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Sensor fault diagnosis for a class of LPV descriptor systems

Carlos-M. Astorga-Zaragoza * **Didier Theilliol** ** **Jean-Christophe Ponsart** ** **Mickael Rodrigues** ***

* *Centro Nacional de Investigación y Desarrollo Tecnológico, Interior Internado Palmira, S/N Col. Palmira C.P.62490, Cuernavaca, Mor., Mexico (e-mail: astorga@cenidet.edu.mx).*
** *CRAN CNRS UMR 7039 Nancy-Universite, France. (e-mail: didier.theilliol@cran.uhp-nancy.fr, jean-christophe.ponsart@cran.uhp-nancy.fr)*
*** *Laboratoire d'Automatique et Génie des Procédés, Université Lyon 1, France. (e-mail: rodrigues@lagep.cpe.fr)*

Abstract: In this paper, a sensor model-based fault diagnosis method for a particular class of linear parameter variant (LPV) systems is developed. The main contribution of this work is to propose an adequate observer that performs fault detection over the whole operating range of the system. The conditions for the existence of an observer are given. Such conditions guarantees the observer convergence and they are proved through a Lyapunov analysis based on Linear Matrix Inequality (LMI) formulation. A generalized observer scheme is performed in order to achieve the fault isolation. The observer is evaluated through numerical simulations.

Keywords: Sensor fault diagnosis, linear time variant system, generalized observer scheme, descriptor system

1. INTRODUCTION

Linear parameter variant (LPV) systems can be used to approximate nonlinear systems and hence systematic and generic available theoretical results for LPV systems can be then applied to derive nonlinear control laws for nonlinear systems. For instance, in (Wei and del Re, 2007) the authors uses the LPV approach to model and control diesel engines. The case of robust fault detection and isolation of LPV systems is presented in (Armeni et al., 2009). However, to the best of the authors' knowledge, there are a very few works where fault detection and isolation (FDI) and fault tolerant control (FTC) approaches are synthesized for this kind of systems associated to model-based observer design. A very few recent papers has been published on this last topic.

The idea of merging descriptor and LPV systems is not new; see for instance (Rehm and Allgöwer, 2000) and more recently (Chadli et al., 2008). The aim of this work is to to develop a sensor fault diagnosis method for LPV systems. In order to achieve this objective, an observer for LPV descriptor systems is synthesized. This observer is an extension of the observer proposed in (Darouach and Boutayeb, 1995) where a method to design full-order observers for LTI descriptor systems has been considered. Then a simple and straightforward method to design an observer for polytopic linear parameter variant (LPV) is presented. The existence conditions of a LPV observer synthesized with appropriate transformations are given. Such conditions guarantee the observer convergence proved through a combined method based on the original approach proposed by Darouach and Boutayeb (1995) and a Lyapunov-like analysis. A bank of residuals within a Generalized Scheme Observer (GOS) is then used to supervise LPV systems. The effectiveness and performance of the proposed scheme are illustrated through a numerical example.

2. PROBLEM FORMULATION

Consider the following continuous-time LPV system

$$\begin{aligned} E\dot{x}(t) &= A(\rho(t))x(t) + B(\rho(t))u(t) \\ y(t) &= Cx(t) \end{aligned} \tag{1}$$

In this paper, the polytopic LPV case is treated, i.e. where the parameter $\rho(t)$ varies in a convex polytope of vertices ρ_i such that $\rho(t) \in Co\{\rho_1, \rho_2, \ldots, \rho_M\}$. In this case, the system (1) can be written as:

$$\begin{aligned} E\dot{x}(t) &= \sum_{i=1}^{M} \varepsilon_i(\rho(t)) \left(A_i x(t) + B_i u(t)\right) \\ y(t) &= Cx(t) \end{aligned} \tag{2}$$

where

$$\sum_{i=1}^{M} \varepsilon_i(\rho(t)) = 1, \;\; \varepsilon_i(\rho(t)) \geq 0 \tag{3}$$

with $x(t) \in \mathbb{R}^n$, $u(t) \in \mathbb{R}^m$, $y(t) \in \mathbb{R}^w$ are the state, the input and the output vectors, respectively, $E \in \mathbb{R}^{q\times n}$, $A_i \in \mathbb{R}^{q\times n}$, $B_i \in \mathbb{R}^{n\times m}$, $C \in \mathbb{R}^{w\times n}$, are constant matrices and $i = 1, \ldots, M$, where M is the total number of functions $\varepsilon_i(\rho(t))$.

Due to abnormal operation or material aging, sensor faults can occur in the system. A sensor fault can be represented by additive and/or multiplicative faults as follows:

$$\omega_j^f = b(t)\omega_j + \omega_0 \tag{4}$$

where ω_j and ω_j^f represent the jth normal and faulty measurements (i.e., $\omega(t) = y(t)$), ω_0 denotes a constant offset and $0 \leq b(t) \leq 1$ denotes a gain degradation of the jth sensor (constant or variable). Therefore, when a sensor fault occurs, the state space representation defined in (2) becomes as:

$$\begin{aligned} E\dot{x}(t) &= \sum_{i=1}^{M} \varepsilon_i(\rho(t)) \left(A_i x(t) + B_i u(t)\right) \\ y(t) &= Cx(t) + Ff(t) \end{aligned} \tag{5}$$

where F represents the sensor fault distribution matrix and $f(t)$ is the faulty vector.

The presence of such faults may lead to performance deterioration, instability of the system or the loss of the process. The next sections are dedicated to the development of an efficient model-based fault diagnosis method in order to provide an efficient monitoring tool in the operator's decision. Firstly, the synthesis of an observer for systems having the form (2) is presented in what follows.

3. OBSERVER SYNTHESIS

Consider the following LPV descriptor system:

$$\begin{aligned} E\dot{x}(t) &= A(\rho(t))x(t) + B(\rho(t))u(t) \\ y(t) &= Cx(t) \end{aligned} \tag{6}$$

with $x(t) \in \mathbb{R}^n$, $u(t) \in \mathbb{R}^m$, $y(t) \in \mathbb{R}^w$ are the state, the input and the output vectors, respectively, $E \in \mathbb{R}^{n\times n}$, $A_i \in \mathbb{R}^{n\times n}$, $B_i \in \mathbb{R}^{n\times m}$, $C \in \mathbb{R}^{w\times n}$, are constant matrices and $i = 1, \ldots, M$, where M is the total number of functions $\varepsilon_i(\rho(t))$.

The following assumptions are considered by Darouach and Boutayeb (1995):

(**A1**) $\mathsf{rank}\, E = r < n$.

(**A2**) $\mathsf{rank} \begin{pmatrix} E \\ C \end{pmatrix} = n$.

Considering Assumption **A2**, there exists a nonsingular matrix Δ:

$$\Delta = \begin{pmatrix} \alpha & \beta \\ \gamma & \xi \end{pmatrix}$$

such that

$$\alpha E + \beta C = I_n \tag{7}$$

$$\gamma E + \xi C = 0 \tag{8}$$

where α, β, γ and ξ are constant matrices of appropriate dimensions which can be found by the Singular Value Decomposition of $\begin{pmatrix} E \\ C \end{pmatrix}$.

As proposed by Darouach and Boutayeb (1995), an observer for the system (2) should have the following form:

$$\begin{aligned} \dot{z}(t) &= \sum_{i=1}^{M} \varepsilon_i(\rho(t)) \left[N_i z(t) + L_{1i} y(t) + G_i u(t) \right. \\ &\quad + \left. L_{2i} y(t)\right] \\ \hat{x}(t) &= z(t) + \beta y(t) + \sum_{i=1}^{M} \varepsilon_i(\rho(t)) K_i \xi y(t) \end{aligned} \tag{9}$$

The general form of this observer is composed by a state $z(t) \in \mathbb{R}^n$. The observer inputs are the measured output of the process $y(t)$ and the process inputs $u(t)$. The matrices N_i, L_{1i}, L_{2i}, G_i and K_i, $i = 1, \ldots, M$ should be determined such that the estimates of the state variables $\hat{x}(t) \in \mathbb{R}^n$ converge asymptotically to $x(t)$. For the sake of simplicity, in what follows, the following notation is used:

$$\Omega(\rho) = \sum_{i=1}^{M} \varepsilon_i(\rho(t))\Omega_i \quad i = 1, \ldots, M$$

Thus, the system (9) is written in the simplified form:

$$\begin{aligned} \dot{z}(t) &= N(\rho)z(t) + L_1(\rho)y(t) + G(\rho)u(t) \\ &\quad + L_2(\rho)y(t) \\ \hat{x}(t) &= z(t) + \beta y(t) + K(\rho)\xi y(t) \end{aligned} \tag{10}$$

The problem is then to give necessary and sufficient conditions such that the system in (9) is an observer for system (2).

3.1 ***Observer design***

Let the observer error be defined as:

$$e(t) = x(t) - \hat{x}(t) \tag{11}$$

Replacing $\hat{x}(t)$ from (10) into (11):

$$e(t) = x(t) - z(t) - \beta C x(t) - K(\rho)\xi C x(t) \tag{12}$$

Replacing βC and ξC from (7) and (8), respectively, into (12):

$$\begin{aligned} e(t) &= x(t) - z(t) - (I_n - \alpha E)x(t) \\ &\quad - K(\rho)(-\gamma E)x(t) \\ &= (\alpha + K(\rho)\gamma)\, Ex(t) - z(t) \end{aligned} \tag{13}$$

Thus

$$\dot{e}(t) = (\alpha + K(\rho)\gamma)\, E\dot{x}(t) - \dot{z}(t) \tag{14}$$

Replacing $E\dot{x}(t)$ and $\dot{z}(t)$ from (2) and (10), respectively, into (14):

$$\begin{aligned}\dot{e}(t) &= (\alpha + K(\rho)\gamma)\,(A(\rho)x(t) + B(\rho)u(t)) \\ &- N(\rho)z(t) - L_1(\rho)y(t) \\ &- L_2(\rho)y(t) - G(\rho)u(t)\end{aligned} \tag{15}$$

By grouping common terms in $x(t)$, $u(t)$ and $z(t)$, (15) becomes:

$$\begin{aligned}\dot{e}(t) &= [K(\rho)\gamma A(\rho) + \alpha A(\rho) - L_1(\rho)C \\ &- L_2(\rho)C]\, x(t) + [K(\rho)\gamma B(\rho) \\ &+ \alpha B(\rho) - G(\rho)]\, u(t) - N(\rho)z(t)\end{aligned} \tag{16}$$

By adding and subtracting the term

$$N(\rho)\,(\alpha + K(\rho)\gamma)\, Ex(t)$$

in (16), it follows that

$$\begin{aligned}\dot{e}(t) &= [(\alpha + K(\rho)\gamma)\, A(\rho) - N(\rho)\,(\alpha + K(\rho)\gamma)\, E \\ &- L_1(\rho)C - L_2(\rho)C]\, x(t) \\ &+ [(\alpha + K(\rho)\gamma)\, B(\rho) - G(\rho)]\, u(t) \\ &+ N(\rho)\, \underbrace{[(\alpha + K(\rho)\gamma)\, Ex(t) - z(t)]}_{e(t)}\end{aligned} \tag{17}$$

It can be seen in (17) that if the following conditions are fulfilled

$$\begin{aligned}0 &= (\alpha + K(\rho)\gamma)\, A(\rho) - N(\rho)\,(\alpha + K(\rho)\gamma)\, E \\ &- L_1(\rho)C - L_2(\rho)C\end{aligned} \tag{18}$$

and

$$G(\rho) = (\alpha + K(\rho)\gamma)\, B(\rho) \tag{19}$$

then (17) reduces to

$$\dot{e}(t) = N(\rho)e(t) \tag{20}$$

Now, replacing αE and γE from (7) and (8), respectively, in (18), it is easy to deduce:

$$\begin{aligned}N(\rho) &= (\alpha + K(\rho)\gamma)\, A(\rho) + [N(\rho)\,(\beta + K(\rho)\xi) \\ &- L_1(\rho)]\, C - L_2(\rho)C\end{aligned} \tag{21}$$

The following assumption ensures that the second term in the right hand side of (21) is zero:

$$L_1(\rho) = N(\rho)\,(\beta + K(\rho)\xi) \tag{22}$$

and then, (21) becomes

$$N(\rho) = K(\rho)\gamma A(\rho) + \alpha A(\rho) - L_2(\rho)C \tag{23}$$

The following theorem is given.

Theorem 1. The system (10) is an observer for the system (2) if the following conditions are satisfied:

(i) sufficient condition:

$$\dot{e}(t) = N(\rho)e(t) \tag{24}$$

is stable

(ii) necessary conditions: given the matrices K_i and L_{2i}, such that the sufficient condition is satisfied, the matrices $N(\rho)$, $G(\rho)$, and $L_1(\rho)$ are deduced by

$$N(\rho) = K(\rho)\gamma A(\rho) + \alpha A(\rho) - L_2(\rho)C \tag{25}$$

$$G(\rho) = (\alpha + K(\rho)\gamma)\, B(\rho) \tag{26}$$

$$L_1(\rho) = N(\rho)\,(\beta + K(\rho)\xi) \tag{27}$$

Proof. Equation (24) is obtained by replacing (26)-(27) in (17). Now, it remains to give the necessary conditions to ensure stability of the system given by (24).

Consider the following Lyapunov function candidate $V(e(t)) = e^T(t)Pe(t)$ with $P = P^T > 0$, where $(\cdot)^T$ denotes matrix transposition. The time derivative of the Lyapunov function along the trajectories of the system (24) is:

$$\begin{aligned}\dot{V}(e(t)) &= \dot{e}^T(t)Pe(t) + e^T(t)P\dot{e}(t) \\ &= e^T(t)\left(N^T(\rho)P + PN(\rho)\right)e(t) \\ &= e^T(t)\sum_{i=1}^{M}\varepsilon_i(\rho(t))\left(N_i^T P + PN_i\right)e(t)\end{aligned} \tag{28}$$

Quadratic stability (Amato, 2006) of the equilibrium point of system (24) is guaranteed if $\dot{V}(e(t)) < 0, \forall e(t) \neq 0$. This condition is satisfied if

$$N_i^T P + PN_i < 0 \tag{29}$$

If there exists an appropriate symmetric matrix P to make possible that $\left(N_i^T P + PN_i\right) < 0$ holds $\forall\ i = 1, \ldots, M$, then it is obvious that (29) holds for any $\varepsilon_i(\rho(t))$. Consequently, the matrix inequality (29) is transformed in a set of M matrix inequalities:

$$\left(N_i^T P + PN_i\right) < 0 \ \ \forall\, i = 1, \ldots, M \tag{30}$$

N_i is deduced from (25) and defined such as:

$$N_i = K_i\gamma\sum_{j=1}^{M}\varepsilon_j(\rho(t))A_j + \alpha A_i - L_{2i}C \tag{31}$$

$\forall\ j = 1, \ldots, M$. Replacing N_i from (31) in (29), the following BMI is obtained:

$$
\begin{aligned}
&A_i^T\alpha^T P + \sum_{j=1}^{M}\varepsilon_j(\rho(t))A_j^T\gamma^T K_i^T P - C^T L_{2i}^T P + \\
&P\alpha A_i + PK_i\gamma\sum_{j=1}^{M}\varepsilon_j(\rho(t))A_j - PL_{2i}C < 0
\end{aligned}
\tag{32}
$$

$\forall\ i = 1,\ldots,M$. The BMI conditions (32) can be transformed into LMI conditions by considering $Q_j = PK_j$ and $R_i = PL_{2i}$. In this way, (32) becomes:

$$
\begin{aligned}
&A_i^T\alpha^T P + \sum_{j=1}^{M}\varepsilon_j(\rho(t))A_j^T\gamma^T Q_i^T - C^T R_i^T + \\
&P\alpha A_i + Q_i\gamma\sum_{j=1}^{M}\varepsilon_j(\rho(t))A_j - R_i C < 0
\end{aligned}
\tag{33}
$$

$\forall\ i = 1,\ldots,M$. By multiplying each LMI (33) by $\sum_{i=1}^{M}\varepsilon_i(\rho(t))$ and adding them all together, the following inequalities are defined:

$$
\begin{aligned}
&\sum_{i=1}^{M}\varepsilon_i(\rho(t))\sum_{j=1}^{M}\varepsilon_j(\rho(t))\left(A_i^T\alpha^T P + A_j^T\gamma^T Q_i^T - C^T R_i^T + \right. \\
&\left. P\alpha A_i + Q_i\gamma A_j - R_i C\right) < 0
\end{aligned}
\tag{34}
$$

Finally, if there exist appropriate matrices P, Q_i and R_i, then it is obvious that (34) holds and consequently the system (24) is stable.

4. GENERALIZED LPV DESCRIPTOR OBSERVER SCHEME

For the purpose of fault diagnosis, the basic idea of this approach is to reconstruct the state of the system from the subsets of measurements. The objective is to build a bank of observers so that each one of them is driven by all inputs and all outputs except the jth measurement variable. y_j is not used in the jth observer due to the fact that y_j is assumed to be corrupted by the fault and therefore does not carry the required information (Frank, 1990). For the synthesis of the observer, the following state space equation is included:

$$
\begin{aligned}
E\dot{x}(t) &= Ax(t) + Bu(t) \\
\zeta_j(t) &= \bar{C}_j x(t)
\end{aligned}
\tag{35}
$$

where $\forall\ j \in [1,\ldots,m]$, $\zeta(t)_j\ j \in \mathbb{R}^{m-1}$ is the output vector without the jth element and $\bar{C}_j$ is the C matrix without the jth row. It can be noted that the jth observer corresponds to the output vector without the jth component.

This fault diagnosis scheme is similar to the well-known Generalized Observer Structure (GOS) but the absence of a sensor fault. In order to reach both the isolation and the estimation of an actuator fault under disturbances, we also consider the case of $j = 0$ needs also to be considered: the observer is based on system (10) where C is not related to the sensor fault-free case.

Under the assumption that $\forall\ j \in [1,\ldots,m]$, the pair $\left(\bar{C}_j, A\right)$ is observable, the bank could be designed. Similarly to the classical approach, a residual evaluation which involves statistical testing such as the limit checking test and the generalized likelihood ratio test, the trend analysis test is performed for each observer of the bank as previously described. An output vector of the statistical test can be built according to a test applied to a set of $m+1$ residuals. The status of the residuals is equal to 0 when the residual signal is closed to zero in some sense and equal to 1 otherwise.

The bank of observers generates an incidence matrix (see Table 1), where each column is called the coherence vector associated with each fault signature:

Table 1. Incidence Matrix

Fault	F_1	F_2	$\cdots$	F_m
$\|y-\hat{y}\|$	1	1	1	1
$\|\zeta_1-\hat{\zeta}_1\|$	0	1	1	1
$\|\zeta_2-\hat{\zeta}_2\|$	1	0	1	1
$\cdots$	1	1	0	1
$\|\zeta_m-\hat{\zeta}_m\|$	1	1	1	0

The bank of observers generates some zero mean residuals, otherwise, only the observer which is insensitive to a sensor fault F_j generates a unique residual with a zero mean. Based on this type of signature, the fault is easily isolated using the GOS structure irrespective of the actuator fault occurrence. Decision-making is then used according to elementary logic which can be described as follows: a fault indicator is equal to one if the residual vector generated by the bank is equal to a column of the incidence matrix, and to zero otherwise. The element which is associated with the indicator being equal to one is then declared to be *faulty*.

5. EXAMPLE

Let us consider a continuous-time LPV descriptor system in fault-free case (2) described by:

$$
E = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}
$$

$$
A_1 = \begin{pmatrix} -5 & 0 & 0 \\ 0 & -4 & 0 \\ 0 & 0 & -4.5 \end{pmatrix}\ A_2 = \begin{pmatrix} -6 & 0 & 0 \\ 0 & -4 & 0 \\ 0 & 0 & -5 \end{pmatrix}
$$

$$
B_1 = \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix}\ B_2 = \begin{pmatrix} 0 \\ -0.5 \\ 2 \end{pmatrix}
$$

$$
C = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}
$$

According to the fulfilled conditions defined in the third section, the design of each LPV observer has been achieved based on a pole placement in LMI region $\mathcal{D}$ as proposed by (Chilali and Gahinet, 1996). A constant matrix K has been considered to synthesize the gain of each observer

which produces a residual $r_j(t)$ equal to $\zeta_j(t) - \hat{\zeta}_j(t)$. The effectiveness of the proposed observer scheme is illustrated with the system studied in open-loop. In fault-free case, the input vector is presented in Fig. 1. As illustrated in Fig. 2, the system is considered on the whole operating conditions.

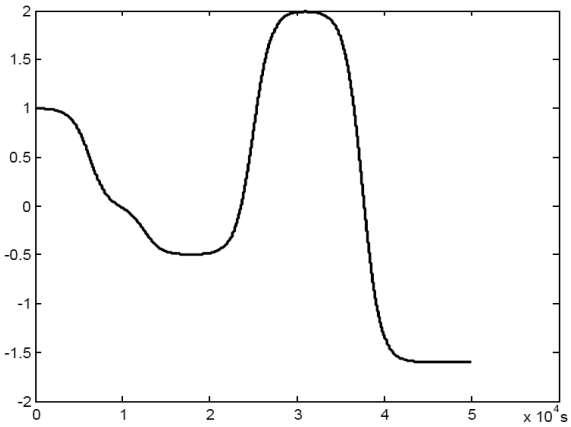

Fig. 1. Dynamic behaviour of the input

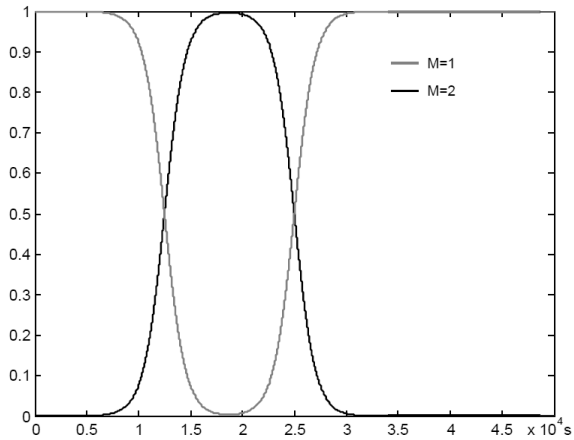

Fig. 2. Dynamic behaviour of the weighting functions

Given the initial conditions $x(0) = [3 \quad 2 \quad 1]^T$, the simulation result of the state space vector $x(t)$ is depicted in Fig. 3 in fault-free case. Under the initial conditions $\hat{x}(0) = [0 \quad 0 \quad 0]^T$, the residual norm vector issued from the two observers (in our case $m = 2$) is close to zero as illustrated in Fig.4. a severe fault is also tested when the first sensor (respectively the second sensor) is out of order at instant 2500s. According to the incidence matrix defined in the previous section, only the observer synthesized in order to be insensitive to a sensor fault on this output provides a residual vector equal to zero means as presented in Fig.5 (respectively in Fig. 6.).

The results show that the bank of observers is very effective in detecting and isolating the fault for the whole operating conditions. The residual norm vector should be evaluated through a classical statistical threshold test in

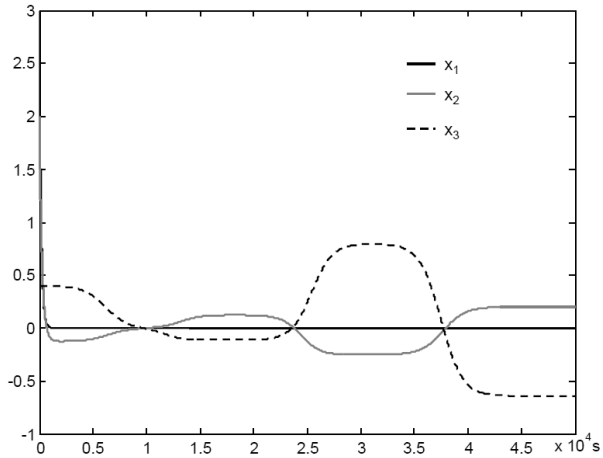

Fig. 3. Dynamic behaviour of the state space vector

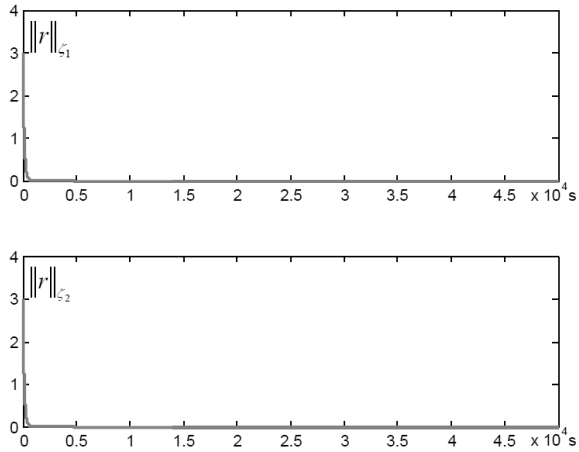

Fig. 4. Residual vector norms in Fault free case

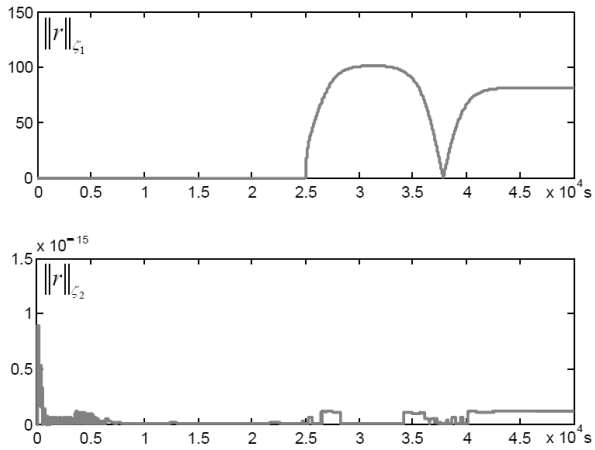

Fig. 5. Residual vector norms with first sensor out of order

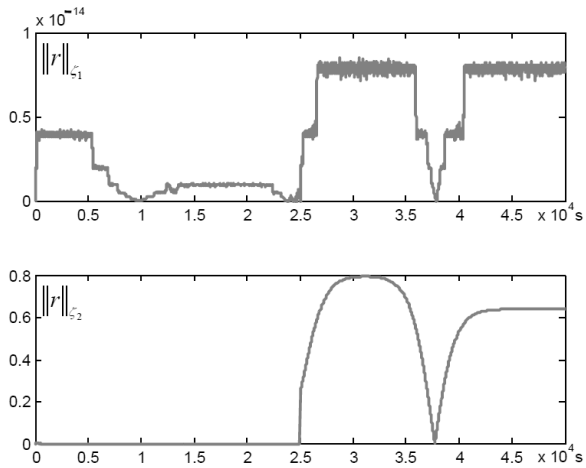

Fig. 6. Residual vector norms with second sensor out of order

order to generate alarms for the operating system. The generalized LPV descriptor observer scheme is able to indicate which sensor is faulty and represents an efficient tool in the operator's decision winding process.

6. CONCLUSION

In this paper, a sensor model-based fault diagnosis method for a particular class of linear parameter variant (LPV) systems is presented. For that, a bank of observers so that each one of them is driven by all inputs and all outputs except the jth measurement variable is designed. Each observer is an observer for polytopic LPV descriptor systems. The observer synthesis is an extension of the work presented in (Darouach and Boutayeb, 1995) where an observer for LTI descriptor systems is reported. Sufficient conditions are stated to ensure the existence and the stability of the proposed observer by using a combined Lyapunov analysis based on LMI formulation. The proposed method is evaluated via simulations using a numerical example.

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