

## ARHITEKT

Let us model the drawing as a graph and try to find the link between the number of vertices  $V$ , the number of edges  $E$  and the number of rooms  $R$  while drawing. If there are multiple strokes between a pair of points, ignore all except the first one.

Initially, we are in the origin and no line segments have been drawn, so  $V=1$  and  $E=R=0$ . When Mirko Jr. draws a line segment, there are two cases:

1. The line segment ends in a point we have not yet been in.  $V$  and  $E$  are incremented, but  $R$  isn't.
2. The line segment ends in a point already visited.  $V$  stays the same, but  $E$  and  $R$  are incremented.

Notice that before and after each stroke the formula  $V - E + R = 1$  holds. This is the famous Euler identity for planar graphs (normally, the entire plane is initially considered to be one room, so the number 2 appears on the right side of the equation instead of 1).

Using this formula, the task is easy to solve: count the number of distinct points we traverse, and also the number of distinct edges in the graph. An additional trick is to multiply all coordinates by 2 and split every stroke into two half-strokes – this elegantly solves the problem of intersecting diagonals of a unit square.

## DORUČAK

Let us first explain in an informal proof why such a line exists; an algorithm to find it follows from the proof.

Let  $(0, b)$  be an arbitrary point on the  $y$ -axis. It is easy to see that a line exists that passes through  $(0, b)$  and cuts the left burek in two equal halves. Its slope can be found using binary search. Let that coefficient be  $a_L(b)$ . Similarly, define a slope  $a_R(b)$  for the same point  $(0, b)$  and the right burek.

We want to find the value of  $b$  such that  $a_L(b) = a_R(b)$ . Then the desired line is  $y = a_L(b) * x + b$ . Let  $a(b) = a_L(b) - a_R(b)$ . We are looking for the value of  $b$  such that  $a(b) = 0$ .

Notice that  $a_L(b)$  and  $a_R(b)$  are continuous functions of  $b$  (without "jumps"). Because of this,  $a(b)$  is also a continuous function of  $b$ .

When  $b$  is negative and sufficiently large in absolute value, then  $a_L(b)$  will be negative, and  $a_R(b)$  will be positive.

When  $b$  is positive and sufficiently large, then  $a_L(b)$  will be positive, and  $a_R(b)$  will be negative.

Because  $a(b)$  is negative for small  $b$ , positive for large  $b$ , and continuous, it has to become zero at some point, and we can find that point using binary search.