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Set-based Multiobjective Fitness Landscapes: definition, properties.

ThRaSh 2010

Sébastien Verel^{1,2} Clarisse Dhaenens^{1,3} Laetitia Jourdan^{1,3}
Arnaud Liefoghe⁴

<http://www.i3s.unice.fr/~verel>

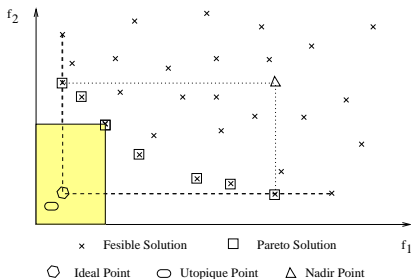
¹DOLPHIN team - INRIA Lille-Nord Europe (France)

²I3S Laboratory - University of Nice-Sophia Antipolis / CNRS (France)

³LIFL - University of Lille 1 / CNRS (France)

⁴University of Coimbra (Portugal)

Multiobjective problem



Multiobjective problem

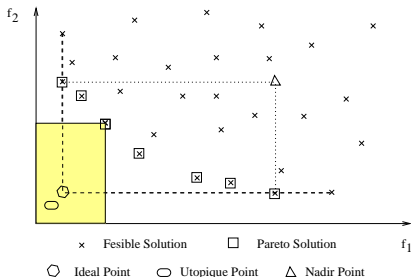
- \mathcal{S} : set of admissible solutions,
- $f : \mathcal{S} \rightarrow \mathbb{R}^p$: multiobjective function to minimize

Dominance (minimization)

x dominate y : $x \prec y$ iff :

- $\forall k = 1..p, f_k(x) \leq f_k(y)$
- $\exists k \in 1..p, f_k(x) < f_k(y)$

Solving a multiobjective problem



Pareto Set (minimization)

$$PS = \{s \in \mathcal{S} \mid \forall z \in \mathcal{S}, z \not\leq s\}$$

Pareto Front

$$PF = \{f(s) \mid s \in PS\}$$

Solving multiobjective problem

Find PS or at least $X \subset PS$
which is a "good" approximation
of PS

Difficulty in multiobjective optimization

What does "difficulty" mean ?

- For a given computational time, X far from a "good" approximation of the Pareto Set (distance in objective space to PS, lack of diversity of X , etc.)
- Time, or number of evaluations, for solving the problem is long : class of complexity

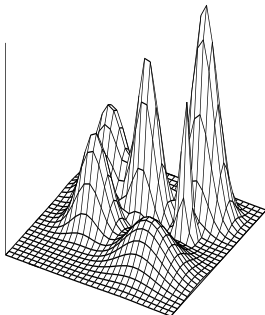
Difficulty in multiobjective optimization

What's make "difficult" a multiobjective problem ?

- The number of objective (criteria)
- The size of the feasible solutions space
- The "difficulty" of the single objective problem (time complexity, ...)
- The correlation between the objectives
- The convexity of the Pareto Front
- The connectedness of the Pareto Set
- ...

Fitness landscape point of view can help...

Fitness landscapes in single-objective optimization



Fitness landscape $(\mathcal{S}, \mathcal{N}, f)$

- \mathcal{S} : set of admissible solutions,
- $\mathcal{N} : \mathcal{S} \rightarrow 2^{\mathcal{S}}$: neighborhood function,
- $f : \mathcal{S} \rightarrow \mathbb{R}$: fitness function.

$$\forall x \in \mathcal{S},$$

$$\mathcal{N}(x) = \{y \in \mathcal{S} \mid P(y = op(x)) > 0\}$$

or

$$\mathcal{N}(x) = \{y \in \mathcal{S} \mid d(y, x) \leq k\}$$

Goal of the fitness landscapes study

- "Geometry" (features) of fitness landscape
⇒ dynamics of a local search algorithm
- Geometry is linked to the problem difficulty :
 - If there are a lot of local optima, the probability to find the global optimum is lower.
 - If the fitness landscape is flat, discovering better solutions is rare.
 - What is the best search direction in the landscape?

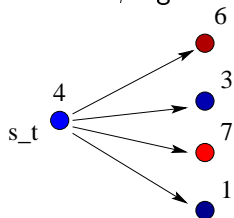
Study of the fitness landscape features
allows to study
the performance of search algorithms

Goal of the fitness landscapes study

- 1 To compare the difficulty of two search spaces :
 - One problem with 2 (or more) possible codings : $(S_1, \mathcal{N}_1, f_1)$ and $(S_2, \mathcal{N}_2, f_2)$
different coding, mutation operator, fitness function, etc.
Which one is easier to solve?
- 2 To choose the algorithm :
 - analysis of global geometry of the landscape
Which algorithm can I use?
- 3 To tune the parameters :
 - *off-line* analysis of structure of fitness landscape
Which is the best mutation operator ? the size of the population ? etc.
- 4 To control the parameters during the run :
 - *on-line* analysis of structure of fitness landscape
Which is the optimal mutation operator according to the estimation of structure ?

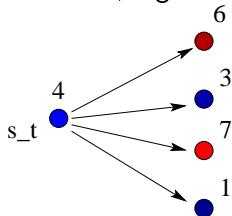
Links with local search

Solutions, eighbors, fitness :

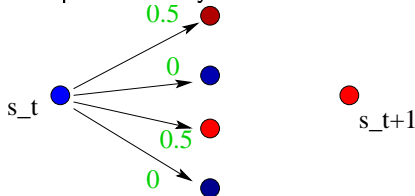


Links with local search

Solutions, eighbors, fitness :



Put prob. from your heuristic :



- Fitness landscape point of view :
before putting a particular heuristic
- Sample the neighborhood to have information on local features of the search space
- From local information :
deduce some global features like general shape of search space, "difficulty", etc.

Multiobjective fitness landscapes (moFiL)

3 ideas to define multiobjective Fitness Landscapes (moFiL)

- Change **nothing** from the mono-objective definition (the most studied from 2003) :
Fitness function is the multiobjective function
- Change the **fitness function** by an indicator such as hypervolume :
Ruggedness can be studied
- Change the **search space** by the set of sets of solutions :
"Real" search space viewed by efficient local searches

Fitness function is the multiobjective function

Multidimensional multiobjective Fitness Landscape $(\mathcal{S}, \mathcal{N}, f)$

- \mathcal{S} : set of admissible solutions,
- $\mathcal{N} : \mathcal{S} \rightarrow 2^{\mathcal{S}}$: neighborhood defined by local operator or distance on \mathcal{S} ,
- $f : \mathcal{S} \rightarrow \mathbb{R}^P$: multiobjective function.

Main idea to analysis of multiobjective fitness landscapes :

- Dominance relation

Overview on previous tools (1)

- **Pareto Set** [AT04] [AT07] :

Number, distribution over landscape

+ : *Information on the final goal*

– : *Information only on the final goal*

- Partition the search space into **dominance levels** [PCS04]
[GD09] :

connection between level of dominance

+ : *Information on structure of search space*

– : *Ways to reach PS? small search space*

Overview on previous tools (2)

- **Connectedness** [EK97] [GKR06] [PS09] :

number of clusters, size of cluster by the graph induced by Pareto set and neighborhood relation (edges if the distance $\leq k$)

+ :If connected, easy way to find the PS

– :Huge PS are intractable, final goal, often not connected

- **Fitness distance correlation, random walk from Pareto loc. opt.** [GD06a] [GD08] [GD06b] :

correlation between distances in obj. and sol. space.

+ :Show the correlation of fitness

*– :The reference solution change, no correlation after few steps,
no scale*

Remarks on multidimensional moFiL

- Majority of works use the concept of dominance :
connectedness, size, estimation of distribution Pareto set,
partition by dominance levels

Limited results :

- No enough information for the design of metaheuristics
- Few understanding of the complexity of multiobjective problems from the fitness landscapes analysis

Change the fitness function by an indicator

Indicator multiobjective Fitness Landscape $(\mathcal{S}, \mathcal{N}, i)$

- \mathcal{S} : set of admissible solutions,
- $\mathcal{N} : \mathcal{S} \rightarrow 2^{\mathcal{S}}$: neighborhood defined by local operator or distance on \mathcal{S} ,
- $i : 2^{\mathcal{S}} \rightarrow \mathbb{R}$: indicator (such as hypervolume, ϵ dominance) on set of solutions

Main idea to analysis of multiobjective fitness landscapes :

- Value of the indicator (hypervolume) of the neighborhood :
$$i(\mathcal{N}(s) \cup \{s\}) = hv(\mathcal{N}(s) \cup \{s\})$$

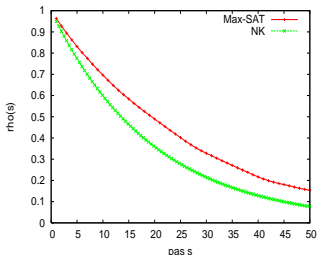
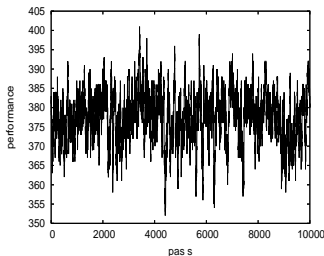
Rugged/smooth fitness landscapes

Autocorrelation of time series of fitnesses $(f(s_1), f(s_2), \dots)$ along a random walk (s_1, s_2, \dots) [Wei90] :

$$\rho(n) = \frac{E[(f(s_i) - \bar{f})(f(s_{i+n}) - \bar{f})]}{\text{var}(f(s_i))}$$

autocorrelation length $\tau = \frac{1}{\rho(1)}$

- small τ : rugged landscape
- long τ : smooth landscape



A model of multiojective problem

Multiojective *MNK*-landscapes with objective correlation

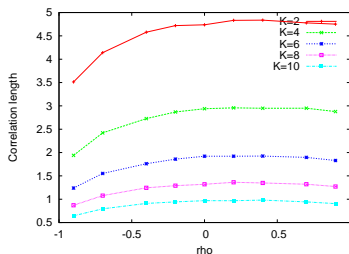
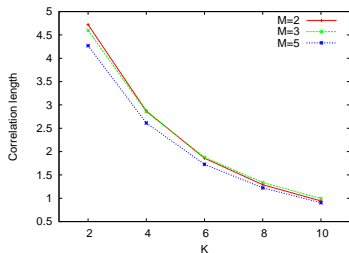
MNK landscapes [AT04]

$$\forall m \in [1, M], f_{NK,m}(s) = \frac{1}{N} \sum_{i=1}^N f_{i,m}(s_i, s_{i_1}, \dots, s_{i_K})$$

- M objectives which are NK – *landscapes*
- Same interaction : same K , and same s_i
- Each fitness component $f_{i,m}$ by extension : $y_{s_i, s_{i_1}, \dots, s_{i_K}}^{i,m}$.

- MNK : y_{\dots}^{im} are randomly and independly drawn from $[0, 1)$.
- in ρMNK : $(y_{s_i, s_{i_1}, \dots, s_{i_K}}^{i1}, \dots, y_{s_i, s_{i_1}, \dots, s_{i_K}}^{iM}) \sim$ multivariate uniform law of dimension M :
objective correlation between objective can be tuned precisely

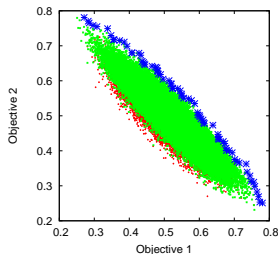
Results on *MNK*-landscapes with objective correlation



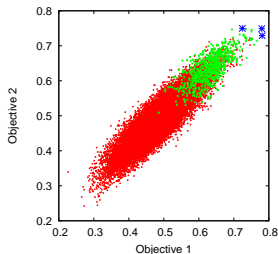
- Dimension has low influence on correlation length
- Objective correlation change slowly the correlation length
- The non-linearity (epistasis) has the main influence

Analysis

- Definition of the ruggedness in indicator moFiL.
- Ruggedness is not correlated to the number of Pareto Local Optima in moFiL.
- Argument in favour to optimize the hypervolume rather than the dominance



$$K = 4, \rho = -0.9$$



$$K = 4, \rho = 0.9$$

Remarks on indicator moFiL

Pros :

- Estimation of non-linearity / ruggedness of the landscapes
- Explain "local" difficulty of multiobjective problem

Cons :

- Metaheuristics manipulate set of solutions, not single solution

Change the search space by the set of sets of solutions

Set based multiobjective Fitness Landscape (Σ_q, N_q, i)

- Σ_q : set of sets of feasible solutions of maximum size q
$$\Sigma_q = \{\sigma \subset \mathcal{S} : |\sigma| \leq q\}, \text{ so } |\Sigma_q| = \binom{|\mathcal{S}|}{q}$$
- N_q : insert, delete, or change one solution in the set
- $i : \Sigma_q \rightarrow \mathbb{R}$: indicator (such as hypervolume, ϵ dominance) on set of solutions

Main idea to analysis of multiobjective fitness landscapes :

- All the standard tools of Fitness Landscapes analysis

Neighborhood N_q : Insert, delete, or change 1 solution

- Neighborhood $N_q : \Sigma_q \rightarrow 2^{\Sigma_q}$:
defined from neighborhood $\mathcal{N} : \mathcal{S} \rightarrow 2^{\mathcal{S}}$
- Neighbors of $\sigma \in \Sigma_q$ are given by 3 operators :
 - Insert one solution which is neighbor of one of solution of σ
 - Delete one solution
 - Change one solution by its neighbor

$$N_q(\sigma) = N_q^i(\sigma) \cup N_q^d(\sigma) \cup N_q^c(\sigma)$$

$N_q^i(\sigma)$: Insert one solution in the set

σ' neighbor of $\sigma = \{s_1, \dots, s_k\}$

- if $k = q$: $\sigma' = \sigma$
- if $0 < k < q$: $\sigma' = \{s_1, \dots, s_k, s_{k+1}\}$ with $s_{k+1} \in \mathcal{N}(s_i)$
- if $k = 0$: $\sigma' = \{s\}$ with $s \in \mathcal{S}$

Sizes

when $0 < k < q$,

- if s_{k+1} is not in σ , $|\sigma'| = |\sigma| + 1$
- $|N_q^i(\sigma)| \leq \sum_{i=1}^k |\mathcal{N}(s_i)|$
 $|N_q^i(\sigma)| \leq k\lambda$ if neighborhood size is cst.

$N_q^d(\sigma)$: Delete one solution from the set

σ' neighbor of $\sigma = \{s_1, \dots, s_k\}$

- if $k = 0$: $\sigma' = \sigma$
- if $0 < k$: $\sigma' = \{s_1, \dots, s_{i-1}, s_{i+1}, \dots, s_k\}$ with $i \in 1..k$

Sizes

when $0 < k$,

- $|\sigma'| = |\sigma| - 1$
- $|N_q^d(\sigma)| = k$

$N_q^c(\sigma)$: Change one solution of the set

σ' neighbor of $\sigma = \{s_1, \dots, s_k\}$

- if $k = 0$: $\sigma' = \sigma$
- if $0 < k$: $\sigma' = \{s_1, \dots, s_{i-1}, s'_i, s_{i+1}, \dots, s_k\}$
with $i \in 1..k$ and $s'_i \in \mathcal{N}(s_i)$

Sizes

when $0 < k$,

- if s'_i is not in σ , $|\sigma'| = |\sigma|$
- $|N_q^c(\sigma)| \leq \sum_{i=1}^k |\mathcal{N}(s_i)|$
 $|N_q^i(\sigma)| \leq k\lambda$ if neighborhood size is cst.
- Total size of $N_q(\sigma)$: $|N_q(\sigma)| \leq k + 2 \sum_{i=1}^k |\mathcal{N}(s_i)|$
 $|N_q(\sigma)| \leq k(2\lambda + 1)$ if neighborhood size is cst.

Links with local search : example of efficient LS

"On set based-based multiobjective optimization" Zitzler et al, 2009

Set based Algo. for MO

Generate initial set P of size q with random solutions

repeat

$P' \leftarrow \text{randomSetMutation}(P)$

$P'' \leftarrow \text{heuristicSetMutation}(P)$

if $P'' \preceq P$ then

$P \leftarrow P''$

else

if $P' \preceq P$ then

$P \leftarrow P'$

end if

end if

until termination criterion is not true

RandomSetMutation

randomly choose $s_1, \dots, s_k \in S$ with $s_i \neq s_j$

randomly select $p_1, \dots, p_k \in P$ with $p_i \neq p_j$

$P' \leftarrow P \setminus \{s_1, \dots, s_k\} \cup \{p_1, \dots, p_k\}$

return P'

heuristicSetMutation

Generate $s_1, \dots, s_k \in S$ based on P (selection and mutation of solutions from P)

$P' \leftarrow P \cup \{s_1, \dots, s_k\}$

while $|P''| > q$ do

for all $a \in P''$ do

$\delta_a \leftarrow i(P'' \setminus \{a\}) - i(P'')$

end for

choose $p \in P''$ with $\delta_p = \min_{a \in P''} \delta_a$

$P'' \leftarrow P'' \setminus \{p\}$

end while

return P''

Tools to analysis set-based multiobjective fitness landscapes

- Density of states :
What can we except from random search ?
- Adaptive walk, local optima :
What is the number of local optima, and size of basin ?
- Random walk, ruggedness :
Is the solution in the neighborhood are random ?
- Fitness cloud, evolvability :
What is the distribution of fitness in the neighborhood ?
Can the solutions go to better fitness values ?
- Neutral walk, neutrality :
Is there a lot of solutions with the same fitness ?
Is there information in and around the neutral networks ?
- Local Optima Network :
What are the links between basins of attraction in the landscapes ?

Conclusion and perspectives

Summary

- Previous works on multidimensional moFiL : Fitness is the multiobjective function based on dominance, information on Pareto Set (goal of optimization)
- Indicator moFiL : Fitness is the value of indicator based on random walk, information on ruggedness
- Set based moFiL : search space is the set of sets
Use the "real" search space and elementary operators on sets

Conclusion and perspectives

Summary

- Previous works on multidimensional moFiL : Fitness is the multiobjective function based on dominance, information on Pareto Set (goal of optimization)
- Indicator moFiL : Fitness is the value of indicator based on random walk, information on ruggedness
- Set based moFiL : search space is the set of sets
Use the "real" search space and elementary operators on sets

Perspectives

- Use it !
- To decide the best encoding and operator for example in multiobjective TVRP (see M-E Marmion)

Non commercial communication

Special Session

Fitness landscapes and metaheuristics

- at international conference META'2010, Djerba Island, Tunisia, October 28-30th, 2010.
- Extended abstract 2 pages
- Post publication for selected papers in COR, ITOR, JMMA
- Deadline may 5, 2010
- web page :
<http://www2.lifl.fr/META10/index.php?n=Main.InfoFIL>

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