Experimental perspectives for systems based on long-range interactions

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Abstract. The possibility of observing phenomena peculiar to long-range interactions, and more specifically in the so-called Quasi-Stationary State (QSS) regime is investigated within the framework of two devices, namely the Free-Electron Laser (FEL) and the Collective Atomic Recoil Laser (CARL). The QSS dynamics has been mostly studied using the Hamiltonian Mean-Field (HMF) toy model, demonstrating in particular the presence of first versus second order out-of-equilibrium phase transitions from magnetized to unmagnetized regimes. Here, we give evidence of the strong connections between the HMF model and the dynamics of the two mentioned devices, and we discuss the perspectives to observe some specific QSS features experimentally. In particular, a dynamical analog of the phase transition is present in the FEL and in the CARL in its conservative regime. Regarding the dissipative CARL, a formal link is established with the HMF model. For both FEL and CARL, calculations are performed with reference to existing experimental devices, namely the FERMI@Elettra FEL under construction at Sincrotrone Trieste (Italy) and the CARL system at LENS in Florence (Italy).

Contents

1. Introduction

Long-range interactions have now been shown to be central in a wide range of scientific contexts such as astrophysics [1], hydrodynamics [2] or nuclear physics [3]. However, the possibilities of investigating the long-range features via dedicated experiments are more restricted: Non-neutral plasmas [4], cold atom and wave-particle systems [5] are among the most serious candidates. The purpose of this paper is to investigate the possibility of using existing set-ups based on the wave-particle interactions to probe long-range features of the dynamics, in particular out-of-equilibrium transitions.

As an introduction to long-range interactions, let us start from the Hamiltonian Mean-Field (HMF) model [6], a paradigmatic system on which many theoretical studies focused. This one-dimensional model describes the interaction of N particles on a circle through a collective field, which depends only on their phase θ_j. This N-body dynamics is described by the following Hamiltonian:

$$H = \sum_{j=1}^{N} \left(\frac{p_j^2}{2} + \frac{\epsilon}{2N} \sum_{k=1}^{N} (1 - \cos(\theta_j - \theta_k)) \right), \tag{1}$$

associated to the canonical bracket in (θ_j, p_j). Here, $\epsilon = \pm 1$ corresponds either to a ferromagnetic (+) or an antiferromagnetic (-) system. In this model, the particles are collectively interacting through the so-called magnetization $\mathbf{M} = Me^{i\phi} = (\sum_j e^{i\theta_j})/N$, since the dynamics of a single particle is given by:

$$\ddot{\theta}_j + \epsilon M \sin(\theta_j - \phi) = 0. \tag{2}$$

Long-range systems can exhibit interesting equilibrium features, such as ensemble inequivalence (see e.g. [7] for the antiferromagnetic two-dimensional version of the HMF model or [8] for a recent review). However, the HMF model mainly revealed itself as a perfect playground to study *out-of-equilibrium* long-range features. Indeed, starting from generic non-stationary initial conditions, the system will typically have a fast transient dynamics until a nearly-stationary state, generally called Quasi-Stationary State (QSS), is reached: Not only this QSS dynamics substantially differs from the equilibrium one, but the system actually stays trapped in it for very long times [6].

More specifically, several authors actually demonstrated that the lifetime of the said QSS diverges when the number of particles in interaction increases. For example, numerical works report that the time of relaxation to equilibrium for the

Hamiltonian Mean-Field model scales as $N^{1.7}$ [9], in a regime of parameters yielding homogeneous QSS. To gain insight into the emergence of QSS, one can resort to a continuous picture, formal limit of the governing discrete Hamiltonian. A rigorous mathematical procedure leads to the Vlasov equation for the evolution of the single particle distribution function, the continuous representation of the particles density in phase space which is recovered when making the number of bodies N tend to infinity. The stability of QSS in the infinite N limit suggests that these latter states can be potentially interpreted as *Vlasov stationary states*, an ansatz that opens up the perspective for further analytical progress, a fact on which we shall return in the following. Operating in this context and explicitly accounting for finite size corrections beyond the idealized Vlasov picture, the authors of [10] proved rigorously that the relaxation of the N-body system towards its deputed equilibrium, as driven by microscopic collision effects, would occur on time scales larger than N, in qualitative agreement with the numerical evidences commented above. Clearly, QSS are supposedly the only regimes which are made experimentally accessible, in all physical situations where a large number of microscopic constituents evolve in mutual interaction. The experimental time of observation is in fact limited, and not sufficient to allow for equilibration. In this perspective, to unravel the puzzle of QSS and so build a comprehensive dynamical picture for their existence and evolution, represents a major challenge, with undoubtedly many practical implications.

An important step forward explaining the presence of the QSS was eventually attained thanks to the theory of violent relaxation of Lynden-Bell (LB) [11]. This is a statistical theory which embeds self-consistently knowledge of the governing Vlasov dynamics. The approach is justified from first principles and allows to resolve the intermediate regime of the discrete N-body evolution, when the system is presumably assimilable to a continuum Vlasov model, before finite size corrections come eventually into play. The theory is based on the maximization of the following entropic functional of the distribution function (DF) $\bar{f}$:

$$s[\bar{f}] = -\int dp d\theta \left[\frac{\bar{f}}{f_0} \ln \frac{\bar{f}}{f_0} + \left(1 - \frac{\bar{f}}{f_0}\right) \ln \left(1 - \frac{\bar{f}}{f_0}\right) \right], \qquad (3)$$

where f_0 describes the initial state of the system, whereas $\bar{f}$ stands for a coarse-grained distribution function of the final state, that one wishes to recover via a predictive approach. The above formulation holds for a two-step initial distribution function (water bag): f at time 0 is equal to either zero or f_0. Whereas the exact evolution according to the Vlasov equation imposes that the DF is only allowed to take 0 and f_0 values at all times, the coarse-grained point-of-view implies a continuous DF $\bar{f}$ that is expected to be valid if one averages over small patches of phase space. As a side comment we notice that the functional (3) can be readily generalized to account for a continuous collection of different density levels, beyond the water-bag hypothesis.

The maximization of s is performed under the macroscopic constraints of normalization, energy and momentum which are conserved by the dynamics. An underlying hypothesis to the theory is that the system explores in an ergodic-like fashion all states allowed by the constraints. The dynamical evolution of the Vlasov equation departs from that of a system sampling the equilibrium microcanonical ensemble, giving rise to different predictions which reflect the out-of-equilibrium nature of the problem. The application of the above predictive strategy to the study of the QSS dynamics of respectively the HMF model [12], free-electron lasers [13] and gravitational systems [14] has confirmed its adequacy.

The LB approach also brought some new insights into the HMF phenomenology. For example, the abrupt change in the QSS magnetization when smoothly tuning the initial state of the system was interpreted as an *out-of-equilibrium phase transition*, which not only depends on the energy of the system - as it happens at equilibrium - but also on the precise way the system is prepared. More specifically, the initial magnetization of the system was shown to have an important role, and phase transitions of both first and second order could be observed depending on the value of this latter parameter. This memory effect - the system keeps track of the detail of its initial state for very long times - makes the QSS dynamics significantly richer than the equilibrium one. For example, as regards the HMF model, an out-of-equilibrium tricritical point was identified, which does not exist at equilibrium.

The purpose of this paper is to determine whether some of the QSS features predicted for the HMF toy-model can be observed in experiments run for a dedicated class of devices. Motivated by this working hypothesis, we shall turn to considering the wide field of wave-particle interaction and focus in particular onto two different experiments, namely the Free-Electron Laser (FEL) and the Collective Atomic Recoil Laser (CARL). In both cases the dynamics reflects the long-range nature of the interaction, along the lines depicted above with reference to the simplified HMF setting. Operating in this framework, we will show that some features of the QSS dynamics, as those previously outlined, may be observed in direct experiments. Moreover, such properties though peculiar to the considered wave-particle dynamics, bear some reminiscent traits of the HMF model, to which both FEL and CARL are intimately connected. Eventually, the associated experimental set-ups are briefly detailed, based on existing machines and current technology.

Section 2 is devoted to FELs. The aim of such devices is to produce high-power short-wavelength light pulses by exploiting the radiation emitted by ultra-relativistic electrons when passing through the static and periodic magnetic field generated by an undulator. Starting from generic initial conditions, the wave power grows to a maximum, and then starts oscillating, keeping a lively exchange of energy with the particles, over times diverging with the number of particles, a characteristics of the QSS. As for the case of HMF, the QSS of a FEL depends not only on the energy of the system, but also on the details of its initial state. Thus, after presenting the FERMI@Elettra FEL, we discuss how to manipulate the electron beam to produce the sought different initial states. Finally, the dynamical transition present in the system is described.

Section 3 is dedicated to discussing the Collective Atomic Recoil Laser (CARL), an experiment where a probe wave is amplified thanks to a grating of cold atoms (back)scattering photons of an incident pump laser beam. As for the FEL, its dynamics is dominated by long-range effects in the one-dimensional limit, an approximation which holds for the CARL experiment based at the European Laboratory for Non-linear Spectroscopy (LENS). We then focus on the conservative regime, when the dynamics formally reduces to that of the FEL: The possibilities to observe for the CARL the QSS phenomenology as depicted for the FEL is investigated. On the other hand, when the wave amplification takes place in a cavity, a damping has to be accomodated for: A formal link between this operational regime of CARL and the HMF dynamics is drawn, as well as the experimental perspectives to detect the associated out-of-equilibrium transitions.

Finally, in Section 4, we discuss the measurements that could be performed for both CARL and FEL in order to unravel the imprint of QSS that indirectly materializes in the existence of distinct out-of-equilibrium regimes.

2. The Free-Electron Laser as a long-range interacting system

FELs are powerful light sources able to deliver coherent pulses of photons over a large and tunable wavelength range. To that aim, ultra-relativistic electrons of energy γ are injected into the periodic magnetic field (of period λ_w and deflection parameter K) produced by an undulator, where they start to wiggle and emit synchrotron radiation around the following wavelength:

$$\lambda = \frac{\lambda_w}{2\gamma^2}\left(1+K^2\right). \quad (4)$$

The light produced by the electrons traps the electrons themselves, resulting in a periodic modulation of the electrons' density (see Fig.1) called bunching: This bunching is the source of the coherent emission. Eventually, under a resonant condition between the electrons and the wave, the strong interplay between coherent emission and particle trapping inside the wave potential leads to the nonlinear growth of the wave (see Fig.1) and to the emission of a powerful light pulse. Due to the high energy

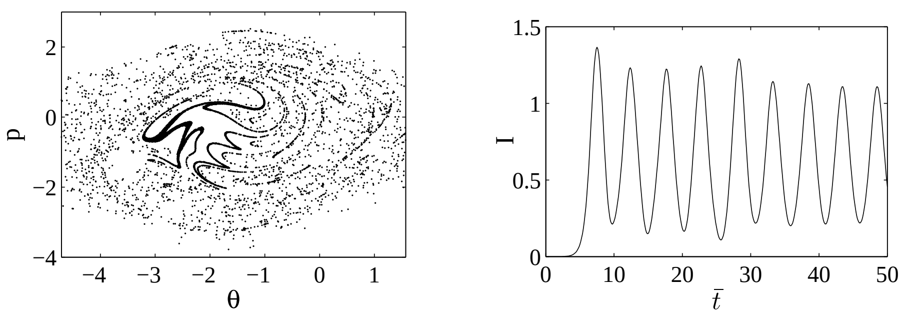

Figure 1. Left: Electron phase-space in the QSS regime (at $\bar{t} = 50$). Right: Normalized laser intensity versus normalized time $\bar{t}$. Simulations performed with $N = 8000$ particles, starting with a waterbag with $b_0 = 0$, $\Delta p = 0.05$, and a negligible intensity $I_0 = 10^{-6}$.

of the electrons (of order 1 GeV typically), the system can be in first approximation considered as one-dimensional, since the angle of the cone of light radiated goes as the inverse of the electrons energy. As for the radiation, it can generally be described by a mean-field wave, leading to the set of equations [15]:

$$\begin{aligned} \frac{d\theta_j}{dt} &= p_j, \\ \frac{dp_j}{dt} &= -\left(Ae^{i\theta_j} + A^* e^{-i\theta_j}\right), \\ \frac{dA}{dt} &= \frac{1}{N}\sum_j e^{-i\theta_j} + i\delta A. \end{aligned} \quad (5)$$

where θ_j is the phase of electron j with respect to the ponderomotive potential, p_j its normalized energy, whereas A stands for the complex amplitude of the synchrotron radiation. The normalized variables are defined as $\theta_j = (k + k_w)z_j - \omega t - \delta\bar{t}$, with z_j the position of particle j along the propagation axis, $p_j = (\gamma_j - \gamma_0)/\rho\gamma_0$, γ_0 the average electron energy, k and ω the radiation wavenumber and frequency, $\delta = (\gamma_0 - \gamma_R)/\rho\gamma_0$ the detuning parameter and γ_R the resonant energy defined by Eq.(4). $\rho = (I/I_A)^{1/3}(\lambda_w a_w/2\pi\sigma)^{2/3}/2\gamma_0$ is the so-called Pierce parameter,