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# Comparison of Robustness Indices and Introduction of a Tolerance Synthesis Method for Mechanisms

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## 1 Introduction

Every engineering design is subject to variations that can arise from a variety of sources, including manufacturing operations, variations in material properties, and the operating environment. When variations are ignored, non robust designs can result, which are expensive to produce or fail in service.

The concept of robust design may be first used by Taguchi. He introduced this concept to improve the quality of a product whose manufacturing process involves significant variability or noise [1]. Robust design aims at minimizing the sensitivity of performances to variations without controlling the causes of these variations.

First, we propose a new robustness index and compare it with three robustness indices used in the literature. Then, we develop a sequential tolerance synthesis method.

## 2 Robust Design Problem

In a robust design problem, the distinction is made between three sets:

- Design Variables (*DV*): nominal values are controllable. However, they are subject to uncontrollable variations because of manufacturing errors, wear, or other uncertainties ;
- Design Environmental Parameters (*DEP*): cannot be adjusted by the designer, they are uncontrollable ;
- Performance Functions (*PF*).

*DV*, *DEP*, *PF*, are grouped in the  $l$ -dimensional vector  $\mathbf{x} = [x_1 \ x_2 \ \dots \ x_l]^T$ , the  $m$ -dimensional vector  $\mathbf{p} = [p_1 \ p_2 \ \dots \ p_m]^T$ , and the  $n$ -dimensional vector  $\mathbf{f} = [f_1 \ f_2 \ \dots \ f_n]^T$ , respectively.

Let us assume a mathematical model between *DV*, *DEP*, and *PF*, as expressed by eq. (1).

$$\mathbf{f} = \mathbf{f}(\mathbf{x}; \mathbf{p}) \quad (1)$$

Robust design aims at rendering *PF* as insensitive to variations in *DV* and *DEP* as possible. Thus, if we

introduce variations  $\delta\mathbf{x}$  and  $\delta\mathbf{p}$  in *DV* and *DEP*, respectively, and use a Taylor expansion of  $\mathbf{f}$  then,

$$\delta\mathbf{f} = \mathbf{J} [\delta\mathbf{x}^T \ \delta\mathbf{p}^T]^T \quad (2)$$

where  $\delta\mathbf{f}$  is the variation in *PF* and  $\mathbf{J}$  is the sensitivity Jacobian matrix of the design.

## 3 Optimal Robustness Index

In order to obtain a robust solution without the knowledge of the variations in *DV* and *DEP*, we need a wise a robustness index. Below, a list of three robustness indices used in the literature :

- $RI_1 = \|\mathbf{J}\|_2 \|\mathbf{J}^{-1}\|_2$ , [2]
- $RI_2 = \|\mathbf{J}\|_{Frob} \|\mathbf{J}^{-1}\|_{Frob}$ , [3]
- $RI_3 = \|\mathbf{J}\|_2$ , [4]

where  $\|\cdot\|_2$  and  $\|\cdot\|_{Frob}$  mean the 2-norm and the Frobenius norm, respectively. Here, we suggest the use of an other robustness index :

- $RI_4 = \|\mathbf{J}\|_{Frob}$

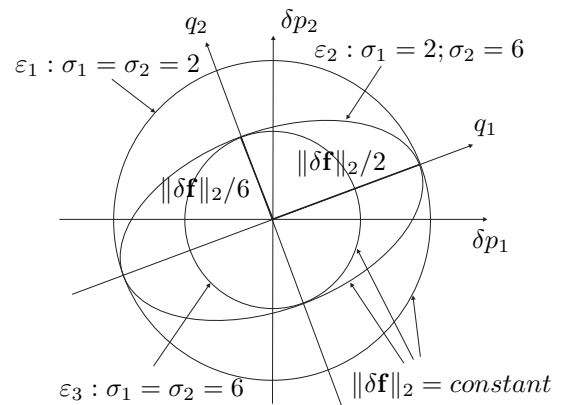


Figure 1: Design Sensitivity Ellipses

In order to illustrate the previous indices, let us compare the robustness of three designs named (1), (2), and (3), respectively. These designs have two *DEP* and variations in their *DV* are supposed to be insignificant.  $\varepsilon_1$ ,  $\varepsilon_2$ , and  $\varepsilon_3$ , depicted in Fig.1, are the design sensitivity

ellipses of designs (1), (2), and (3), respectively. The inclusion of  $\varepsilon_2$  in  $\varepsilon_1$  means that (1) is more robust than (2). Likewise, the inclusion of  $\varepsilon_3$  in  $\varepsilon_2$  means that (2) is more robust than (3).

Table 1 depicts the values of  $RI_1$ ,  $RI_2$ ,  $RI_3$ , and  $RI_4$  corresponding to designs (1), (2), and (3), respectively. Whatever the index, the smaller it is, the more robust the design is supposed to be. However, according to  $RI_1$  and  $RI_2$ , the robustness of designs (1) and (3) are similar, and (3) is more robust than (2). According to  $RI_3$ , (1) is more robust than (2) and (3), but the robustness of (2) and (3) are similar. Finally,  $RI_4$  makes the difference between the robustness of all the designs accurately.

Table 1: Values of robustness indices

Robustness index	Design		
	(1)	(2)	(3)
$RI_1$	1	3	1
$RI_2$	1	1.67	1
$RI_3$	2	6	6
$RI_4$	2	3.16	6

In short, the minimization of  $RI_1$  and  $RI_2$  assures an homogeneity of the influence of variations in  $DV$  and  $DEP$  on  $PF$ , i.e.: an isotropic design, but not a minimum sensitivity of  $PF$  to variations in  $DV$  and  $DEP$ . Therefore, we had better use  $RI_3$  or  $RI_4$  in a robust design problem. Moreover,  $RI_4$  is suitable for an optimization robust design problem because of its analytical form.

## 4 Tolerance Synthesis Method

The dimensional tolerances of a mechanism are usually fixed according to various parameters such as the manufacturing process, the performance tolerances, and the manufacturing cost. Here, we assume that the cost a mechanism decreases when its dimensional tolerances increase.

We suggest the use of a sequential tolerance synthesis method. First, robustness index  $RI_4$  is used to compute the nominal values of  $DV$ :  $\bar{\mathbf{x}} = [\bar{x}_1 \ \bar{x}_2 \ \dots \ \bar{x}_l]^T$ . Then, assuming that  $\|\delta\mathbf{f}\|_2$  has to be smaller than  $C$ , the optimal tolerances of  $DV$ ,  $\Delta x_{i,opt}$ , are computed by solving the following optimization problem:

$$\begin{cases} \max_{\mathbf{u}} \prod_{i=1}^l |u_i| \\ \text{s.t. } U(u_1, u_2, \dots, u_l) \in \xi(C) \\ u_i \cdot \text{sign}(V_i) \geq 0, \quad i = 1, \dots, l \\ |u_i| \geq \Delta x_{i,min}, \quad i = 1, \dots, l \end{cases}$$

where

- $\mathbf{V}$  is the eigenvector corresponding to the maximum singular value of the sensitivity Jacobian matrix of the mechanism and  $V_i$  is its  $i^{th}$  component ;

- $\xi(C)$  is the design sensitivity ellipse of the mechanism, corresponding to  $\|\delta\mathbf{f}\|_2$  equal to  $C$ .

The problem aims at finding the largest tolerance box of the design of a mechanism without rejects, which is included in  $\xi(C)$ . Besides, it assures that each dimensional tolerance  $\Delta x_i$  is higher than a minimum dimensional tolerance  $\Delta x_{i,min}$ , which depends on the manufacturing process and  $\bar{x}_i$ .

For instance, Fig.2 depicts all the possible positions of  $U$  when  $l = 2$  and  $V_1, V_2$  are negative and positive, respectively, and the optimal tolerance box.

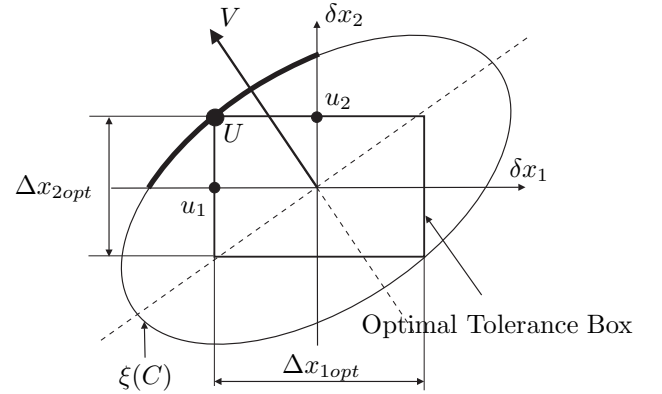


Figure 2: Tolerance Synthesis

## 5 Conclusions

A new robustness index was proposed. It was compared with three other robustness indices used in the literature. It turns out that the new index is an optimal criterion in a optimization robust design problem. Moreover, a sequential tolerance synthesis method was introduced.

## References

- [1] G. Taguchi. *On Robust Technology Development, Bringing Quality Engineering Upstream*. ASME Press, 1993.
- [2] K.L. Ting and Y. Long. *Performance Quality and Tolerance Sensitivity of Mechanisms*. ASME Journal of Mechanical Design, Vol.118, march, 1996, 144-150.
- [3] K. Al-Widyan and J. Angeles. *A Model-Based Formulation of Robust Design*. ASME Journal of Mechanical Design, accepted for publication, 2005.
- [4] S. Caro, F. Bennis and P. Wenger. *Tolerance Synthesis of Mechanisms : A Robust Design Approach*. ASME Journal of Mechanical Design, accepted for publication, 2005.