

## Solutions

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### KVADRAT

The key here is finding the magic sum of the square (the sum of each row, column and diagonal). If there is at least one complete row, column or diagonal, then the magic sum is obvious.

However, when the three unknown elements are on one of the two diagonals, then the magic sum is one half of all the known elements. This is easy to prove: the sum of all elements in the magic square is three times the magic sum. One diagonal (whose sum should also be the magic sum) is missing so the sum of all known elements is twice the magic sum.

Having found the magic sum, we proceed with filling the gaps.

### SVEMIR

A simpler version of the problem is that in one dimension, where the black box is located somewhere on a line. We can use binary search to find the location of the black box by maintaining an interval of possible solutions  $[a, b]$  and then:

- jump to the beginning of the interval  $a$ , discarding the feedback from our sensors
- jump to the end of the interval  $b$
- if the second move has brought us closer to the box then the box is in the second half of the interval, otherwise it is somewhere in the first half

Since the number of possible positions of the black box in three dimensions is  $10^{27}$ , some sort of binary search is obviously needed here as well. What is less than obvious is that we can perform a binary search on each individual coordinate, while keeping the other two coordinates of a point fixed. This allows us to find the box in less than  $3 \cdot 2 \cdot \log_2(10^9) = 180$  moves!

### JMBG

We can split the solution into three parts:

- calculating in how many ways each of the 19 remainders can be attained in the first (date) part of the number
- the same for the second part of the number, consisting of arbitrary digits
- combining the two sub-solutions

For the first part we generate all legal dates matching the pattern. We can use dynamic programming to solve the second part. Combining the solutions means determining in how many ways the control digit can be attained from the first and second parts. If the control digit is arbitrary we sum over all ten possible digits.