



HAL
open science

Cohesive zone model and bipotential formulation: application to a pile/soil interface

Nazihe Terfaya, Michel Raous, Abdelmadjid Berga

► **To cite this version:**

Nazihe Terfaya, Michel Raous, Abdelmadjid Berga. Cohesive zone model and bipotential formulation: application to a pile/soil interface. 3rd Euro Mediterranean Symposium on Advances in Geomaterials and Structures 2010, May 2010, Djerba, Tunisia. pp.1-6. <hal-00466596>

HAL Id: hal-00466596

<https://hal.science/hal-00466596v1>

Submitted on 2 Oct 2023

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire HAL, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.



Distributed under a Creative Commons CC BY 4.0 - Attribution - International License

Cohesive zone model and bipotential formulation: application to a pile/soil interface

Nazihe Terfaya^{1,2}, Michel Raous¹, Abdelmadjid Berga²

¹ Laboratoire de Mécanique et d'Acoustique, CNRS, Marseille, France

E-mail: raous@lma.cnrs-mrs.fr

² Laboratoire de Fiabilité des MATériaux et des Structures, Université de Béchar, Béchar, Algérie

E-mail: terfaya@lma.cnrs-mrs.fr, bergaabdelmadjid@yahoo.fr

Abstract. The cohesive zone model developed by Raous, Cangémi, Cocou and Monerie includes strict unilateral contact to avoid interpenetration, initial adhesion progressively decreasing when the load increases, and Coulomb's friction which is progressively activated when adhesion decreases. Because of its implicit character, the Coulomb friction law is non-associated, and the notion of superpotential with normality rule cannot be used anymore. To overcome this difficulty, G. De Saxcé introduced the class of implicit standard materials based on the concept of bipotential. In the present work a specific bipotential adapted to the coupling of unilateral contact, friction and adhesion is constructed, and a numerical method is proposed and implemented. An application is presented, for modeling soil/pile interface behavior, on the simulation of pull-out experiments of a pile.

Keywords: friction, adhesion, bipotential, soil/pile interface.

1 Introduction

In order to model the behaviour of complex interfaces, a model coupling adhesion, friction and unilateral contact was proposed in (Raous et al, 1999) and extended in (Raous and Monerie, 2002) (the RCCM model). This model is based on the adhesion intensity variable, introduced by Fremond (Frémond, 1988), which is a surface damage variable and takes its values between 0 and 1 (0 is no adhesion and 1 is total adhesion). This model gives a smooth transition from total adhesion to the usual Coulomb friction law with unilateral contact. Because of the non smooth character of the interface law set as multivalued applications (strict Signorini conditions and strict Coulomb law after collapse of the adhesion), the analysis regards non smooth mechanics. Because of its implicit character, the Coulomb friction law is non-associated, and the notion of a superpotential with normality rule, used in classical plasticity, cannot be used anymore. To overcome this non-associated character (loss of the normality rule) G. De Saxcé (De Saxcé et al., 1998) introduced the class of implicit standard materials based on the concept of bipotential. A major result of the implicit standard material theory was the possibility of representing many non-standard dissipative laws by a suitable pseudo-potential depending on the dual variables, internal variable rates and associated variables. The contact law is then written as a differential inclusion and the choice of a convenient bipotential allows us

to uncover a normality rule. The concept of bipotential is based on an extension of the Fenchel inequality, and is a generalization of the notion of pseudo-potential which enables us to write the evolution laws as implicit normality rules. In the present work, a specific bipotential adapted to the coupling of unilateral contact, friction and adhesion is constructed and a numerical method, based on a previous work of Terfaya and Berga in contact and plasticity (Berga et al., 2003) (Terfaya et al., 2006), is proposed and implemented to solve the problem.

As an application to Civil Engineering, the numerical simulation of a pull-out experiment of a pile is presented.

2. The RCCM model

The RCCM model has been successfully used for composite materials (matrix-fibre interfaces, ductile cracks) (Raous and Monerie, 2002), steel-concrete interfaces (pull out of reinforced concrete) (Raous and Karray, 2006) and interfaces in masonry construction (Fouchal et al, 2006). The model is based on an adhesion intensity variable, introduced first by Frémond (Frémond, 1988), which is a surface damage variable that takes its value between zero (no adhesion) and one (perfect adhesion). The introduction of a damageable stiffness of the interface ensures a smooth transition between the two contact conditions (initial adhesion and final frictional sliding) during the competition between friction and adhesion. The contact of two deformable bodies is considered.

Let \mathbf{u} denote the gap between the two solids and \mathbf{R} the contact force. They are separated into normal and tangential components as follows (\mathbf{n}^1 is the normal vector to the contact interface):

$$\mathbf{u} = u_n \cdot \mathbf{n}^1 + \mathbf{u}_t \quad (1)$$

$$\mathbf{R} = R_n \cdot \mathbf{n}^1 + \mathbf{R}_t \quad (2)$$

In the framework of continuum thermodynamics, the contact zone is assumed to be a material surface and the local constitutive laws are obtained by choosing the two specific forms (3) and (4) for the free energy and the dissipation potential associated to the surface. Details on the thermodynamic analysis can be found in (Raous et al, 1999). The relative displacements (u_n , u_t), and the adhesion intensity β are chosen as state variables, and the contact force R and the decohesion force G are the associated thermodynamic forces.

$$\Psi_s(u_n, u_t, \beta) = \frac{C_n}{2} u_n^2 \beta^2 + \frac{C_t}{2} u_t^2 \beta^2 - wh(\beta) + I_{\mathfrak{R}^+}(u_n) + I_{[0,1]}(\beta) \quad (3)$$

$$\Phi(\dot{u}_t, \dot{\beta}, u_n, R_n, \beta) = \mu(1 - \beta) \left| R_n^r - C_n u_n \beta^2 \right| \|\dot{u}_t\| + \frac{\alpha}{2} (\dot{\beta})^2 + I_{c^-}(\dot{\beta}) \quad (4)$$

In (3), the indicator functions $I_{\mathfrak{R}^+}$ and $I_{[0,1]}$ impose the unilateral conditions and the condition $\beta \in [0, 1]$, respectively. In (4), the indicator function I_{c^-} imposes the condition $\frac{\partial \beta}{\partial t} \leq 0$ (which means that adhesion cannot be regenerated).

The state laws and the complementary laws are then written in the sense of differential inclusions in order to obtain the contact behaviour laws

$$(R_n^r, R_t^r) \in \partial_u \Psi(u_n, u_t, \beta) \text{ and } -G_\beta \in \partial_\beta \Psi(u_n, u_t, \beta), \quad (5)$$

$$R_n = R_n^r \text{ and } (R_t^r, G_\beta) \in \partial_{(\dot{u}_t, \dot{\beta})} \phi(\dot{u}_t, \dot{\beta}, R_n, u_n, \beta). \quad (6)$$

The response of the interface is described by the following relations:

- unilateral contact (Signorini conditions) with adhesion

$$R_n - C_n u_n \beta^2 \geq 0; \quad u_n \leq 0; \quad (R_n - C_n u_n \beta^2) u_n = 0 \quad (7)$$

- Coulomb friction with adhesion

$$\begin{cases} R_t^r = C_t u_t \beta^2; \quad R_n^r = R_n \\ \left\| R_t - R_t^r \right\| \leq \mu \left| R_n - C_n u_n \beta^2 \right| \end{cases} \quad (8)$$

$$\begin{cases} \left\| R_t - R_t^r \right\| < \mu \left| R_n - C_n u_n \beta^2 \right| \Rightarrow \dot{u}_t = 0 \\ \left\| R_t - R_t^r \right\| = \mu \left| R_n - C_n u_n \beta^2 \right| \Rightarrow \exists \lambda \geq 0, \dot{u}_t = -\lambda \frac{(R_t - R_t^r)}{\left\| R_t - R_t^r \right\|} \end{cases}$$

- evolution of the intensity of adhesion

$$\begin{cases} \alpha \dot{\beta} = -(wh'(\beta) - (C_n u_n^2 + C_t u_t^2) \beta)^- & \text{si } \beta \in [0, 1[\\ \alpha \dot{\beta} = -(wh'(\beta) - (C_n u_n^2 + C_t u_t^2) \beta)^- & \text{si } \beta = 1 \end{cases} \quad (9)$$

The constitutive parameters of the model are:

- C_n and C_t are the initial stiffness of the interface,
- w is the decohesion energy (as long as that threshold is not reached, adhesion stays to be complete and the behavior of the interface is elastic with the initial stiffness C_n and C_t),
- μ is the friction coefficient,
- α is the viscosity associated to the evolution of the adhesion.

3. The bipotential model

In order to generalize the concept of pseudo-potential (introduced by J.-J. Moreau for multivalued constitutive laws) to non associated laws, G. De Saxcé (De Saxcé, 1998) introduced the concept of bipotential and the one of Implicit Standard Materials (ISM). This generalizes the Fenchel's inequality and permits to recover a flow rule subnormality for non-standard behaviors (specially soils and unilateral contact with dry friction).

3.1 Concept of bipotential

The Coulomb's law is non-associated and the concept of pseudo-potential cannot be used. De Saxcé and Feng (De Saxcé et al., 19998) have generalized the normal dissipation rule by constructing a unique function depending on the velocities \dot{u} and on the forces R , called bipotential and here denoted b :

$$b : V \times F \mapsto [-\infty, +\infty] : (\dot{u}, R) \mapsto b(\dot{u}, R) \quad (10)$$

b is biconvex, namely convex with respect to \dot{u} , when R is fixed, and convex with respect to R , when \dot{u} is fixed. Furthermore, the bipotential satisfies the fundamental inequality:

$$\forall(\dot{u}, R) \in V \times F, b(\dot{u}, R) \geq \dot{u}.R \quad (11)$$

This generalization of the Fenchel inequality is the cornerstone of the formulation. The couples (\dot{u}, R) related by the dissipative law are said extremal when equality is reached:

$$b(\dot{u}, R) = \dot{u}.R \quad (12)$$

Then any extremal pair satisfies:

$$\begin{cases} a) b(\dot{u}', R) - b(\dot{u}, R) \geq R(\dot{u}' - \dot{u}) & \dot{u} \in V \\ b) b(\dot{u}, R') - b(\dot{u}, R) \geq x(R' - R) & R' \in F \end{cases} \quad (13)$$

This is equivalent to the differential inclusions characterizing an implicit subnormality law:

$$\dot{u} \in \partial_R b(\dot{u}, R) \quad \text{and} \quad R \in \partial_{\dot{u}} b(\dot{u}, R) \quad (14)$$

The relations (14) define a multivalued constitutive law and its inverse.

3.2 Bipotential coupling contact, friction and adhesion

Let $K_{\bar{\mu}}$ be the Coulomb's cone, which defines the set of admissible forces satisfying:

$$K_{\bar{\mu}} = \left\{ (\bar{R}_n, \bar{R}_t), / \left\| \bar{R}_t \right\| - \bar{\mu} \left| \bar{R}_n \right| \leq 0 \right\} \quad (15)$$

where: $\bar{R}_n = R_n - C_n \beta^2 u_n$, $\bar{R}_t = R_t - C_t \beta^2 u_t$, and $\bar{\mu} = \mu(1 - \beta)$.

The relative velocity \dot{u} is not normal to the Coulomb's cone, because the normal relative velocity \dot{u}_n is equal to zero. Normality does not apply.

A change of variable for the sliding velocity is introduced and the following bipotential (16) is constructed. It is composed of two parts, one controlling the interface law and the other one controlling the adhesion evolution:

$$b(-\dot{u}, \dot{\beta}, \bar{R}, G_\beta) = I_{K_{\bar{\mu}}}(\bar{R}) + I_{\mathfrak{R}^-}(-\dot{u}_n) + \mu(1 - \beta) \cdot \bar{R}_n \left\| -\dot{u}_t \right\| + \frac{\alpha \dot{\beta}^2}{2} + I_{C^-}(\dot{\beta}) + \frac{(G_\beta^-)^2}{2\alpha} \quad (16)$$

where \dot{u} is the relative velocity, β the intensity of adhesion, R the contact force and G_β the thermodynamic force associated to the state variable β . The parameter α is the adhesion viscosity and μ is the friction coefficient. I_S denotes the indicator function of the specified sets S. It has been shown that this bipotential verifies the suitable properties of biconvexity and satisfies (11) (12) and (13). Then the contact law and the equation for the evolution of β , which are explicitly given by expression (7) (8) and (9), are deduced from the state and complementary laws expressed in term of differential inclusions:

$$\bar{R}_n \in \partial_{-\dot{u}_n} b(-\dot{u}, \dot{\beta}, \bar{R}, G_\beta) \Rightarrow R_n - C_n \beta^2 u_n \in \partial_{-\dot{u}_n} I_{\mathfrak{R}^-}(-\dot{u}_n) \quad (17)$$

$$\bar{R}_t \in \partial_{-\dot{u}_t} b(-\dot{u}, \dot{\beta}, \bar{R}, G_\beta) \Rightarrow R_t - C_t \beta^2 u_t \in \mu(1 - \beta) \left| R_n - C_n \beta^2 u_n \right| \partial_{-\dot{u}_t} \left\| -\dot{u}_t \right\| \quad (18)$$

$$\begin{aligned} G_\beta \in \partial_{\dot{\beta}} b(-\dot{u}, \dot{\beta}, \bar{R}, G_\beta) &\Rightarrow G_\beta \in \alpha \dot{\beta} + \partial_{\dot{\beta}} I_{C^-}(\dot{\beta}) \\ &\Rightarrow G_\beta - \alpha \dot{\beta} \in \partial_{\dot{\beta}} I_{C^-}(\dot{\beta}) \end{aligned} \quad (19)$$

3.3 Local algorithm

The bipotential leads to a single displacement variational principle and a single inequality. The unilateral contact with adhesion and the friction with adhesion are coupled. An

augmented Lagrangian Method (Fortin et al.1982) is used. Its leads to the following implicit equation:

$$\bar{R} = Proj(\tau, K_{\bar{\mu}}) \quad (20)$$

where τ is the modified augmented contact reaction defined by:

$$\tau = \bar{R} - \rho [\dot{u}_t + (\dot{u}_n + \bar{\mu} \|\dot{u}_t\|)n] \quad (21)$$

The parameter ρ is a positive number, chosen according to the eigenvalue of the contact flexibility matrix. The equality (20) means that \bar{R} is the projection of τ onto the closed convex Coulomb cone $K_{\bar{\mu}}$. The local problem is treated in a reduced system by means of reliable and efficient predictor-corrector algorithm. For solving the implicit equation (20), the Uzawa algorithm is used and it leads to an iterative process involving one predictor-corrector step:

$$\text{- Predictor: } \tau^{i+1} = \bar{R}^i - \rho [\dot{u}_t^i + (\dot{u}_n^i + \bar{\mu} \|\dot{u}_t^i\|)n] \quad (22a)$$

$$\text{- Corrector: } \bar{R}^{i+1} = Proj(\tau^{i+1}, K_{\bar{\mu}}) \quad (22b)$$

4- Simulation of the pile-soil interface during a pull-out experiment

The simulation of pull-out experiments of a pile conducted by R. Franck and P. Mestat on the sites of Plancoët (Frank et al. 2000) is presented. The RCCM model is used to describe the pile/soil interface. The pile has a diameter $D = 0.27\text{m}$ and a length $L=13\text{m}$. Linear elasticity is considered both for the pile ($E=3.E06 \text{ kN/m}^2$, $\nu=0.3$) and for the soil ($E=3.E04 \text{ kN/m}^2$ and $\nu=0.33$) according to (Frank and al. 2000). The finite element mesh and the boundary conditions are given on figures 1 and 2. Axisymmetric three-node elements have been used in the computations carried on using the SYMEF finite element code developed at the FIMAS in Béchar. For the in-situ soil condition prior installation, the soil is considered to be normally consolidated and the initial lateral stress coefficient K_0 is taken to be 0.43. The pull-out is simulated by applying displacement increments on the pile head. The initial normal stresses due to the pile installation are taken into account by applying a prescribed displacement on the pile shaft.

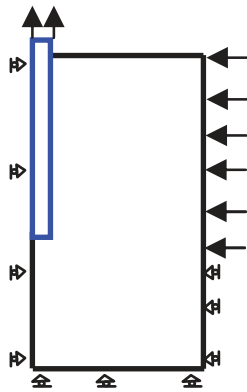


Figure 1: Pull-out of a pile

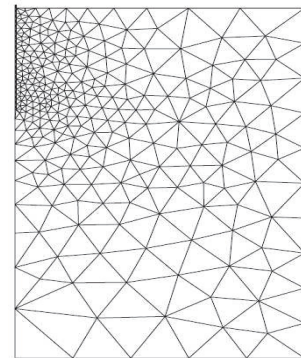


Figure 2: Finite element mesh

On figures 3 and 4, results of the numerical simulations are presented and compared to the experiment results given in (Frank et al. 200).

Results, as the ones presented on figures 3 and 4, have shown the strong influence of the characteristics of the soil/pile interface on the behavior of the pile, and the necessity of

coupling adhesion to frictional contact for the interface when evaluating the load bearing capacity.

Figure 3: Results with classical Coulomb's law

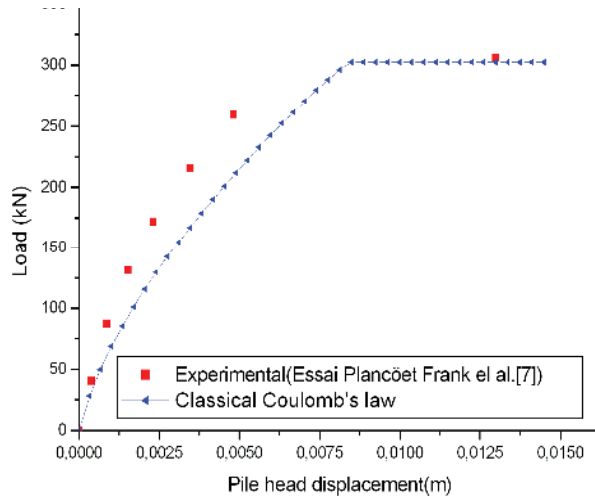
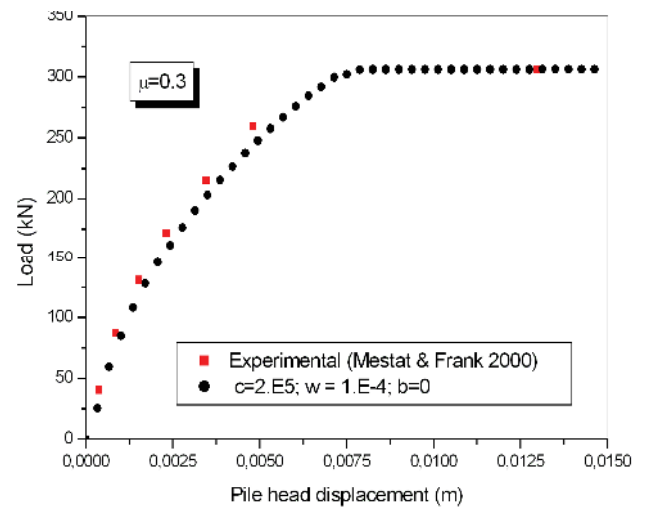


Figure 4: Results with Interface model



5 Conclusion

In this work, a bipotential formulation for the RCCM model coupling contact, friction and adhesion has been presented. It has shown that the contact law on the interface can be derived from a unique bipotential. A numerical method is deduced. The convenient agreement observed between simulation and experimental results for a pull-out test demonstrates the ability of the model to simulate the behavior of pile-soil interfaces.

References

- Berga A., Terfaya N. (2003) Modélisation de la plasticité des sols et du phénomène de contact avec frottement par l'approche de matériaux standard implicite, Colloque International GECOTEC, 11-12 Octobre 2003, Alger, Algérie.
- De Saxcé G., Feng Z. Q. (1998) The bipotential method : a constructive approach to design the complete contact law with friction and improved numerical algorithms, *Math. Comput. Modelling*, 28, No 4-8, 225-245.
- Fouchal F., Lebon F., Titeux I. (2006) Contribution to the modelling of interfaces in masonry construction, *AGS06 Advances on Geomaterials and Structures*, Hammamet, 3-5 May 2006, Tunisia.
- Frank R., Mestat P. (2000) Aspects expérimentaux et numériques du frottement unilatéral des pieux“, *Mec. Ind.* , 1, 651–666.
- Frémond M. (1988), ‘Contact with adhesion’, In *Moreau J-J and Panagiotopoulos P.D (eds): Non Smooth Mechanics and applications, CISM Courses and Lecture, 302*, Springer, pp. 177-221.
- Fortin M., Glowinski R. (1982) *Méthodes de Lagrangien augmenté, application à la résolution numérique des problèmes aux limites*, Dunod, Paris.
- Raous M., Cangémi L., Cocu. M. (1999) A consistent model coupling adhesion, friction and unilateral contact, *Computer Methods in Appl. Mech. and Engng*, 177, 383–399.
- Raous M., Monerie.Y. (2002) Unilateral contact, friction and adhesion in composite materials, *In Martins, J. - Marques M.(Eds): Contact Mechanics*, Kluwer, 333-346.
- Raous M., Karray M.A. (2009) Model coupling friction and adhesion for steel-concrete interfaces, *International Journal of Computer Application in Technology*, 34, No. 1, pp.42-51.
- Terfaya N., Berga A. (2006) Modélisation des problèmes de contact avec frottement sec de Coulomb par la méthode du bipotentiel, *Rapport interne*, Université de Béchar, 2006.