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## Set-based Multiobjective Fitness Landscapes: definition, properties

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# Set-based Multiobjective Fitness Landscapes: definition, properties.

ThRaSh 2010

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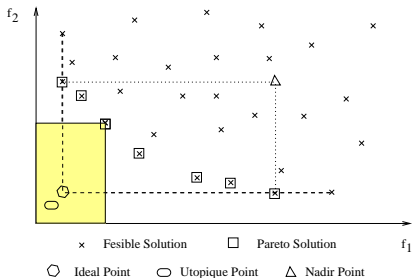
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# Multiobjective problem



## Multiobjective problem

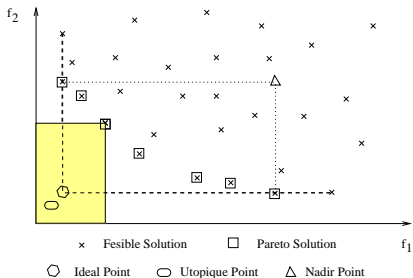
- $\mathcal{S}$  : set of admissible solutions,
- $f : \mathcal{S} \rightarrow \mathbb{R}^p$  : multiobjective function to minimize

## Dominance (minimization)

$x$  dominate  $y$  :  $x \prec y$  iff :

- $\forall k = 1..p, f_k(x) \leq f_k(y)$
- $\exists k \in 1..p, f_k(x) < f_k(y)$

## Solving a multiobjective problem



## Pareto Set (minimization)

$$PS = \{s \in \mathcal{S} \mid \forall z \in \mathcal{S}, z \not\prec s\}$$

## Pareto Front

$$PF = \{f(s) \mid s \in PS\}$$

## Solving multiobjective problem

Find PS or at least  $X \subset PS$   
 which is a "good" approximation  
 of PS

# Difficulty in multiobjective optimization

## What does "difficulty" mean ?

- For a given computational time,  $X$  far from a "good" approximation of the Pareto Set (distance in objective space to PS, lack of diversity of  $X$ , etc.)
- Time, or number of evaluations, for solving the problem is long : class of complexity

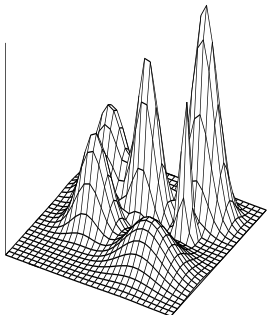
# Difficulty in multiobjective optimization

## What's make "difficult" a multiobjective problem ?

- The number of objective (criteria)
- The size of the feasible solutions space
- The "difficulty" of the single objective problem (time complexity, ...)
- The correlation between the objectives
- The convexity of the Pareto Front
- The connectedness of the Pareto Set
- ...

Fitness landscape point of view can help...

# Fitness landscapes in single-objective optimization



## *Fitness landscape* $(\mathcal{S}, \mathcal{N}, f)$

- $\mathcal{S}$  : set of admissible solutions,
- $\mathcal{N} : \mathcal{S} \rightarrow 2^{\mathcal{S}}$  : neighborhood function,
- $f : \mathcal{S} \rightarrow \mathbb{R}$  : fitness function.

$\forall x \in \mathcal{S},$

$$\mathcal{N}(x) = \{y \in \mathcal{S} \mid P(y = op(x)) > 0\}$$

or

$$\mathcal{N}(x) = \{y \in \mathcal{S} \mid d(y, x) \leq k\}$$

# Goal of the fitness landscapes study

- "Geometry" (features) of fitness landscape  
⇒ dynamics of a local search algorithm
- Geometry is linked to the problem difficulty :
  - If there are a lot of local optima, the probability to find the global optimum is lower.
  - If the fitness landscape is flat, discovering better solutions is rare.
  - What is the best search direction in the landscape?

Study of the fitness landscape features  
allows to study  
the performance of search algorithms

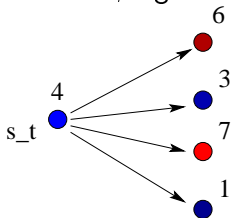


# Goal of the fitness landscapes study

- 1 To compare the difficulty of two search spaces :
  - One problem with 2 (or more) possible codings :  $(S_1, \mathcal{N}_1, f_1)$  and  $(S_2, \mathcal{N}_2, f_2)$   
different coding, mutation operator, fitness function, etc.  
Which one is easier to solve?
- 2 To choose the algorithm :
  - analysis of global geometry of the landscape  
Which algorithm can I use?
- 3 To tune the parameters :
  - *off-line* analysis of structure of fitness landscape  
Which is the best mutation operator ? the size of the population ? etc.
- 4 To control the parameters during the run :
  - *on-line* analysis of structure of fitness landscape  
Which is the optimal mutation operator according to the estimation of structure ?

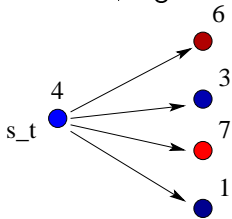
# Links with local search

Solutions, eighbors, fitness :

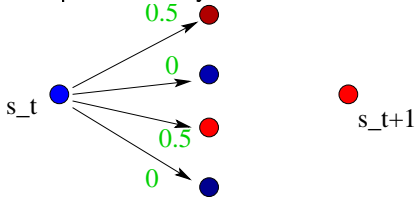


# Links with local search

Solutions, eighbors, fitness :



Put prob. from your heuristic :



- Fitness landscape point of view :  
before putting a particular heuristic
- Sample the neighborhood to have information on local features of the search space
- From local information :  
deduce some global features like general shape of search space, "difficulty", etc.

# Multiobjective fitness landscapes (moFiL)

## 3 ideas to define multiobjective Fitness Landscapes (moFiL)

- Change **nothing** from the mono-objective definition (the most studied from 2003) :  
Fitness function is the multiobjective function
- Change the **fitness function** by an indicator such as hypervolume :  
Ruggedness can be studied
- Change the **search space** by the set of sets of solutions :  
"Real" search space viewed by efficient local searches

# Fitness function is the multiobjective function

## *Multidimensional multiobjective Fitness Landscape $(\mathcal{S}, \mathcal{N}, f)$*

- $\mathcal{S}$  : set of admissible solutions,
- $\mathcal{N} : \mathcal{S} \rightarrow 2^{\mathcal{S}}$  : neighborhood defined by local operator or distance on  $\mathcal{S}$ ,
- $f : \mathcal{S} \rightarrow \mathbb{R}^p$  : multiobjective function.

Main idea to analysis of multiobjective fitness landscapes :

- Dominance relation

## Overview on previous tools (1)

- **Pareto Set** [AT04] [AT07] :

Number, distribution over landscape

*+ :Information on the final goal*

*– :Information only on the final goal*

- Partition the search space into **dominance levels** [PCS04]  
[GD09] :

connection between level of dominance

*+ :Information on structure of search space*

*– :Ways to reach PS? small search space*

## Overview on previous tools (2)

- **Connectedness** [EK97] [GKR06] [PS09] :

number of clusters, size of cluster by the graph induced by Pareto set and neighborhood relation (edges if the distance  $\leq k$ )

*+ :If connected, easy way to find the PS*

*– :Huge PS are intractable, final goal, often not connected*

- **Fitness distance correlation, random walk from Pareto loc. opt.** [GD06a] [GD08] [GD06b] :

correlation between distances in obj. and sol. space.

*+ :Show the correlation of fitness*

*– :The reference solution change, no correlation after few steps,  
no scale*

# Remarks on multidimensional moFiL

- Majority of works use the concept of dominance :  
connectedness, size, estimation of distribution Pareto set,  
partition by dominance levels

## Limited results :

- No enough information for the design of metaheuristics
- Few understanding of the complexity of multiobjective problems from the fitness landscapes analysis



## Change the fitness function by an indicator

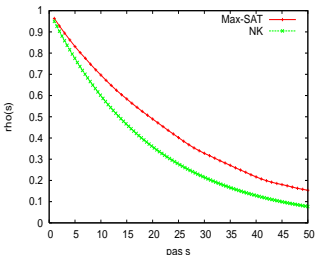
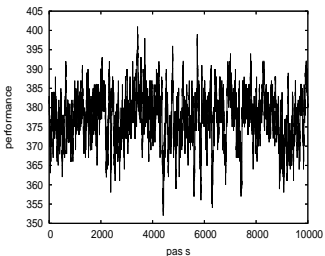
### *Indicator multiobjective Fitness Landscape $(\mathcal{S}, \mathcal{N}, i)$*

- $\mathcal{S}$  : set of admissible solutions,
- $\mathcal{N} : \mathcal{S} \rightarrow 2^{\mathcal{S}}$  : neighborhood defined by local operator or distance on  $\mathcal{S}$ ,
- $i : 2^{\mathcal{S}} \rightarrow \mathbb{R}$  : indicator (such as hypervolume,  $\epsilon$  dominance) on set of solutions

Main idea to analysis of multiobjective fitness landscapes :

- Value of the indicator (hypervolume) of the neighborhood :  
$$i(\mathcal{N}(s) \cup \{s\}) = hv(\mathcal{N}(s) \cup \{s\})$$

# Rugged/smooth fitness landscapes



Autocorrelation of time series of fitnesses  $(f(s_1), f(s_2), \dots)$  along a random walk  $(s_1, s_2, \dots)$  [Wei90] :

$$\rho(n) = \frac{E[(f(s_i) - \bar{f})(f(s_{i+n}) - \bar{f})]}{\text{var}(f(s_i))}$$

autocorrelation length  $\tau = \frac{1}{\rho(1)}$

- small  $\tau$  : rugged landscape
- long  $\tau$  : smooth landscape

# A model of multiobjective problem

Multiobjective *MNK*-landscapes with objective correlation

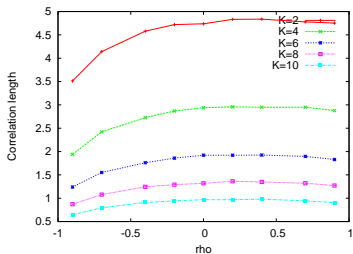
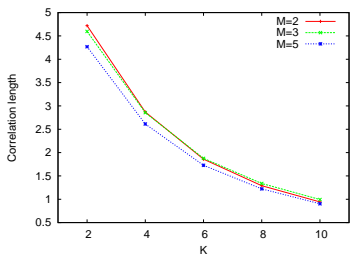
## *MNK* landscapes [AT04]

$$\forall m \in [1, M], f_{NK,m}(s) = \frac{1}{N} \sum_{i=1}^N f_{i,m}(s_i, s_{i_1}, \dots, s_{i_K})$$

- $M$  objectives which are  $NK$  – landscapes
- Same interaction : same  $K$ , and same  $s_i$
- Each fitness component  $f_{i,m}$  by extension :  $y_{s_i, s_{i_1}, \dots, s_{i_K}}^{i,m}$ .

- $MNK$  :  $y_{\dots}^{im}$  are randomly and independently drawn from  $[0, 1)$ .
- in  $\rho MNK$  :  $(y_{s_i, s_{i_1}, \dots, s_{i_K}}^{i1}, \dots, y_{s_i, s_{i_1}, \dots, s_{i_K}}^{iM}) \sim$  multivariate uniform law of dimension  $M$  :  
objective correlation between objective can be tuned precisely

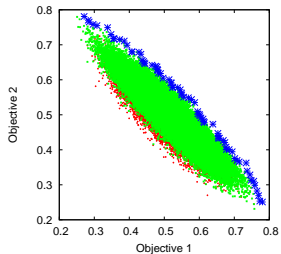
# Results on *MNK*-landscapes with objective correlation



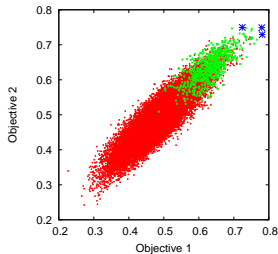
- Dimension has low influence on correlation length
- Objective correlation change slowly the correlation length
- The non-linearity (epistasis) has the main influence

# Analysis

- Definition of the ruggedness in indicator moFiL.
- Ruggedness is not correlated to the number of Pareto Local Optima in moFiL.
- Argument in favour to optimize the hypervolume rather than the dominance



$$K = 4, \rho = -0.9$$



$$K = 4, \rho = 0.9$$

## Remarks on indicator moFiL

Pros :

- Estimation of non-linearity / ruggedness of the landscapes
- Explain "local" difficulty of multiobjective problem

Cons :

- Metaheuristics manipulate set of solutions, not single solution

# Change the search space by the set of sets of solutions

## *Set based multiobjective Fitness Landscape* ( $\Sigma_q, N_q, i$ )

- $\Sigma_q$  : set of sets of feasible solutions of maximum size  $q$

$$\Sigma_q = \{\sigma \subset \mathcal{S} : |\sigma| \leq q\}, \text{ so } |\Sigma_q| = \binom{|\mathcal{S}|}{q}$$

- $N_q$  : insert, delete, or change one solution in the set
- $i : \Sigma_q \rightarrow \mathbb{R}$  : indicator (such as hypervolume,  $\epsilon$  dominance) on set of solutions

Main idea to analysis of multiobjective fitness landscapes :

- All the standard tools of Fitness Landscapes analysis

# Neighborhood $N_q$ : Insert, delete, or change 1 solution

- Neighborhood  $N_q : \Sigma_q \rightarrow 2^{\Sigma_q}$  :  
defined from neighborhood  $\mathcal{N} : \mathcal{S} \rightarrow 2^{\mathcal{S}}$
- Neighbors of  $\sigma \in \Sigma_q$  are given by 3 operators :
  - Insert one solution which is neighbor of one of solution of  $\sigma$
  - Delete one solution
  - Change one solution by its neighbor

$$N_q(\sigma) = N_q^i(\sigma) \cup N_q^d(\sigma) \cup N_q^c(\sigma)$$



$N_q^i(\sigma)$  : Insert one solution in the set

$\sigma'$  neighbor of  $\sigma = \{s_1, \dots, s_k\}$

- if  $k = q$  :  $\sigma' = \sigma$
- if  $0 < k < q$  :  $\sigma' = \{s_1, \dots, s_k, s_{k+1}\}$  with  $s_{k+1} \in \mathcal{N}(s_i)$
- if  $k = 0$  :  $\sigma' = \{s\}$  with  $s \in \mathcal{S}$

## Sizes

when  $0 < k < q$ ,

- if  $s_{k+1}$  is not in  $\sigma$ ,  $|\sigma'| = |\sigma| + 1$
- $|N_q^i(\sigma)| \leq \sum_{i=1}^k |\mathcal{N}(s_i)|$   
 $|N_q^i(\sigma)| \leq k\lambda$  if neighborhood size is cst.

$N_q^d(\sigma)$  : Delete one solution from the set

$\sigma'$  neighbor of  $\sigma = \{s_1, \dots, s_k\}$

- if  $k = 0$  :  $\sigma' = \sigma$
- if  $0 < k$  :  $\sigma' = \{s_1, \dots, s_{i-1}, s_{i+1}, \dots, s_k\}$  with  $i \in 1..k$

### Sizes

when  $0 < k$ ,

- $|\sigma'| = |\sigma| - 1$
- $|N_q^d(\sigma)| = k$

$N_q^c(\sigma)$  : Change one solution of the set

$\sigma'$  neighbor of  $\sigma = \{s_1, \dots, s_k\}$

- if  $k = 0$  :  $\sigma' = \sigma$
- if  $0 < k$  :  $\sigma' = \{s_1, \dots, s_{i-1}, s'_i, s_{i+1}, \dots, s_k\}$   
with  $i \in 1..k$  and  $s'_i \in \mathcal{N}(s_i)$

## Sizes

when  $0 < k$ ,

- if  $s'_i$  is not in  $\sigma$ ,  $|\sigma'| = |\sigma|$
- $|N_q^c(\sigma)| \leq \sum_{i=1}^k |\mathcal{N}(s_i)|$   
 $|N_q^i(\sigma)| \leq k\lambda$  if neighborhood size is cst.
- Total size of  $N_q(\sigma)$  :  $|N_q(\sigma)| \leq k + 2 \sum_{i=1}^k |\mathcal{N}(s_i)|$   
 $|N_q(\sigma)| \leq k(2\lambda + 1)$  if neighborhood size is cst.

# Links with local search : example of efficient LS

"On set based-based multiobjective optimization" Zitzler et al, 2009

## Set based Algo. for MO

Generate initial set  $P$  of size  $q$  with random solutions

repeat

$P' \leftarrow \text{randomSetMutation}(P)$

$P'' \leftarrow \text{heuristicSetMutation}(P)$

if  $P'' \preceq P$  then

$P \leftarrow P''$

else

if  $P' \preceq P$  then

$P \leftarrow P'$

end if

end if

until termination criterion is not true

## RandomSetMutation

randomly choose  $s_1, \dots, s_k \in S$  with  $s_i \neq s_j$

randomly select  $p_1, \dots, p_k \in P$  with  $p_i \neq p_j$

$P' \leftarrow P \setminus \{s_1, \dots, s_k\} \cup \{p_1, \dots, p_k\}$

return  $P'$

## heuristicSetMutation

Generate  $s_1, \dots, s_k \in S$  based on  $P$  (selection and mutation of solutions from  $P$ )

$P' \leftarrow P \cup \{s_1, \dots, s_k\}$

while  $|P''| > q$  do

for all  $a \in P''$  do

$\delta_a \leftarrow i(P'' \setminus \{a\}) - i(P'')$

end for

choose  $p \in P''$  with  $\delta_p = \min_{a \in P''} \delta_a$

$P'' \leftarrow P'' \setminus \{p\}$

end while

return  $P''$

# Tools to analysis set-based multiobjective fitness landscapes

- Density of states :  
*What can we expect from random search ?*
- Adaptive walk, local optima :  
*What is the number of local optima, and size of basin ?*
- Random walk, ruggedness :  
*Is the solution in the neighborhood are random ?*
- Fitness cloud, evolvability :  
*What is the distribution of fitness in the neighborhood ?*  
*Can the solutions go to better fitness values ?*
- Neutral walk, neutrality :  
*Is there a lot of solutions with the same fitness ?*  
*Is there information in and around the neutral networks ?*
- Local Optima Network :  
*What are the links between basins of attraction in the landscapes ?*

## Conclusion and perspectives

### Summary

- Previous works on multidimensional moFiL : Fitness is the multiobjective function based on dominance, information on Pareto Set (goal of optimization)
- Indicator moFiL : Fitness is the value of indicator based on random walk, information on ruggedness
- Set based moFiL : search space is the set of sets  
Use the "real" search space and elementary operators on sets

## Conclusion and perspectives

### Summary

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- Set based moFiL : search space is the set of sets  
Use the "real" search space and elementary operators on sets

### Perspectives

- Use it !
- To decide the best encoding and operator for example in multiobjective TVRP (see M-E Marmion)

# Non commercial communication

## Special Session

### Fitness landscapes and metaheuristics

- at international conference META'2010, Djerba Island, Tunisia, October 28-30th, 2010.
- Extended abstract 2 pages
- Post publication for selected papers in COR, ITOR, JMMA
- Deadline may 5, 2010
- web page :  
<http://www2.lifl.fr/META10/index.php?n=Main.InfoFiL>



## For Further Reading



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## For Further Reading



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