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# Emergence of rogue waves from optical turbulence

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**Abstract:** We show the emergence of rogue wave events from optical turbulence. Depending on the amount of incoherence in the system, we identified different turbulent regimes that exhibit intermittent rogue events or sporadic bursts of light.

**OCIS codes:** (190.0190) Nonlinear optics; (190.5530) Pulse propagation and temporal solitons; (030.1640) Coherence

## 1. Introduction

There has been a tremendous activity devoted to the identification and the understanding of the mechanisms underlying the intricate process of optical rogue wave generation. The previous works [1-4] have been focused on the study of two well-known classes of solutions of the NonLinear Schrödinger Equation (NLSE), namely soliton or quasi-soliton solutions and, on the other hand, Akhmediev Breathers (AB) solutions. It turns out that, as a rather general rule, the collision of (quasi-)solitons or the collision of AB, have been recognized as the essential mechanisms responsible for the generation of optical rogue waves [2-4]. We propose here a novel approach for the understanding of the generation of optical rogue waves, which is based on the study of the impact of the amount of incoherence in the nonlinear system.

## 2. Model

We consider the one-dimensional NLSE in the presence of Third-Order Dispersion (TOD):

$$i \frac{\partial u}{\partial z} = -\frac{1}{2} \frac{\partial^2 u}{\partial t^2} + i\sigma \frac{\partial^3 u}{\partial t^3} - |u|^2 u \quad (1)$$

We normalized the problem with respect to the nonlinear length  $L_0 = 1/\gamma P$  and time  $\tau_0 = (\beta_2 L_0)^{1/2}$ , where  $\gamma$  is the nonlinear coefficient,  $P$  the average power of the field and  $\beta_2$  the second-order dispersion coefficient. In these units, the normalized TOD coefficient reads  $\sigma = \beta_3 / (6 L_0^{1/2} \beta_2^{3/2})$ ,  $\beta_3$  being the TOD coefficient. The NLSE conserves the ‘energy’ (Hamiltonian)  $H = H_L + H_{NL}$ , which has a linear kinetic contribution  $H_L = \int k(\omega) |u(\omega)|^2 d\omega$  and a nonlinear contribution  $H_{NL} = -1/2 \int |u(t)|^4 dt$ , where  $k(\omega) = \omega^2/2 - \sigma \omega^3$  is the linear dispersion relation. *The Hamiltonian is known to provide a natural measure of the amount of incoherence in the system* [5]. Eq.(1) is known to admit quasi-soliton solutions: because of the TOD term ( $\sigma \neq 0$ ), the standard soliton slowly loses its power through radiation.

In the context of ‘(quasi-)soliton turbulence’ [6], a nonintegrable Hamiltonian wave system exhibits a thermalization process characterized by an irreversible evolution of the wave towards a specific equilibrium state, in which a (quasi-)soliton structure remains immersed in a sea of small scale fluctuations. We study here the transition from the coherent quasi-soliton regime to the fully turbulent regime by increasing the energy  $H$  of the system.

## 3. Results

We performed intensive numerical simulations of Eq.(1). Contrary to previous studies [2-4], we are interested here in the long term evolution of the system (typically  $z \sim 2000 L_0$ ). Figure 1a reports the average of the maximum intensity peak detected in the temporal window in the last stage of propagation (over  $500 L_0$ ) as a function of the energy  $H$ , keeping the power  $N = \int |u(t)|^2 dt$  constant. We remark that there exists a threshold value for the energy,  $H_c \sim 15$ , above which quasi-soliton structures are no longer generated. In analogy with wave condensation [5], this means that the system is ‘too incoherent’ (‘too hot’) to generate a coherent structure when  $H > H_c$ .

More precisely, we identified in Fig. 1a three different regimes: For  $H \leq 5$ , we recover the quasi-soliton turbulent regime (i): a large amplitude quasi-soliton immersed in a sea of small-scale fluctuations eventually emerges from multiple inelastic collisions. Figure 1b represents the space-time intensity pattern over the last 10 nonlinear lengths. The corresponding Probability Distribution Functions (PDF) of the intensity of the field, and of the maxima of the intensity, are reported in Fig. 1c. We point out that, because of the large localization of the power in the quasi-soliton, the field statistics deviates substantially from the Gaussian statistics, i.e.  $f_I(I) = \exp(-I)$  with our normalized units. This observation is corroborated by the PDF of the intensity maxima, which is centered at a high value ( $\sim 30$ ) and exhibits a *narrow width and symmetric shape*, thus confirming the persistence of the high power quasi-soliton structure.

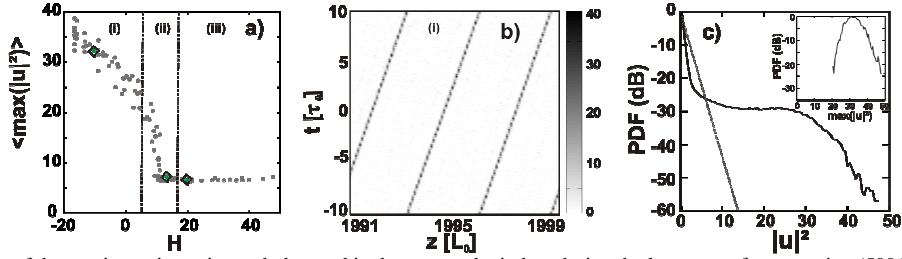


Fig. 1: (a) Average of the maximum intensity peak detected in the temporal window during the last stage of propagation ( $500 L_0$ ) as a function of the Hamiltonian  $H$ , for  $\sigma = 0.02$ . The green dots (or diamonds) in (a) are analyzed separately. (b) Space-time intensity pattern showing a coherent quasi-soliton propagating in the midst of small-scale fluctuations for  $H = -10.3$  (Regime (i)). (c) PDF of the intensity of the field and of the maxima of the intensity (inset). The gray line stands for Gaussian statistics of the field amplitude.

For  $5 \leq H \leq 15$ , the system enters into the second regime (ii), in which the quasi-soliton structures exhibit a kind of *intermittent dynamics*, i.e. the quasi-soliton appears and disappears erratically during the propagation. Note that, despite such intermittent-like behavior, the trajectory of the quasi-soliton may still be identified in the space-time intensity pattern (see Fig. 2a1). As for the regime (i), the field does not exhibit a Gaussian statistics. However, the PDF of the maxima of the intensity gets relatively *broader and asymmetric* (see Fig. 2a2). This is due to the intermittent (non-persistent) character of the quasi-soliton structure, whose significant fluctuations tend to favor the high intensity tail of the PDF. The resulting asymmetry in the PDF bears a strong resemblance with the typical L-shaped probability distributions that characterize freak-wave extreme events.

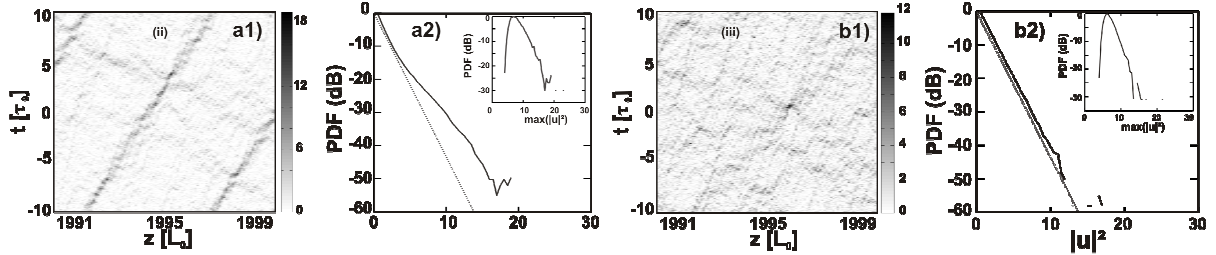


Fig. 2: (a1,b1) Space-time intensity pattern for  $H = 13.3$  (Regime (ii)) and  $H = 19.5$  (Regime (iii)). (a2,b2) Corresponding PDF of the intensity of the field and of the maxima of the intensity (insets).

Finally, for  $H \geq 15$  the system enters into the fully incoherent and weakly nonlinear regime of propagation (iii), in which quasi-soliton structures are no longer generated. The corresponding spatiotemporal intensity pattern reveals that *very short-lived rogue wave* events may still emerge from the turbulent field, although these extreme events become rare (see Fig. 2b1). The generation of these *sporadic bursts of light* in regime (iii) is no longer related to coherent and deterministic quasi-soliton structures. This is corroborated by the statistics of the field, which approaches a Gaussian statistics, as expected for an incoherent system of weakly nonlinear waves. We verified that few extreme events identified in regimes (ii) and (iii) verify the standard criterion usually employed to define *hydrodynamic rogue waves* [7]. For higher values of  $H$ , the system becomes essentially linear, i.e.  $|H_{NL}| \ll |H_L|$ .

In conclusion, we provided some general physical insights into the spontaneous emergence of extreme events from turbulent fluctuations. The new optical rogue waves reported in the regime (ii) and, more importantly in regime (iii), seem to exhibit properties analogous to genuine hydrodynamic rogue wave events, a feature that needs to be analyzed in more detail so as to draw a substantiated analogy between them.

#### 4. References

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