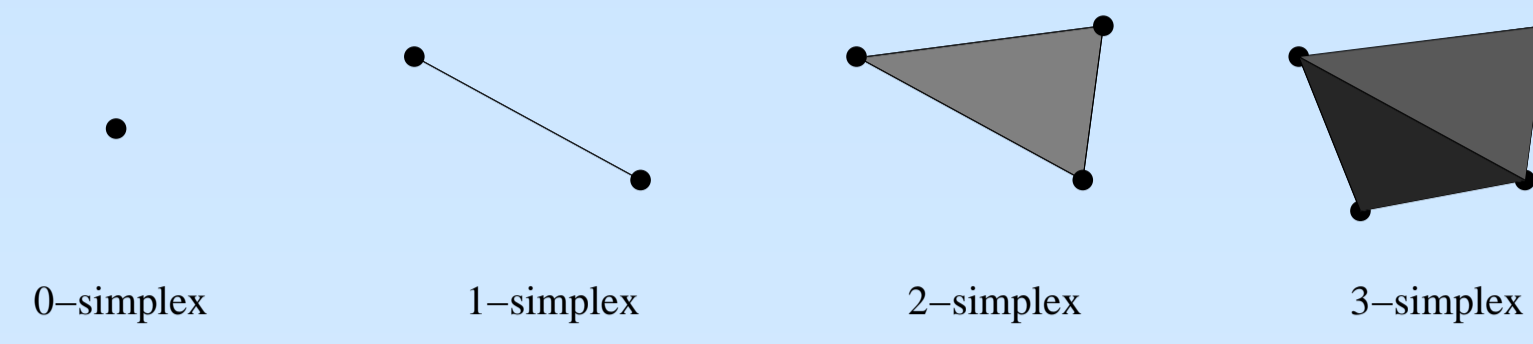
$$p_x(X) = \frac{\mathbb{1}_{[0,t]}(X)}{t}$$

## References

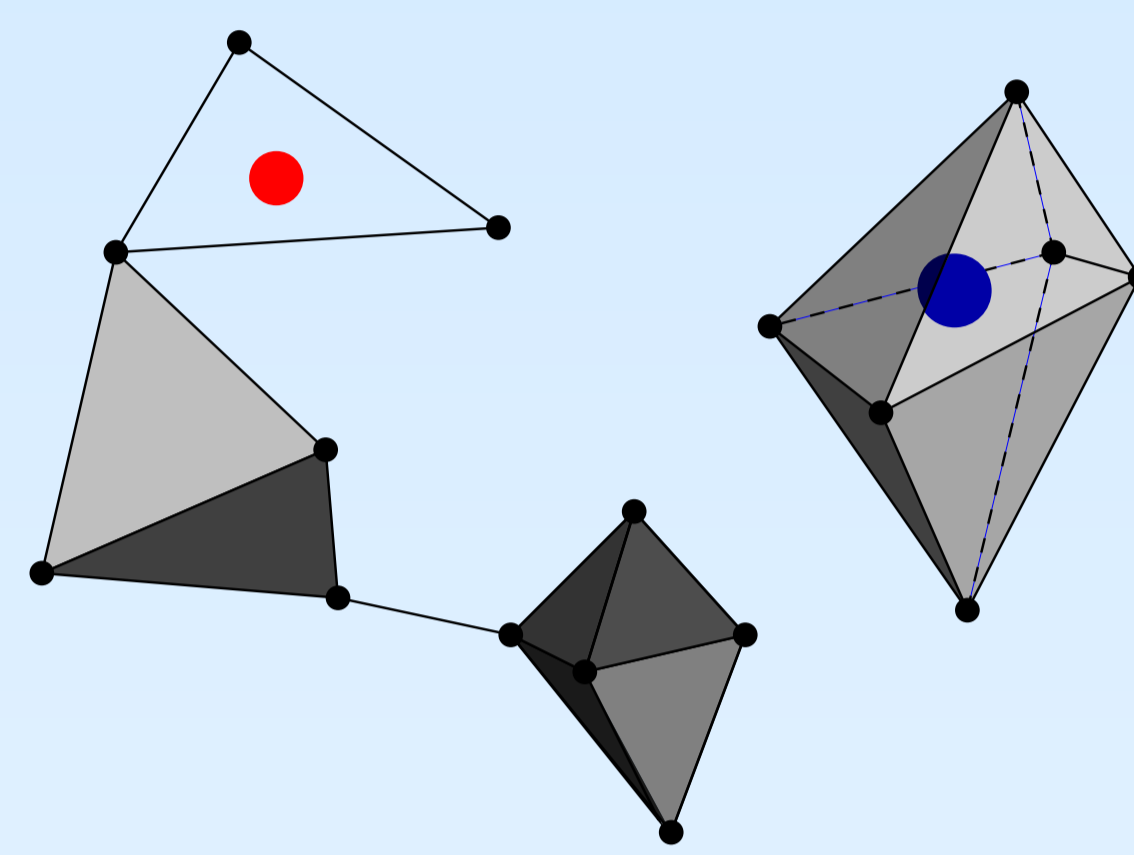
- [1] R. Ghrist, A. Muhammad. Coverage and Hole-Detection in Sensor Networks Via Homology In *Fourth International Conference on Information Processing in Sensor Networks (IPSN'05)*, UCLA, 2005.
- [2] C. Houdré, N. Privault. Concentration and deviation inequalities in infinite dimensions via covariance representations *Bernoulli*, 2002.

## Simplicial Homology

Simplicial complexes are structures composed by elements named simplices, which can be seen by  $d$ -dimensional filled spaces. Exemples of simplexes are given below:



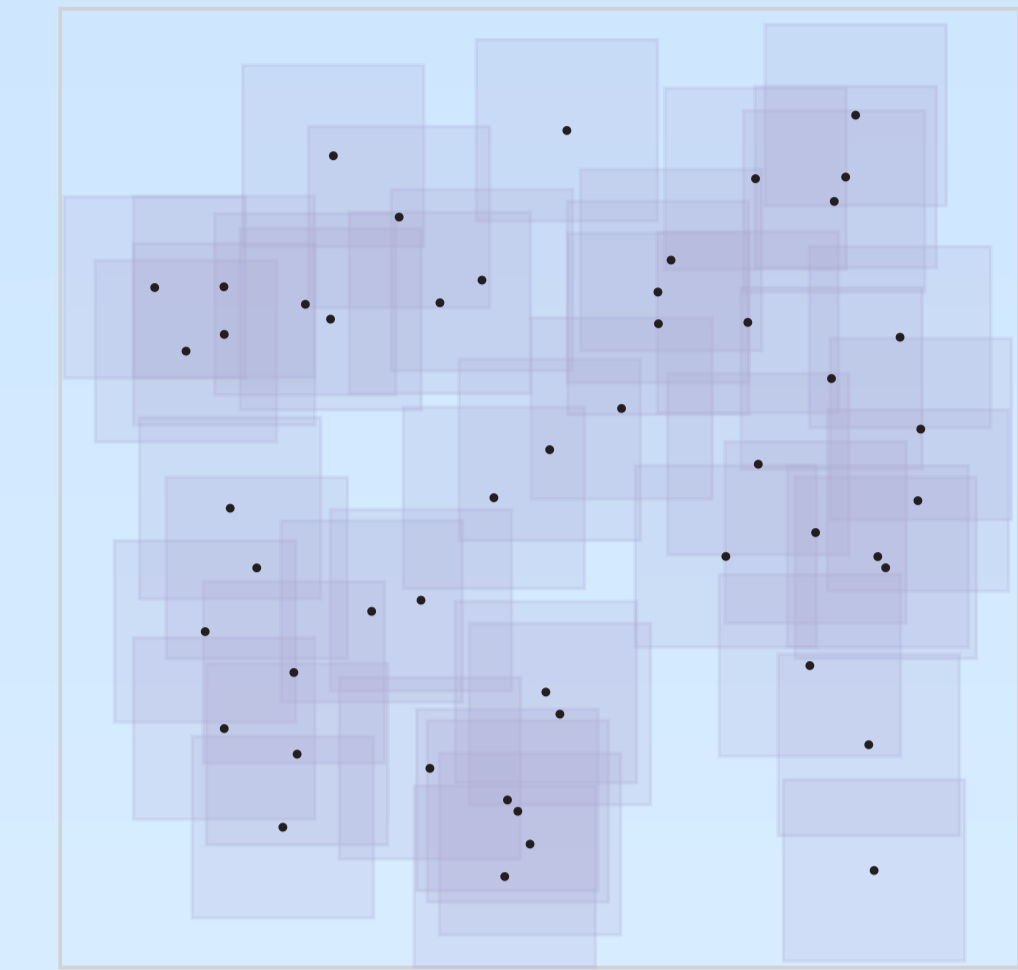
$\beta_d$  denotes the number of  $d$ -dimensional holes. Particularly,  $\beta_0$ ,  $\beta_1$  and  $\beta_2$  measure, respectively, the connectiveness, the number of holes and the number of voids of a complex.



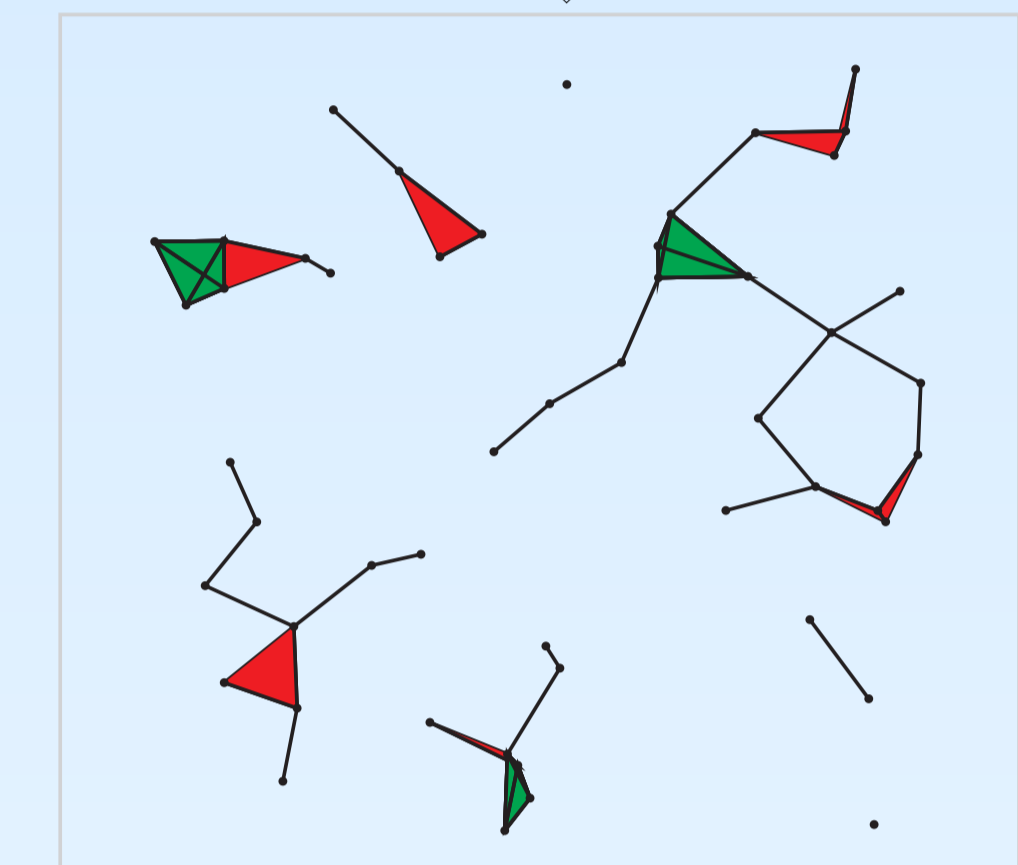
Two connex components:  $\beta_0 = 2$   
The 2-dimensional being "Red Point" is trapped, we have one void:  $\beta_1 = 1$   
The 3-dimensional being "Blue Point" is trapped, we have one hole:  $\beta_2 = 1$

## Interpreting a Sensor Network

We can represent the topology of a sensor network by its Rips complex, which is obtained when we consider that whenever  $k + 1$  points are 2 by 2 closer than  $\epsilon$  between them, they create a  $k$ -simplex.



• Sensor  
■ Coverage



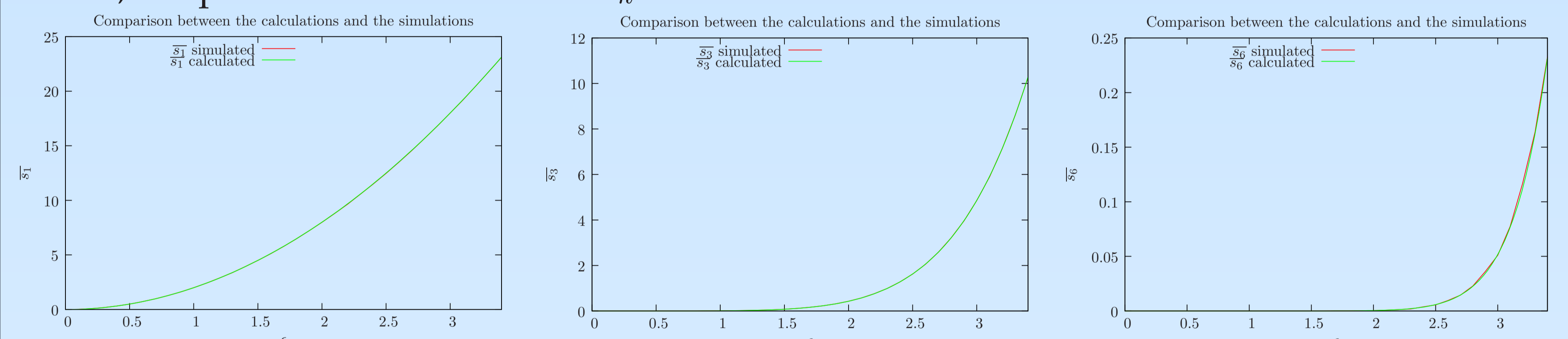
• 0-simplex  
— 1-simplex  
■ 2-simplex  
■ 3-simplex

## Results: Mean of $k$ -simplexes, $\bar{s}_k$ , and Euler's Characteristic, $\bar{\chi}$

It is possible to calculate the mean of  $k$ -simplices given the size of individual coverage  $\epsilon$ , the density of sensors  $\lambda$ , the dimension  $d$  and the sizes of the  $d$ -torus,  $a$ :

$$\bar{s}_{k-1} = \frac{\lambda^k a^d}{k!} k^d (\epsilon^{k-1})^d, \quad \epsilon < a/3$$

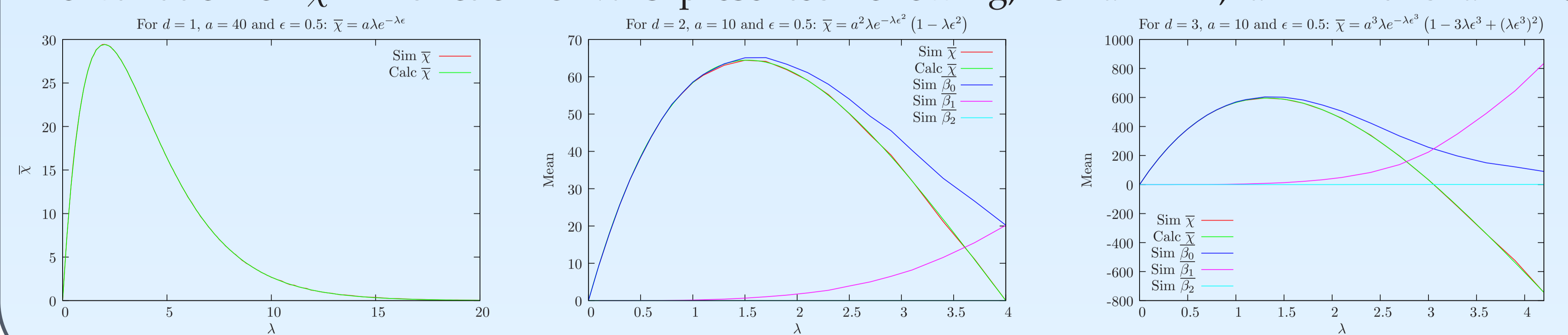
Below, we present the variation of  $\bar{s}_k$  in function of  $\epsilon$  for  $a = 100$  and  $\lambda = 0.10$  in two dimensions.



Let  $B_d$  be the Bell's polynomial. Using the mean of  $k$ -simplices, we can calculate the mean of the Euler's Characteristic  $\bar{\chi}$ :

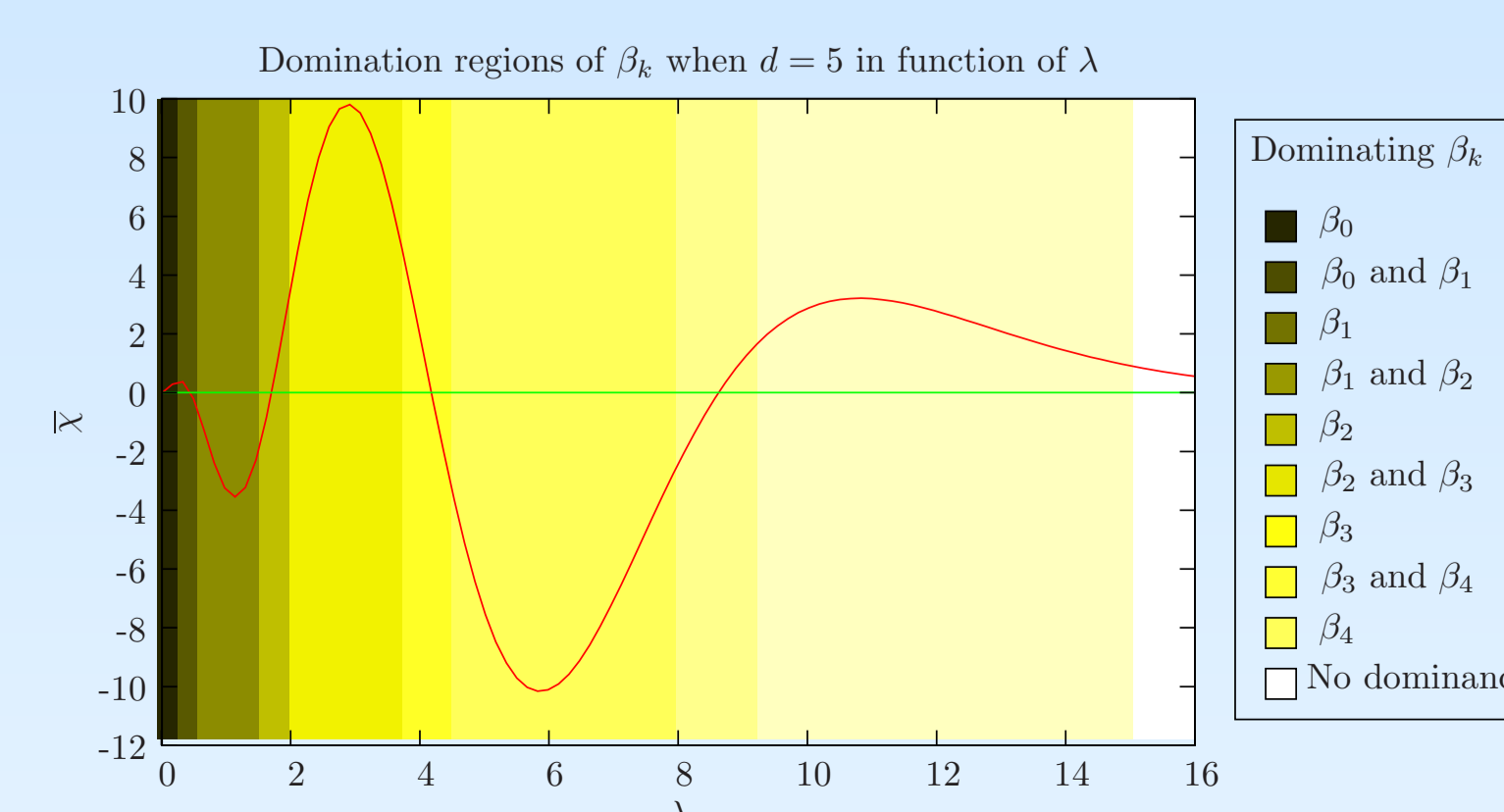
$$\bar{\chi} = \frac{a^d \lambda e^{-\lambda \epsilon^d}}{-\lambda \epsilon^d} B_d(-\lambda \epsilon^d)$$

The variation of  $\bar{\chi}$  in function of  $\lambda$  is presented following, for  $d = 1$ ,  $d = 2$  and  $d = 3$ :



## Conjecture: $\beta_i$ dominance region

Based on simulations and analytical expressions, we can conjecture that, given a density of points, there are at most two dominating types of holes.



## Results: Concentration Inequality

Since a compensated Poisson point process can be seen as a martingale, we can use a concentration inequality to find a superior limit for  $P(\beta_0 \geq c)$  in two dimensions

