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### A combined closed loop optimal design of experiments and online identification control approach

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Outline : motivations, proposed method, case study

- Some needs for identification [Ljung, 1999] :
  - to get a simulator or do advanced control : need a process model
  - to manipulate the model : need to have numerical value of all its parameters
  - to identify the model parameters : need "rich" input / output (u(t) / y<sub>p</sub>(t)) data
- Identification based on real experiments =
  - 1/ tune (on/off line) the process input to run the experiments
  - + 2/ with the collected input-output data, extract the value of the unknown constant value of the model parameters  $\theta$
  - these 2 tasks are usually done separately

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- Let us define the parametric sensitivity of the process outputs  $y_p$  w.r.t the model parameters with unknown constant value  $\theta : \frac{\partial y_p}{\partial \theta}(t)$
- If the norm (to be chosen) of this sensitivity  $\frac{\partial y_{\rho}}{\partial \theta}(t)$  is "small" or "null", then, for identification :
  - 1. whether, this parameter  $\theta$  does not influence the output  $y_p(t)$ = bad model structure
  - 2. whether, the measure y<sub>p</sub>(t) is not well chosen for the identification of θ=bad output
  - 3. whether, the input signal u(t) does not sufficiently excite the parametric sensitivity =bad input

#### Hypothesis

We tackle here the case : good model structure + good output + need to design the closed loop optimal input for combined online identification.

- 2 main ideas for design of control for identification.
  - 1/ adjust, in an optimal manner [Pronzato, 2008], on line and in closed loop, the value of the input u(t)
  - + 2/ jointly estimate the constant value of the unknown model parameter  $\theta$
- Use : the input-output data + a dynamic model
- Deduce : an observer, a sensitivity model
- Use an internal model control (IMC) structure
- Use a model predictive controller (MPC) to optimize the input + handle operating constraints (input, state, output) during the identification task
- To simplify the presentation : SISOSP case.

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#### Advantages :

- many theoretical papers published
- +4000 applications in the world [Qin et Badgwell, 2003]
- Idea :
  - use the model to predict the future process behavior
  - optimize any specified criteria
  - take account for constrains on measures/estimations
  - closed loop control approach

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- Time discretization :  $t = k \times T_s$  (*k*=current time index)
- At each k, over the prediction horizon N<sub>p</sub> : maximize the norm of the sensitivity <sup>∂yp</sup>/<sub>∂θ</sub> to get "rich" data for identification
- During its use for identification, maintain the process inside a prescribed operating zone =
  - constrains on the input (physical limitations of the actuator) :

$$\left\{ \begin{array}{l} u_{min} \leq u(t) \leq u_{max} \ , \ \forall \ t \geq t_i \\ \Delta u_{min} \leq \frac{du(t)}{dt} \leq \Delta u_{max} \ , \ \forall \ t \geq t_i \end{array} \right.$$

• constrains on the states and/or output (operating zone, safety, maintain product quality, ...) :

$$c_i(x_p(t), y_p(t), u(t)) \le 0, \ \forall \ t \ge t_i, \ \forall \ i \in I^{n_c} = \{1, ..., n_c\}$$
(2)

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The constrained optimization problem for identification is initially :

$$(COPI_{I}) \begin{cases} \max_{\tilde{u}} J(\tilde{u}) = \sum_{j=k+1}^{k+N_{p}} \left(\frac{\partial y_{p}}{\partial \theta}(j)\right)^{2} \\ \tilde{u} = \{p_{u}(1), ..., p_{u}(n_{pu})\} \\ u(j) = f_{u}(p_{u}(i), j, k) \ i \in [1, n_{pu}] \\ \forall k > 0, \ \forall \ j \in [k, k+N_{p}-1] \\ u_{min} \le u(j) \le u_{max} \\ \Delta u_{min} \le \frac{u(j)-u(j-1)}{T_{s}} \le \Delta u_{max} \\ \forall \ k > 0, \ \forall \ j \in [k+1, k+N_{p}], \forall \ i \in I^{n_{c}} : \\ c_{i}\left(x_{p}(j), y_{p}(j), u(j-1)\right) \le 0 \end{cases}$$
(3)

This optimization problem can not be solved on-line.

- Issue : formulation based on signals into the future  $j : x_p$ ,  $y_p, \frac{\partial y_p}{\partial \theta}$ 
  - Values not available at k into the future j.
  - u and  $y_p$  are the only signals available at k.

So, such problem may not yet be solved at k

- Solution : at each *k*, approximate these signals into the future *j* with :
  - the measures  $y_p$  and u + the on-line estimations
  - integration of the model and sensitivity model over the future N<sub>p</sub>
  - internal model control structure

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#### New method for coupled on-line optimal design of experiment and identification Step 1 : Introduce the model

 $(M) \begin{cases} \dot{\mathbf{x}}(t) = f(\mathbf{x}(t), \alpha, \theta, \mathbf{u}(t)), & t > t_i \\ \mathbf{y}(t) = h(\mathbf{x}(t), \alpha, \theta, \mathbf{u}(t)), & t > t_i \\ \mathbf{x}(t) = \mathbf{x}^{t_i}, & t = t_i \end{cases}$ (4)

- state  $x \in \mathbb{R}^n$ , input  $u \subset \mathbb{R}$  and the output  $y \in \mathbb{R}$
- $\alpha$  : vector of the model parameters with known value
- $\theta$  : the model parameter with unknown value
- (*f*, *h*) = dynamic model (nonlinear or linear)

Hypothesis

In the system (4), f and h are  $C^1$ .

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# New method for coupled on-line optimal design of experiment and identification

Step 2 : Introduce the observer

- Use the large theory on observers for nonlinear systems [Besançon, 2007]
- Build an observer (O), based on the model (M) and on the process measures y<sub>p</sub> and u.
- (0) estimates on line (at each k) :
  - the unknown elements of the process state :  $\hat{x}_p(k)$
  - and the unknown model parameter :  $\hat{\theta}(k)$ .

#### Hypothesis

#### The state of the observer (O) is observable.

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## New method for coupled on-line optimal design of experiment and identification

Step 3 : Introduce the sensitivity model

Based on the model (*M*) and on the observer (*O*), the sensitivity model ( $M_{\theta}$ ) is :

$$(M_{\theta}) \begin{cases} \dot{x}_{\theta}(t) = \frac{\partial}{\partial t} \frac{\partial f}{\partial \mathbf{x}} \frac{\partial x}{\partial \theta}(t) = \dots \\ \dots f_{\theta}(\mathbf{x}_{\theta}(t), \mathbf{x}(t), \alpha, \hat{\theta}(t), \mathbf{u}(t)), & t > t_{i} \\ \mathbf{y}_{\theta}(t) = \frac{\partial h}{\partial \mathbf{x}} \frac{\partial x}{\partial \theta}(t) = \dots \\ \dots h_{\theta}(\mathbf{x}_{\theta}(t), \mathbf{x}(t), \alpha, \hat{\theta}(t), \mathbf{u}(t)), & t > t_{i} \\ \mathbf{x}_{\theta}(t) = \mathbf{x}_{\theta}^{t_{i}}, & t = t_{i} \end{cases}$$
(5)

- state x<sub>θ</sub> = ∂x/∂θ ∈ ℝ<sup>n</sup> : sensitivity of the state of the model (M) w.r.t. the parameter θ,
- output y<sub>θ</sub> = ∂y/∂θ ∈ ℝ : sensitivity of the output of the model (M) w.r.t. the parameter θ.

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## New method for coupled on-line optimal design of experiment and identification

Step 4 : Approximate the signals into the future

- Using :
  - the measures  $y_p$  and u at k
  - the estimations of (O) at k
  - the prediction of the 2 models (*M*) and ( $M_{\theta}$ ) over  $N_p$  at *j*
- Approximate the signals necessary in the (COPI<sub>1</sub>) :

$$\begin{cases} \forall k \ge 0, \ \forall \ j \in [k+1, k+N_p] :\\ x_p(j) = x_m(j) + (\hat{x}_p(k) - x_m(k))\\ y_p(j) = y_m(j) + (y_p(k) - y_m(k))\\ \frac{\partial y_p}{\partial \theta}(j) = \frac{\partial y_m}{\partial \theta}(j) + (\frac{\partial y_p}{\partial \theta}(k) - \frac{\partial y_m}{\partial \theta}(k)),\\ \forall k \ge 0 :\\ \theta = \hat{\theta}(k), \end{cases}$$
(6)

#### New method for coupled on-line optimal design of experiment and identification Step 5 (end) : get the final MPC problem

Constrained optimization problem for identification based on the model :

$$(COPI_{M}) \begin{cases} \max_{\tilde{u}} J(\tilde{u}) = \sum_{j=k+1}^{k+N_{p}} \left( \frac{\partial y_{m}}{\partial \theta}(j) + \left( \frac{\partial y_{p}}{\partial \theta}(k) - \frac{\partial y_{m}}{\partial \theta}(k) \right) \right)^{2} \\ \tilde{u} = \{p_{u}(1), ..., p_{u}(n_{pu})\} \\ u(j) = f_{u}(p_{u}(i), j, k) \ i \in [1, n_{pu}] \\ \forall k > 0, \ \forall \ j \in [k, k + N_{p} - 1] \\ u_{min} \le u(j) \le u_{max} \\ \Delta u_{min} \le \frac{u(j) - u(j-1)}{T_{s}} \le \Delta u_{max} \\ \forall \ k > 0, \ \forall \ j \in [k+1, k+N_{p}], \forall \ i \in I^{n_{c}} : \\ c_{i}(x_{m}(j) + (\hat{x}_{p}(k) - x_{m}(k)), ... \\ ..., y_{m}(j) + (y_{p}(k) - y_{m}(k)), u(j-1)) \le 0 \\ + \text{ on line resolution of } (M), (M_{\theta}) \text{ and } (O). \end{cases}$$

$$(7)$$

This model based optimization problem may be solved on-line.

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Objectives : underline the procedure and advantages of the proposed approach (without optimizing observer/controller parameter settings) on a simple case.

Nonlinear model (state-parameter, input-parameter) of saponification :

$$(M) \begin{cases} \dot{x_1}(t) = -\theta_1 x_1^2(t) + \frac{u(t)}{\alpha_1} (\alpha_2 - x_1(t)), \ t > 0\\ y(t) = x_1(t), \ t > 0\\ x_1(t) = x_1^0, \ t = 0 \end{cases}$$
(8)

- $x_1(t)$  : concentration of ethyl acetate inside the reactor
- u(t) : input flow rate of sodium hydroxide .
- Known constant parameters : constant volume of the reaction α<sub>1</sub>, input concentration α<sub>2</sub>
- Unknown constant parameter : reaction rate  $\theta_1$

Based on [Gauthier et al., 1992], the augmented state  $x_o = [x_{o \ 1} \ x_{o \ 2}]^T$  allows to estimate  $[\hat{x}_{p \ 1} \ \hat{\theta}_1]^T$ :

$$(O) \begin{cases} \dot{x}_{o}(t) = A_{o}(y_{p}(t))x_{o}(t) + B_{o}(u(t)) - S_{o}^{-1}C^{T}(Cx_{o}(t) - y_{p}(t)) \\ \dot{S}_{o}(t) = -\gamma_{o}S_{o}(t) + A_{o}^{T}(y_{p}(t))S_{o}(t) - S_{o}(t)A_{o}(y_{p}(t)) + C^{T}C \\ (9) \end{cases}$$

where  $\gamma_o$  is the observer tuning parameter and where :

$$\begin{cases}
A_o(y_\rho(t)) = \begin{pmatrix} 0 & -y_\rho^2(t) \\ 0 & 0 \end{pmatrix} \\
B_o(u(t)) = \begin{pmatrix} \frac{u(t)}{\alpha_1}(\alpha_2 - y_\rho(t)) \\ 0 \end{pmatrix} & (10) \\
C = \begin{bmatrix} 1 & 0 \end{bmatrix} \\
S_o : \text{SPD matrix}
\end{cases}$$

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The sensitivity model ( $M_{\theta}$ ) is written, where the state is  $x_{\theta} = x_{\theta 1}$ :

$$(M_{\theta}) \begin{cases} \dot{x}_{\theta \ 1}(t) = -(2x_{1}(t)x_{0 \ 2}(t) + \frac{u(t)}{\alpha_{1}})x_{\theta \ 1}(t) - x_{1}^{2}(t), \ t > 0\\ y_{\theta}(t) = y_{\theta \ 1}(t) = x_{\theta \ 1}(t), \ t > 0\\ x_{\theta \ 1}(t) = 0, \ t = 0 \end{cases}$$
(11)

It is explicitly coupled to (M) and (O).

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- Maximize the sensitivity of the concentration of ethyl acetate y<sub>p</sub> w.r.t. the reaction rate θ<sub>1</sub> to be estimated on-line.
- 2 cases are simulated :
  - Case 1 :
    - magnitude and velocity constrains on the control,
    - a free behavior is allowed for the output
  - Case 2 (green/red) :
    - magnitude and velocity constrains on the control,
    - the output must now be maintained below a prescribed maximum threshold (operating constrains).
  - These 2 cases allow to evaluate the approach in a constrained situation.

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#### Case study Numerical results (1)



- Case 1 : control is always saturated in magnitude or velocity.
- Case 2 (green/red) : during the 5 first minutes = case 1. After 5 minutes, the control changes due to the need to satisfy the output constraint.

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Parameter estimation

- Case 1 & Case 2 : estimation of the unknown process parameter is efficient, is spite of an initial error of 30%
- Case 1 & Case 2 : trajectory of estimation are similar (in these numerical conditions)
- Case 2 & not Case 1 : allows to maintain the reactor inside the specified region.

### Conclusions and perspectives

- We have a new generic on-line approach :
  - it helps to automate the optimal design of experiment coupled to the parametric identification
  - = jointly 1/ determine on-line the optimal control to apply + 2/ identify on-line the value of the unknown parameter
- Hypothesis have a physical sense :
  - 1/ for the initial formulation of the problem
  - 2/ model (functions are  $C^1$ )
  - 3/ design of the observer (strongest assumption)
- Optimization problem that may be solved by any software (cf. ODOE4OPE.univ-lyon1.fr)
- Perspectives : particular classes of models, multi-variable cases, unstable systems, non uniformly observable systems, robustness analysis, experimental applications, ...

Myself (resume, links to open archives/journal papers ...) : tinyurl.com/dufourpascal

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Time for discussion !!!

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#### Case study Numerical results (3)



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