

Solutions

KONTROLA

We can divide passengers into two groups: those who have had their ticket examined (type A) and those that haven't (type B).

When the conductor examines tickets, all type B passengers become type A.

We can simulate the train ride through all N stations always having type B passengers get off first, then type A passengers if necessary. This gives the largest number of passengers getting away without having their tickets examined.

The minimum is obtained by having type A passengers get off first.

STOLNI

The solution to this problem is simulation, moving the handles any way which allows the ball to go through.

KOCKICE

The easiest approach would probably be to recursively try and put one letter at a time in each column. The time complexity of this solution is $O(3^N)$.

The key observation is that we don't need to remember the entire column, but only the current 'run' of letters at the top: for each column we keep track of the top letter and how many times it appears consecutively (if there are more than 5 letters, we can ignore the excess).

The state of the game can be described with the septuplet (index, letter1, howmany1, letter2, howmany2, letter3, howmany3, letter3), where index is the index of the next letter to be dropped. The total number of states can be up to about $1000 \cdot (26 \cdot 5)^3$, which is over 2 billion.

To further reduce the number of states, we observe that the order of the columns doesn't matter and that we can permute them as we feel fit. By always considering the column where we dropped the last letter as the first column, we eliminate the need for the letter1 part of the state, which reduces the maximum number of states to about 100 million.

A dynamic programming solution over the described space state yields the solution. The memory limit prohibits us from implementing it using recursion: we need to iteratively process the state space as each letter arrives, keeping only states with the last and current letters in memory. This effectively reduces the first dimension of the state from N to 2 as far as memory is concerned.