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Large-scale bottleneck effect in two-dimensional turbulence

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Abstract. The bottleneck phenomenon in three-dimensional turbulence is generally associated with the dissipation range of the energy spectrum. In the present work, it is shown by using a two-point closure theory, that in two-dimensional turbulence it is possible to observe a bottleneck at the large scales, due to the effect of friction on the inverse energy cascade. This large-scale bottleneck is directly related to the process of energy condensation, the pile up of energy at wavenumbers corresponding to the domain size. The link between the use of friction and the creation of space-filling structures is discussed and it is concluded that the careless use of hypo-friction might reduce the inertial range of the energy spectrum.

I. INTRODUCTION

In high Reynolds number three-dimensional turbulence at large wavenumbers k, the compensated energy spectrum $k^{5/3}E(k)$ often shows a little positive bump, before it drops down rapidly in the dissipation range. This pre-dissipative bump region was first observed in closure computations [1], but no physical explanation was given. The bump was also observed in experimental work [2, 3] and direct numerical simulations [4]. A physical explanation of the effect was first given in the work by Herring *et al.*[5]. They explain that, because the wavenumbers above the Kolmogorov scale are damped by viscosity, some of the nonlocal interactions are damped, which causes the transfer of the energy flux to be less effective, leading to a pile-up of energy. A more detailed investigation of this effect was performed by Falkovich [6], who predicted its wavenumber-dependence, and who named it the *bottleneck*. In that work it is also inferred that the bottleneck should become more pronounced when hyperviscosity is used. This was illustrated in three-dimensional turbulence[7] and two-dimensional turbulence with a direct energy cascade [8] such as magnetohydrodynamic turbulence.

In the present work, we show that in two-dimensional turbulence it is possible to observe a bottleneck at the *large* scales, due to the effect of friction on the inverse energy cascade. The possibility of large-scale bottlenecks was advanced by Lohse and Mueller-Groeling [9]. This range is in three-dimensional turbulence generally affected by a forcing which supplies energy to the flow, complicating the observation of a possible large-scale bottleneck.

An alternative interpretation of the bottleneck in three-dimensional turbulence was recently proposed by Frisch and coworkers [10]. They relate the bottleneck to statistical equilibrium, and view it as an incomplete thermalization. Indeed, it was stated that if the Laplacian ∇^2 in the viscous term of the Navier-Stokes equations is replaced by a hyperviscous term $\nabla^{2\alpha}$, with the *dissipativity* α tending to infinity, a Galerkin truncated system is obtained. In Galerkin-truncated 3D inviscid turbulence, energy piles up at the highest wavenumbers, leading to wavemodes in equipartition at the tail of the spectrum. The simultaneous observation of a $k^{-5/3}$ Kolmogorov inertial range energy spectrum and this equipartition range, proportional to k^2, was recently observed in direct numerical simulations[11] (DNS) and closure calculations [12]. In between these two ranges a pseudo-dissipation range was observed, caused by nonlocal interactions between the thermal bath and the end of the inertial range.

Since we are interested in the large scales in the present work, we discuss the influence of infrared wavenumber truncation, due to the finite domain-size. For three-dimensional turbulence, this truncation influences decaying turbulence and increases the energy decay rate, if the size of the integral lengthscale becomes comparable to the domain-size [13]. Steady turbulence, forced at a wavenumber $k_i > k_0$, in which k_0 is the smallest wavenumber of the domain, called the *fundamental*, will establish an infrared region proportional to k^2 between k_0 and k_i, independent of the exact choice of k_0 [14].

In two space-dimensions, the effect of infrared truncation is very different. The absence of vortex stretching induces a double cascade with energy cascading to the large scales and enstrophy going to the small scales. In this case, the large scales are not directly influenced by forcing. The energy spectrum is not necessarily decreasing for k tending to zero, if no energy-sink is present at low k. In an infinite domain, the energy will continue to cascade to smaller and smaller wavenumbers. In the case of a finite domain-size, the energy, cascading backwards, piles up at the fundamental. This condensation was predicted by Kraichnan [15] and observed numerically [16]. The condensation process was studied in detail by Smith and Yakhot [17, 18], who showed that the statistics of the velocity field remain close to Gaussian until the condensate starts to form. At later times the statistics become non-Gaussian. We note that the condensate state is not the only possible state of finite-size two-dimensional turbulence in the absence of an infrared energy sink. Indeed Tran and Bowman [19, 20] showed that if the in-

verse energy cascade is weak, *i.e.*, if only a small part of the injected energy cascades to small wavenumbers, the rest being dissipated around the forcing scale, no condensation takes place, but a gradual steepening of the spectrum occurs, when the cascade reaches the smallest wavenumbers, correponding to the box-size. This steepening results in a k^{-3} range at late times. This scenario was also observed by Chertkov *et al.* [21]. In the present work we consider the case in which the energy cascade is strong and in which condensation takes place at the fundamental, when no energy sink is present.

II. THE LINK BETWEEN HYPOFRICTION AND ENERGY CONDENSATION

To avoid condensation in two-dimensional turbulence, large-scale friction is often used. Friction acts as an energy sink. The influence of friction, which in principle acts throughout the spectrum, can be concentrated at the smallest wavenumbers by using hypofriction, analogous to hyperviscosity at the highest wavenumbers. The Navier-Stokes equations in this case become:

$$\frac{\partial}{\partial t} + (u \cdot \nabla)u = -\nabla p + \nu \nabla^2 u + f \quad -\mu k_f^{2\alpha}(-\nabla^2)^{-\alpha} u; \qquad \nabla \cdot u = 0, \tag{1}$$

in which u is the velocity vector, p the pressure, ν the viscosity, f an isotropic forcing term localized in spectral space around wavenumber k_i, μ the friction coefficient, k_f the friction wavenumber, and we shall call α the *frictionality*, in analogy with the dissipativity in hyperviscous flows[10].

In the presence of hypofriction, an energy sink is present at low k (e.g. [22, 23]). In this case a steep fall-off of the energy spectrum at low k can be created for large α. If the solution of the hypofrictional case converges to a Galerkin truncated case if $\alpha \to \infty$, in analogy with the work of Frisch *et al.*[10], a pile-up of energy can be expected around k_f. In the case of linear friction ($\alpha = 0$), no noticable bottleneck is observed (e.g. [24]). The spectral slope $d\, ln(E(k))/d\, ln(k)$, is a continuously decreasing function of k for $k_0 < k \ll k_i$ with k_0 the fundamental and k_i the wavenumber where energy is introduced by the forcing.

In the present study, using a closure model, we will systematically vary the order of the frictionality from $\alpha = 0$ to $\alpha = 128$. We will investigate the appearance of bottlenecks, its influence on the inertial range scaling and the convergence of this bottleneck to an infrared wavenumber truncation.

Some evidence for large-scale bottlenecks can be found in literature [22, 23]. In these studies hypofriction was used with values 8 and 5 respectively. The compensated energy spectra do show a pile up of energy at the small wavenumbers. A link with the creation of coherent structures is made but no systematic study of the influence of α on the large-scale bottleneck was reported in these studies.

Our interpretation, in the light of the present study, is that the bottlenecks in these works are in fact incomplete condensates, in analogy with the interpretation of Frisch *et al.*[10], that the bottleneck in 3D turbulence is an incomplete thermalization. The coherent structures that are observed are closely related to the structures, or condensates, that would be observed in a truncated solution.

Even though numerical resources increase rapidly and two-dimensional turbulence is computationally accessible at high Reynolds numbers, a systematic DNS study of the influence of α would be very expensive. Furthermore the small wavenumber range of an energy spectrum converges more slowly to a statistically steady state than the small scales. It was shown that for this kind of studies, two-point closure is a convenient tool[10, 12]. For the case of two-dimensional turbulence, it was pointed out [25] that in the absence of coherent vortices, closure approaches compare well to direct numerical simulation. The inverse cascade is known to be free from large coherent structures and its statistics are close to Gaussian [26]. In direct numerical simulations, if friction is absent, the pile up of energy at the fundamental will eventually give rise to the creation of two counter-rotating vortices of the size of the domain.

In the long-time limit, the statistics become highly non-Gaussian [17] so that the use of the model, which are based on an expansion about Gaussianity, will probably not be justified [27–29]. The time before this happens is supposed to be reasonably well predicted by closure. It is this time-interval, in which pile-up of kinetic energy at small wavenumbers takes place, and which precedes the formation of coherent structures, on which we concentrate in the present work.

III. OBSERVATION OF LARGE-SCALE BOTTLENECKS

We use the Eddy-Damped Quasi Normal Markovian (EDQNM) closure. It was used to study two-dimensional turbulence by Leith [30] and Pouquet *et al.*[31]. In this closure, the evolution equation for the energy spectrum is solved

$$\left[\partial_t + \nu k^2 + \mu (k_f/k)^{2\alpha}\right] E(k,t) = T_{NL}(k,t) + F(k). \tag{2}$$

The nonlinear transfer T_{NL} is

$$T_{NL}(k,t) = \frac{4}{\pi} \iint_\Delta \Theta_{kpq} \frac{xy - z + 2z^3}{\sqrt{1-x^2}} \left[k^2 p E(p,t) E(q,t) - k p^2 E(q,t) E(k,t)\right] \frac{dp dq}{pq}. \tag{3}$$

The forcing is a time-independent energy input of rate c, localized at $k = k_i$, $F(k) = c\, \delta(k - k_i)$. In equation (3), Δ is a band in p, q-space so that the three wave-vectors

k, p, q form a triangle. x, y, z are the cosines of the angles opposite to k, p, q in this triangle. The characteristic time Θ_{kpq} is defined as:

$$\Theta_{kpq} = \frac{1 - exp(-(\eta_k + \eta_p + \eta_q) \times t)}{\eta_k + \eta_p + \eta_q}, \quad (4)$$

in which η is the eddy damping. In the present study we use the following expression for the damping coefficient

$$\eta_k = \lambda \sqrt{\int_0^k s^2 E(s,t) dS} + \nu k^2 + \mu (k_f/k)^{2\alpha}. \quad (5)$$

In expression (5), three contributions to the eddy damping are present. The first two are the standard contributions introduced in the EDQNM closure. For λ we use the value 0.4 [32]. The third contribution is an ad-hoc modification of the damping to take into account the presence of friction at large scale. A rigorous derivation of the modification induced by the friction as done for the rotating case [33] or using an improved eddy-damping [34] is not attempted in the present work. The numerical set-up is similar to the one in [12]. The wavenumbers are spaced as $k_p = k_0 r^{p-1}$ with k_0 the fundamental and $r > 1$ a constant which determines the number of wavenumbers per decade. The spatial resolution for the computations reported here is 40 wavenumbers per decade which corresponds to a resolution of $F \approx 12$ wavenumbers per octave. Simulations are reported for $\alpha = 0, 1, 2, 4, 8, 32, 128$ as well as a friction-free simulation truncated at $k_0 = k_f$.

For all runs but this last one, a steady state was obtained, for which the energy spectrum is shown in Figure 1. For the friction-free (truncated) case, a steady state will not be obtained until the energy-pile up at the fundamental is balanced by the viscous dissipation at this wavenumber, which is very small in the present case as the Reynolds number is large. The fundamental would then accumulate an amount of energy which is enormous compared to the cases with small frictionality. For a better comparison between the different cases, in the following a spectrum is shown for which the energy pile-up at the fundamental is comparable to the pile-up for $\alpha = 128$. In Figure 2 we observe that for $\alpha > 0$ a bottleneck is present around $k_f \approx 3$. The size of this bottleneck increases with α. For values of $1 \leq \alpha < 8$, the bottleneck has the form of a bump. For $\alpha \geq 8$, the bottleneck changes its shape towards a peak and for the highest values of α used in the present work, the bottleneck collapses with the energy spectrum of the frictionless truncated case. The $k^{-5/3}$ inertial range is influenced by this bottleneck, not only on the bottleneck itself, where the spectrum is steeper than $k^{-5/3}$, but also in the region with wavenumbers larger than the bottleneck, and extending for about one decade, the spectrum deviates and is slightly shallower than $k^{-5/3}$. The friction-free truncated case, and the high-α friction cases, show much similarity (at small k) with the inviscid truncated 3D case (at high k) studied in [12]. A depletion of the compensated energy spectrum is present in the vicinity of the peak (for $k > k_f$ in the 2D case, and for $k < k_{th}$ in the 3D case, where k_{th} is the wavenumber marking the beginning of the equipartition range). The effect can, in both cases, be explained by nonlocal interactions, as was done in the three-dimensional case [12].

It is interesting to investigate whether this depleted region in the compensated energy spectrum corresponds to the emergence of a new scaling regime. Dimensional analysis based on the assumption of a constant flux of energy and a sweeping timescale related to the advection of the small eddies by the velocity associated with the kinetic energy contained in the condensate, one can deduce a scaling proportional to $k^{-3/2}$. It is observed in the inset of Figure 2 that a small wavenumber range shows a spectral exponent close to this value (1.53). This range is however too short to make conclusive statements.

It was shown by Smith and Yakhot [17], that when the condensate becomes large enough, the statistics of the velocity field become strongly non-Gaussian. One could then ask, if the present closure approach, based on an assumption of maximum randomness [35], is still valid. To estimate the extend to which the closure gives results for the energy spectrum close to results of DNS, we compare our results with DNS results from literature [16]. In that study the coexistence of a large peak at the fundamental and a $k^{-5/3}$ range for larger wave-numbers is observed. The value of the energy accumulated at the fundamental is one order of magnitude above its inertial range value. In Figure 2 the peak value is also one order of magnitude above its inertial range value. So at least qualitatively, the coexistence of a large peak and the $k^{-5/3}$ inertial range is observed in DNS as well as in closure. However this validation remains qualitative and the present results could only be fully supported by a detailed and quantitative comparison with DNS. Such a comparison is beyond the scope of the present paper.

When the bottleneck becomes large enough, a small kink is observed in the spectrum around $k = 50$. This comes from the discretization. Nonlocal interactions are not resolved if the ratio of the smallest wavenumber to the middle wavenumber of an interacting triad is smaller than[31] $a = 2^{1/F} - 1$. For elongated triads, the middle wavenumber and the longest wavenumber are approximately of the same size. The largest wavenumber k directly interacting with k_f can then be shown to be $k \approx 50$ for $F = 12$. This clearly illustrates the nonlocal origin of the pseudo-friction range, created by the bottleneck. In principle, the effect of the truncation can be removed by refining the discretization. To allow nonlocal interactions over the full 4 decades of the simulation, the number of necessary gridpoints can be estimated to be roughly 700 per octave which would make the simulations prohibitively expensive. Another solution would be to explicitly compute the first-order contributions of the nonlocal interactions [31] by expanding the nonlinear transfer with respect to the parameter a, which is small for very nonlocal interactions. This procedure has the disadvantage not to conserve energy or to mix-up the different

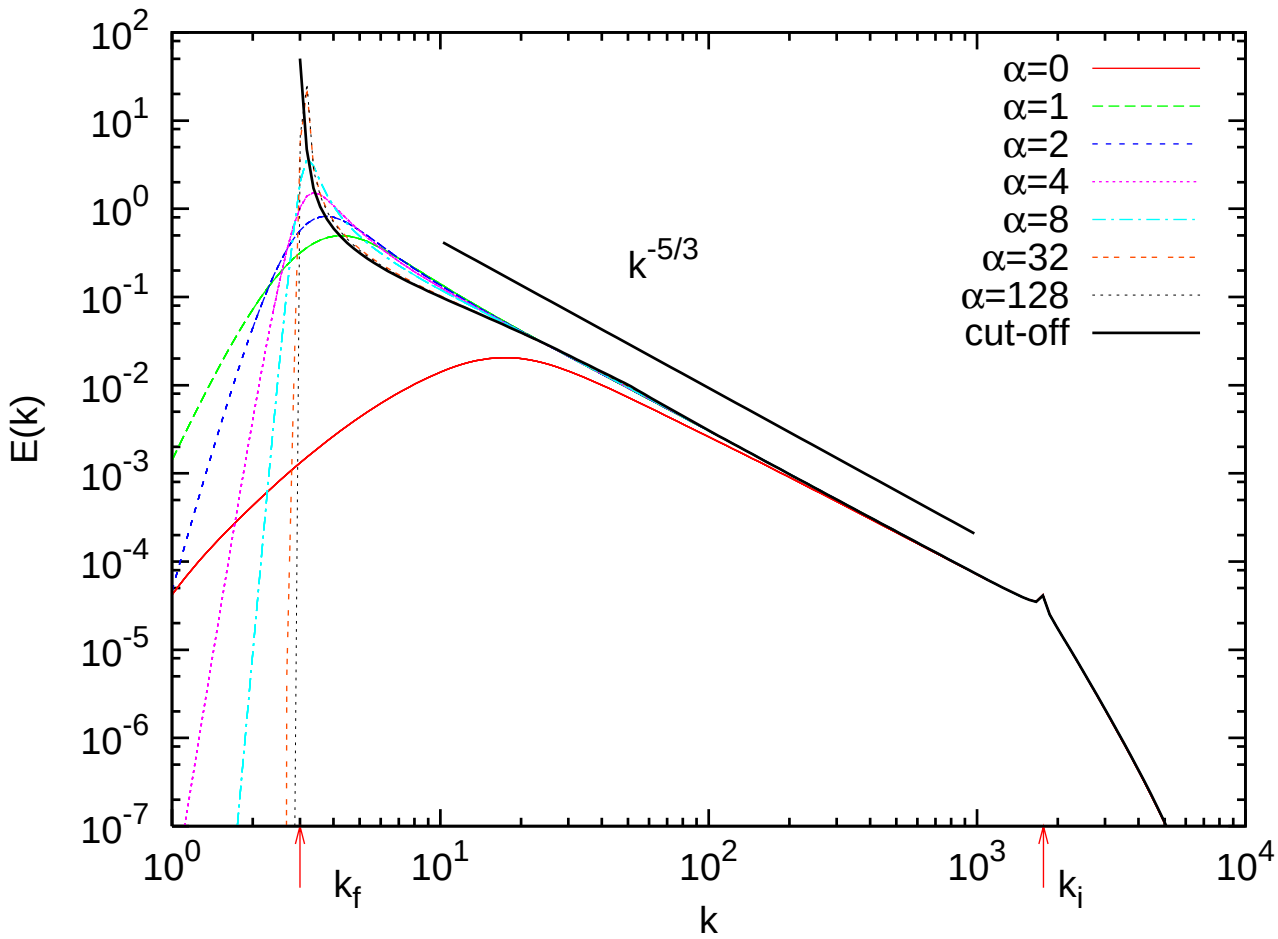

FIG. 1: Steady-state energy spectra with frictionality ranging from 0 to 128 and a friction free case truncated at the low wavenumbers at $k = k_f$.

orders of the expansion. The improvement expected to be minimal with respect to the presented results, these solutions will not be attempted here.

IV. CONCLUSION

The present results show that the bottleneck effect is not only observable at the small scales, but it can also be created, be it artificially, at the large scales. It shows that by increasing α, the energy spectrum tends to the condensed solution predicted by Kraichnan [15]. This constitutes a logical extension of the ideas of Frisch *et al.*[10] to the inverse cascade in two-dimensional turbulence. It also shows that pile-up of energy at small wavenumbers influences the inertial range scaling by nonlocal interactions. The present work thereby confirms the ideas by Sukoriansky *et al.*[36, 37], that large-scale friction can influence all scales of two-dimensional turbulence, since the pile-up of energy at the small scales might trigger the creation of coherent structures. These coherent structures can induce a k^{-3} scaling[21, 22], which camouflates the inertial range. The use of hypofriction, intended to enlarge the inertial range, can thus have the opposite effect.

A detailed DNS study would be necessary to confirm the effects observed in the present study. Even at low or moderate resolution, trends could probably be observed and answer the question whether the predictions of the present closure, based on an expansion about Gaussianity, are valid for the flow here considered in which coherent large scale structures are present. In the authors' opinion, in this type of flow, as is the case for many types of turbulent fields, the simultaneous use of Direct Numerical Simulations at low and moderate Reynolds numbers and of closures at high Reynolds numbers will probably constitute the best strategy to gain understanding of the physics of turbulence dynamics. Such a study is however left for future work.

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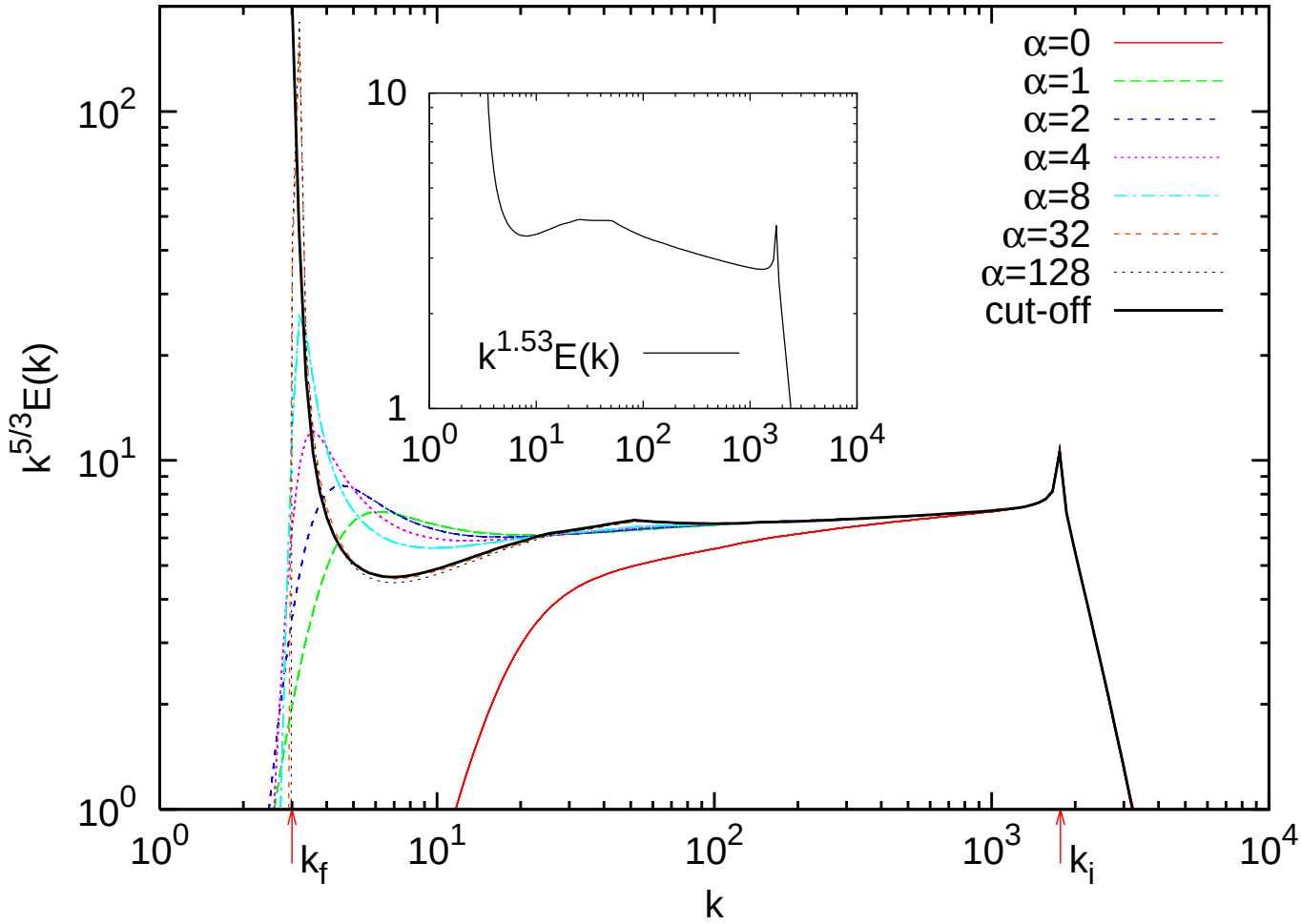

FIG. 2: The same spectra as in Figure 1 compensated by $k^{5/3}$ and, in the inset, compensated by $k^{1.53}$.

Pauls, Samriddhi Sankar Ray, Jian-Zhou Zhu and two anonymous referees.

[1] J.C. André and M. Lesieur. Influence of helicity on the evolution of isotropic turbulence at high Reynolds number. *J. Fluid Mech*, **81**, 187, (1977).

[2] P. Mestayer. Local isotropy and anisotropy in a high-Reynolds-number turbulent boundary layer. *J. Fluid Mech.*, **125**, 47, (1982).

[3] S.G. Saddoughi and S.V. Veeravalli. Local isotropy in turbulent boundary layers at high Reynolds number. *J. Fluid Mech.*, **268**, 333, (1994).

[4] R.M. Kerr. Velocity, scalar and transfer spectra in numerical turbulence. *J. Fluid Mech.*, **211**, 309, (1990).

[5] J.R. Herring, D. Schertzer, M. Lesieur, G.R. Newman, J.P. Chollet, and M. Larcheveque. A comparative assessment of spectral closures as applied to passive scalar diffusion. *J. Fluid Mech.*, **124**, 411, (1982).

[6] G. Falkovich. Bottleneck phenomenon in developed turbulence. *Phys. Fluids*, **6**, 1411, (1994).

[7] V. Borue and S.A. Orszag. Forced three-dimensional homogeneous turbulence with hyperviscosity. *Europhys. Lett.*, **29**, 687, (1995).

[8] D. Biskamp, E. Schwarz, and A. Celani. Nonlocal bottleneck effect in two-dimensional turbulence. *Phys. Rev. Lett.*, **81**, 4855, (1998).

[9] D. Lohse and A. Mueller-Groeling. Bottleneck effects in turbulence: scaling phenomena in r versus p space. *Phys. Rev. Lett.*, **74**, 1747, (1995).

[10] U. Frisch, S. Kurien, R. Pandit, W. Pauls, S.S. Ray, A. Wirth, and J.-Z. Zhu. Hyperviscosity, Galerkin truncation and bottlenecks in turbulence. *Phys. Rev. Lett.*, **101**, 144501, (2008).

[11] C. Cichowlas, P. Bonaïti, F. Debbasch, and M. Brachet. Effective dissipation and turbulence in spectrally truncated euler flows. *Phys. Rev. Lett.*, **95**, 264502, (2005).

[12] W. J. T. Bos and J.-P. Bertoglio. Dynamics of spectrally truncated inviscid turbulence. *Phys. Fluids*, **18**, 071701, (2006).

[13] H. Touil, J.P. Bertoglio, and L. Shao. The decay of turbulence in a bounded domain. *J. Turbul.*, **3**, 049, (2002).

[14] M. Lesieur. *Turbulence in fluids.* Kluwer Dordrecht, (1990).

[15] R.H. Kraichnan. Inertial ranges in two-dimensional turbulence. *Phys. Fluids*, **10**, 1417, (1967).

[16] M. Hossain, W.H. Matthaeus, and D.C. Montgomery. Long-time states of inverse cascades in the presence of a maximum length scale. *J. Plasma Phys.*, **30**, 478, (1983).

[17] L.M. Smith and V. Yakhot. Bose condensation and small-scale structure generation in a random force driven 2d turbulence. *Phys. Rev. Lett.*, **71**, 352, (1993).

[18] L.M. Smith and V. Yakhot. Finite-size effects in forced two-dimensional turbulence. *J. Fluid Mech*, **274**, 115, (1994).

[19] C.V. Tran and J.C. Bowman. On the dual cascade in two-dimensional turbulence. *Phys. D*, **176**, 242, (2003).

[20] C.V. Tran and J.C. Bowman. Robustness of the inverse energy cascade in two-dimensional turbulence. *Phys. Rev. E*, **69**, 036303, (2004).

[21] M. Chertkov, C. Connaughton, I. Kolokolov, and V. Lebedev. Dynamics of energy condensation in two-dimensional turbulence. *Phys. Rev. Lett.*, **99**, 084501, (2007).

[22] V. Borue. Inverse energy cascade in stationary two-dimensional homogeneous turbulence. *Phys. Rev. Lett.*, **72**, 1475, (1994).

[23] H.P. Huang, B. Galperin, and S. Sukoriansky. Anisotropic spectra in two-dimensional turbulence on the surface of a rotating sphere. *Phys. Fluids*, **13**, 225, (2001).

[24] J. Paret and P. Tabeling. Intermittency in the two-dimensional inverse cascade of energy: Experimental observations. *Phys. Fluids*, **10**, 3126, (1998).

[25] J.R. Herring and J.C. McWilliams. Comparison of direct numerical simulation of two-dimensional turbulence with two-point closure: the effects of intermittency. *J. Fluid Mech.*, **153**, 229, (1985).

[26] G. Boffetta, A. Celani, and M. Vergassola. Inverse energy cascade in two-dimensional turbulence: deviations from Gaussian behavior. *Phys. Rev. E*, **61**, R29, (2000).

[27] R.H. Kraichnan and H. Chen. Is there a statistical mechanics of turbulence? *Phys. D*, **37**, 160, (1989).

[28] H. Chen, J.R. Herring, R.M. Kerr, and R.H. Kraichnan. Non-gaussian statistics in isotropic turbulence. *Phys. Fluids A*, **1**, 1844, (1989).

[29] J.-P. Bertoglio, K. Squires, and J. Ferziger. EDQNM closure: A homogeneous simulation to support it. a quasi-homogeneous simulation to disprove it. In *Proceedings of the 1987 Summer Program of the CTR*, page 53, Stanford University, (1987).

[30] C.E. Leith. Atmospheric predictability and two-dimensional turbulence. *J. Atmos. Sci.*, **28**, 145, (1971).

[31] A. Pouquet, M. Lesieur, J.C. André, and C. Basdevant. Evolution of high Reynolds number two-dimensional turbulence. *J. Fluid Mech.*, **72**, 305, (1975).

[32] M. Lesieur and J.R. Herring. Diffusion of a passive scalar in two-dimensional turbulence. *J. Fluid Mech.*, **161**, 77, (1985).

[33] F. Bellet, F.S. Godeferd, J.F. Scott, and C. Cambon. Wave turbulence in rapidly rotating flows. *J. Fluid Mech*, **562**, 83, (2006).

[34] W.J.T. Bos and J.-P. Bertoglio. A single-time two-point closure based on fluid particle displacements. *Phys. Fluids*, **18**, 031706, (2006).

[35] R.H. Kraichnan. The structure of isotropic turbulence at very high Reynolds numbers. *J. Fluid Mech.*, **5**, 497, (1959).

[36] S. Sukoriansky, B. Galperin, and A. Chekhlov. Large scale drag representation in simulations of two-dimensional turbulence. *Phys. Fluids*, **11**, 3043, (1999).

[37] S. Sukoriansky and B. Galperin. Subgrid- and supergrid-scale parameterization of turbulence in quasi-two-dimensional barotropic flows and the phenomenon of negative viscosity. In J. Simpson H. Baumert and J. Sundermann, editors, *Marine Turbulence, Theories, Observations, and Models.*, 458–471. CUP, (2005).