



## Online resources in mathematics: teachers' geneses and didactical techniques

Laetitia Bueno-Ravel, Ghislaine Gueudet

### ► To cite this version:

Laetitia Bueno-Ravel, Ghislaine Gueudet. Online resources in mathematics: teachers' geneses and didactical techniques. *International Journal of Computers for Mathematical Learning*, 2009, 14 (1), pp.1-20. 10.1007/s10758-009-9143-0 . hal-00459548

**HAL Id: hal-00459548**

**<https://hal.science/hal-00459548>**

Submitted on 11 Jul 2013

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

## **Online resources in mathematics : Teachers' geneses and didactical techniques.**

Laetitia Bueno-Ravel, Ghislaine Gueudet

*Address: IUFM de Bretagne, site de Rennes. 153 rue Saint Malo. CS 54 310 35043 RENNES CEDEX FRANCE*

*Affiliation: CREAD, IUFM de Bretagne UBO*

Phone: 00 33 (0)2 99 54 64 35

Fax: 00 33 (0)2 99 54 82 28

e-mail: [Laetitia.Bueno-Ravel@bretagne.iufm.fr](mailto:Laetitia.Bueno-Ravel@bretagne.iufm.fr)

web page: [http://cread.bretagne.iufm.fr/article.php3?id\\_article=139](http://cread.bretagne.iufm.fr/article.php3?id_article=139)

**Abstract:** Teachers increasingly draw on online resources for their courses. On one side, these resources influence the teaching they propose; on the other side, resources are modified by teachers in their appropriation processes. Understanding these phenomena is an important issue for educational research, in mathematics in particular. The instrumental approach (Guin, Ruthven & Trouche 2005) has been used in many research works to understand how students learn with technology. We propose here to enlarge its scope to the study of teachers using online resources. We study instrumental geneses, and instrumented techniques, of teachers working with an e-exercise basis.

**Keywords:** *Instrumental genesis, instrumented technique, e-exercise basis, teachers' resources.*

**Abbreviations:**

Mathenpoche, MEP; Genesis of Professional Use of Technologies by Teachers, GUPTEn; e-exercise basis, EEB.

## 1. Introduction

Teachers increasingly draw on online resources for their courses. Lessons plans, exercises, assessment texts... can be found on web sites all over the world. A teaching whose elaboration involves such resources is likely to be influenced by their mathematical content, by their authors' choices. Similar statements can be formulated about textbooks, and some of the resources available on the web do not tremendously differ from paper resources. However, many digital resources offer specific possibilities: digital textbooks integrate web links, and texts not limited by size constraints; online exercises have specific interactive features etc. We consider that the generalized use of online resources and the specificity of some of these resources require the development, in mathematics education research, of a theoretical approach permitting to capture the associated teaching phenomena.

The theoretical elements we propose, and the case study we use to display our approach, were developed within a French national research project called GUPTEn (which holds for, in French: Genesis of Professional Uses of Technology by Teachers, Lagrange *et al.* 2007). This project aims at studying, in particular, the way teachers evolve towards stabilized uses of technological tools. We do not study here the evolution aspect, but stable behaviors of teachers who have taken up an online resource, and use it in their courses, in association with other resources: curriculum material, students' sheets etc.

We focus here mostly on specific online resources: e-exercises bases (shortened EEB in what follows). These resources consist of exercises classified according to their mathematical content, to their difficulty, and/or to the mathematical tools they require. These exercises are associated with an environment which consists of suggestions, correction, explanations, tools for the resolution of the exercise, score etc. (for more details about the possible features of an e-exercises resource, see Cazes *et al.* 2007). EEB can be used by students on their own; but they are also resources for the teacher, as the class textbook for example (similarities and differences between EEBs and textbooks are an interesting topic; we will mention some of these, but this point is not central here).

The examples developed in this paper involve a specific EEB, called "Mathenpoche"<sup>1</sup> ("Maths in the pocket", shortened as MEP in what follows; figure 1 and 2 display examples of

---

<sup>1</sup><http://mathenpoche.sesamath.net>

MEP's screens). A teacher inscribed as "MEP's user" can constitute groups of students, and choose different contents for these groups amongst MEP's exercises. After a work on MEP, the marks of each student are recorded in two MEP's files, one bound to the student and the other to the teacher. We call this last one the session's sheet; it provides the teacher with all the marks reached during the session, and more precisely the indication of success or failure for each question encountered (a MEP's exercise comprises 5 or 10 questions, the mark is exactly the number of right answers to these questions).

MEP is in France a very popular part of the available curriculum material (Remillard 2005). Its massive adoption by teachers is probably linked with the fact that, on the opposite of what happens with ICT tools like spreadsheet, CAS etc. it does not require a tremendous change in the usual "activity structure" (Monaghan 2004), or "activity format" (Ruthven 2008) of the teachers.

Despite this apparently natural integration, MEP is likely to influence the teacher's practices, as any other kind of curriculum material. Thus the questions we study here can be formulated as follows:

- For a given teaching objective, which are the resources used by the teacher, which is the place and role of MEP within these resources?
- How does the teacher modify this material in his/her appropriation process?
- How does MEP, and the associated resources, influence the teacher's practice? Which are the reasons for the choices observed?

In order to study these questions, we use the theoretical framework provided within mathematics didactics by the instrumental approach (Guin *et al.* 2005). Recent developments (Gueudet & Trouche, to appear) propose to enlarge the scope of this approach, usually dedicated to the use of ICT, to the study of teachers' professional activity, and to any kind of curriculum material considered as an artifact for the teacher.

We retain here such a perspective; we expose it in section 2.

The instrumental genesis framework can lead to an interpretation of the teachers' behavior in terms of schemes. However, because of the importance of the institutional constraints for the teachers' choices, we retain here an institutional perspective. It leads us to analyze the teacher's activity in terms of didactic tasks and techniques; this is exposed in section 3. In section 4, we focus on a particular case study, about a teaching of trigonometry in grade 9.

## 2. Instrumental genesis for teachers

The instrumental approach has been developed and used in mathematics didactics by many authors (Guin *et al.* 2005). We recall here briefly its main principles. The instrumental approach is grounded in cognitive ergonomy. Rabardel (1995) stresses the difference between an artifact, which is a given object, and an instrument. The instrument is a psychological construct, constituted of an artifact and of a psychological component defined through the notion of scheme. A scheme is considered here as an invariant organization of behavior for a given class of situations (Vergnaud 1998). The instrument built by the subject comprises the artifact, and the schemes organizing the activity of the subject. The instrument's building process is called the instrumental genesis. This genesis is a dual process. On one side, for a given class of situations, the subject builds a scheme of use of the artifact; the subject's knowledge guides the way the artifact is used, which sometimes differs from the artifact's designer expectations. This is called the instrumentalization process. On the other side, the artifact features constrain the subject's activity: this is the instrumentation process.

In the instrumental approach, an artifact can be any kind of resource, and there is naturally no restriction about who may be a subject. An EEB is an artifact for teachers. Within a given class of situations, the teacher, in a genesis process, elaborates an instrument built from the EEB and associated resources. This genesis encompasses an instrumentation process (the features of the EEB shape the teacher's action), and an instrumentalization process (the appropriation by the teacher shapes the instrument built).

We give here examples of instrumentation and instrumentalization processes for teachers with the EEB used in our examples, MEP. Some instrumentation/instrumentalization processes intervene on general dimensions of the teachers' action, while others are attached to very specific mathematical content. Thus we propose here examples corresponding to more or less general levels.

### Examples of instrumentation processes with MEP

A general instrumentation process which can be observed in the geneses involving MEP concerns assessment. MEP is providing marks for the students. But it does not offer the possibility to sort out automatically the best mark for several tries on the same exercise; neither to construct a global mark for several exercises. MEP is widely used by teachers; but almost never for assessment purposes (we observed this through a questionnaire proposed to

MEP's users), while similar EEBs, offering more marking possibilities<sup>2</sup> are used this way. This can be interpreted as the sign of an instrumentation process.

Let us give another instrumentation example, concerning this time a very precise mathematical content. We observed two teachings of trigonometry in grade 9 classes (details about one of these sequences are given in section 4). In these classes, the three trigonometric ratios:  $\sin$ ,  $\cos$ ,  $\tan$  are introduced in a right-angled triangle. They are not presented as functions; functions are introduced in France only in grade 10, and thus not known by grade 9 students. However, the students can use the " $\sin$ " key on their calculator to compute the sine of a given angle; and even the " $\sin^{-1}$ " key to deduce an angle from a given sine value. But some teachers, and some textbooks, prohibit the writing of " $\sin^{-1}$ " by the students. In such cases, students are supposed to write things like " $\sin(a) = 0,8$ , thus  $a \approx \text{arcsin}(0,8)$ " and writing " $a = \sin^{-1}(0,8)$ " is forbidden. There is no mathematical obstacle here: the angles considered are between 0 and  $90^\circ$ , the sine function has a reciprocal on this domain. But according to some teachers, the students sometimes mix up between an angle and its sine, writing things like " $\sin(0,5) = 30$ ". And the introduction of  $\sin^{-1}$  might worsen this situation. The official curriculum does not take position in this debate. The teachers we observed usually adopt a " $\sin^{-1}$  prohibition" position. MEP adopts an intermediate position (figure 1).

<p>Exercices : 1 2 3 4 5 6</p> <p>Exercice n°5 : Calcul de l'angle</p> <p>Question N°1 : Complète.</p> <p>Le triangle SQG est rectangle en Q tel que  <math>SG = 32</math> et <math>SQ = 5</math>. Alors :</p> <p><math>\sin(\widehat{SGQ}) = \frac{SQ}{SG}</math></p> <p><math>\sin(\widehat{SGQ}) = \frac{5}{32}</math></p> <p>On effectue à la calculatrice : <math>\sin^{-1}(5/32)</math></p> <p><math>\widehat{SGQ} \approx \text{ }^\circ</math> (arrondir au degré près)</p> <p>Valider</p> <p>Mon score :</p>	<p>(Translation of the text)</p> <p>Exercise 5: computing an angle.</p> <p>Question 1. Fill in</p> <p>The triangle SQG is right-angled in Q, and such that <math>SG=32</math> and <math>SQ=5</math>. Hence: .....</p> <p>We compute with the calculator by typing <math>\sin^{-1}(5/32)</math>.....</p> <p>SGQ <math>\approx</math> .....</p>
---	---

**Figure 1.** Use of the notation " $\sin^{-1}$ " in MEP

In MEP, the notation " $\sin^{-1}$ " is attached to the use of the calculator, but not figured as a calculator key. In the example of figure 1, " $\sin^{-1}(5/32)$ " is completely written (but included in a sentence and not in an equation providing the angle)

<sup>2</sup> For example WIMS, <http://wims.unice.fr/wims/>

Students who worked on such MEP screens started to use  $\sin^{-1}$  themselves. Under these conditions, the choice of the teachers was the following: no official position presented during the course; and remarks like “this must not be written” when “ $\sin^{-1}$ ” appears in a student’s sheet. This can be interpreted as an instrumentation process: the prohibited notation is used in MEP, thus the teacher changes her usual practice. She does not say during her course that the notation must not be used; however, she can not fully accept it, and corrects it in the students’ sheets.

### Examples of instrumentalization processes with MEP

For each exercise, MEP proposes help after a first wrong answer. The help comprises presentations of methods, considered as very interesting by many teachers. Thus some teachers developed an unexpected use of the help, with projection devices for the whole class. These teachers were forced to start an exercise and make a deliberate mistake, to display the help. We interpret this as an instrumentalization process. Eventually a special page providing access to all the helps has even been elaborated on the general website hosting MEP<sup>3</sup>.

Naturally the instrumentalization process does not necessarily lead to an intervention of the resource’s designers, or of anyone else than the user. As we mentioned it above, MEP proposes sessions’ sheets comprising for each student the recording of his or her success or failure for each question tackled, and the mark out of 5 or 10 for each exercise. But it does not sort out the best mark for one exercise tackled several times. One of the teachers we observed built her own session’s sheet on a paper. She uses MEP’s session sheet, and for each student and each exercise only keeps the best score, which is then written on her paper session’s sheet. She has built this way an instrument for assessment whose material part comprises MEP and her paper session’s sheet.

In the above given examples, other resources are clearly involved with MEP in the genesis processes. The instrumentation process around the use of the “ $\sin^{-1}$ ” notation involves also the personal calculators used by the students, with their “ $\sin^{-1}$ ” keys; the students’ sheet, where “ $\sin^{-1}$ ” is written. In the last instrumentalization example, the class of situations can be described as “building and collecting marks for the students”, and the teacher builds an instrument from MEP and her paper session’s sheet.

---

<sup>3</sup> Sesamath, <http://www.sesamath.net/>

In the approach we develop here, we never consider an EEB, or any other kind of artifact or resource (we use both words as synonymous here) by itself, but sets of resources. This choice is probably a natural consequence of our initial focus on online resources. Most online resources associate indeed several materials: for example a spreadsheet with possible exercises, and even examples of possible scenarios for its use in class. In the case of MEP, the website hosting it also proposes dynamic geometry software, a spreadsheet, propositions for scenarios in class, some of them specially designed to integrate a numerical whiteboard etc. It even proposes a complete digital textbook, with a traditional course and exercises, but also propositions for using all the available material we mentioned. And naturally, uses of MEP in class will also involve paper and pencil devices, a whiteboard...

Gueudet and Trouche (to appear) introduce the terms “resources” and “documents”: a set of resources, for a given class of situations, generates a document entailing the resources and a scheme of use of these resources. They consider any kind of resources, not restricted to curriculum material. We adopt here a similar perspective. We will also speak of resources, instead of artifacts, to emphasise their various natures; but we go on using the term instrument instead of document, because of our special focus on EEBs.

In the subgroup we constituted within the GUPTEn project, we worked with five teachers who used MEP in their classes and described very precisely the way they used it, their scenarios in use of MEP (Gueudet 2006, Trouche and Guin 2006). We observed along the two years of the group's work evolutions of these scenarios. These evolutions led to stable uses and regularities in the teacher's activity witnessing the existence of an underlying scheme, outcome of the genesis process. We do not describe here these evolutions of scenarios; they are detailed in Bueno-Ravel and Gueudet (2008). Genesis processes comprise evolutions and stable periods, continuity and ruptures. We focus here on stable uses of sets of resources. Analyzing these uses means to situate them within other possible uses, in particular uses expected by the institution. In her study about spreadsheets, Haspekian (2005) identifies a gap between the way the secondary school institution expects the spreadsheet to be used and the way it is actually used by teachers. Things are different about the use of EEBs, which is not officially planned by the institution. For this reason we consider various possible uses, and identify within these uses the institution's influence. Because of the importance of institutional aspects in our study, we refer to the anthropological approach to didactics (Chevallard 1992). Thus we consider didactic tasks instead of classes of situations, and




analyze the genesis processes in terms of instrumented techniques (Lagrange 2000), which are here didactic techniques developed by the teacher. We detail this choice and its consequences in the next section.

The instrumental approach for teachers has similarities with other approaches used to study the teacher's activity in a technology-rich environment, in particular with Monaghan's use of the Saxe's model (Monaghan 2004). The central elements of the model are the teachers "emergent goals" which are connected with four parameters: activity structure, prior understandings, conventions and artifacts, and social interactions. The resources are artifacts, thus belong to the "conventions and artifact" parameter. Apart from this direct link, the other articulations are more intricate (we point such articulations in our analyzes). For example, didactical types of tasks are linked both with the "activity structure", and with "emergent goals".

We think that the instrumental approach is likely to bring specific results about the teachers' in class and out of class activity for at least two reasons. It devotes a particular attention to institutional constraints; and it takes into account the transformations of the artifacts which happen through their appropriation by teachers. These reasons will appear more clearly in the examples provided. However, the articulations between the instrumental approach and Saxe's model certainly require a further theoretical work.

### 3. Didactic tasks and techniques

In the anthropological approach to didactics, Chevallard (2002) considers that any human activity consists in carrying through a given task  $t$ , belonging to a given type of task  $T$ , with a given technique  $\blacktriangle$ ; the discourse used to explain and justify this technique is a technology  $\neg$ , grounded in a theory  $\rightarrow$ . The whole set of four elements  $[T, \blacktriangle, \neg, \rightarrow]$  is called a praxeology . The anthropological approach has been mostly used up to now, and especially within the instrumental approach, to study mathematical tasks, and thus mathematical praxeologies, often called mathematical organizations. But Chevallard approach ranges over more general human activities, and the teachers' activity in particular (however the theory level is not often studied in the analysis of the teachers' activity; we will not investigate it here). For example, "introducing the sine in a right-angled triangle in a grade 9 class" is a didactical type of task. It entails sub-types of tasks like "choosing the succession of mathematical organizations

*to be presented*". For example, if cosine is already known, the teacher can choose just to recall cosine, and use an analogy to introduce sine. This depends strongly on the official curriculum. In such a case the technique entails "*use cosine and analogies*", but certainly also other aspects which can only be determined through a thorough observation. The technology comprises "*use analogies with old knowledge to introduce new notions*", "*link the different trigonometric ratios*". Another sub-type of task of a different nature is: "*choosing a didactical organization for the introduction of the sine*". This means deciding: how much time will be devoted to this introduction, will it be a course by the teacher, or exercises followed by a course, will the student receive a prepared sheet...

We always consider "*choosing a didactical organization*" together with "*choosing a mathematical organization*". These two types of tasks are indeed strongly interwoven. Chevallard (2002) speaks of the mathematics and didactics co-determination, and considers that the whole teacher's activity can be described in terms of these two types of tasks, which refine into sub-tasks as more precise mathematical (and thus didactical) organizations are considered.

In our study, we retain three main types of tasks for the teacher: "*Choosing sub-groups of students*", "*Choosing mathematical organizations*" (for each sub-group), and "*Choosing didactical organizations*" (for each subgroup and each mathematical organization). The choice of sub-groups can be considered as a sub-task of "*Choosing didactical organizations*". We retain it here as a specific task, because the analysis of the scenarios chosen by the teachers of our group revealed that the EEB was massively used to organize and manage different teachings for different sub-groups in the class (which is usually not very frequent at secondary school in France).

### **"Choosing sub-groups of students"**

This type of task can be considered as a sub-type of the task "managing the class heterogeneity". The choice of the teacher can be "doing the same for the whole class"; in this case the teacher does not encounter this type of task. But our observations of scenarios indicate that teachers often constitute sub-groups when they organize a teaching with an EEB. Most EEBs, MEP in particular, offer indeed interesting possibilities to design a suitable teaching to students' particular needs (see appendix 1) Then the teacher must choose the number of groups he/she will constitute, and their composition. Such a choice can involve several resources: the record of the students' marks from the beginning of the year, the

teacher's knowledge of the students' affinities; an EEB can also intervene, if the teacher refers to marks obtained on the EEB.

### **“Choosing the mathematical organizations for a given course (and a given subgroup)”**

When preparing a course, the teacher has to define exactly which mathematical contents (and thus which mathematical organizations) will be taught. Several resources may intervene in this choice: the official curriculum, the class textbook, and also an EEB like MEP. Moreover, teachers can be influenced by the EEB when designing exercises' sheets for students.

### **“Choosing a didactical organization (for a given mathematical content and a chosen subgroup)”**

Choice of mathematical contents and supports and choice of didactical organizations are articulated. For a sequence comprising different sessions a teacher has to plan these sessions' nature: revision, discovery, synthesis, training, assessment, support sessions etc. These sessions need to be articulated within the sequence. It is also necessary to plan the activity structure for each session, and to decide what will be done during the class or at home. An EEB can be used for homework, if the students have out of class Internet access.

Moreover, about these three tasks we point the following elements:

- Each of these tasks is present both in the teacher's in class and out of class activity. For example, some of the mathematical organizations presented in a session are planned. But some are chosen in class, according to observed difficulties, or if more time than planned is available. The genesis processes happen in the whole teachers' professional activity, and an important part of this activity happens out of class.
- These three tasks can be considered at different time scales, from the whole school year to a short class episode. We have chosen here to focus on two time scales: the scale of the sequence and the scale of the session. We intend this way to make precise analyzes, without missing the wholeness of teachers' practices.

For each type of task, we try to determine the associated didactical techniques. About mathematical techniques, Artigue (2002) explains:

“A technique is a manner of solving a task, and as one goes beyond the body of routine tasks for a given institution, each technique is a complex assembly of reasoning and routine work. I would like to stress that techniques are most often perceived and evaluated in terms of pragmatic value, that is to say, by focusing on

their productive potential (efficiency, cost, field of validity). But they have also an epistemic value, as they contribute to the understanding of the objects they involve, and thus techniques are a source of questions about mathematical knowledge.”(P.248)

Similar statements can be formulated about the way teachers perform didactical tasks (we consider here only teachers with several years of professional experience; specific phenomena probably happen with novice teachers). It entails routine work and reasoning to adapt to what happens in class, to unexpected or new conditions... Pragmatic value of didactical techniques can be observed through the progress of the teaching, the evolution of the students' knowledge... And they certainly also have an epistemic value; they play an important part in the teacher's professional development. For example, about the “*Introducing the sine in a right-angled triangle*” didactical type of task, a teacher can develop a technique starting by “*recall the definition and properties of cosine*”. This can be done through a traditional course, but can also be instrumented by an EEB, proposing exercises involving cosine. With the EEB, these exercises can even be proposed out of class; the teacher can follow the students' scores, and then decide to recall cosine only for some students, or ask some students to expose cosine properties for the whole class, according to what they saw on the EEB, on the textbook... In this case the EEB, and the technique instrumented by it, leads to a professional evolution: start the introduction of a new notion by a diagnostic about related previous knowledge.

Constituting, a priori, an inventory of all the potential teachers' techniques is impossible. We identify teachers' techniques through the observation of regularities in ways of accomplishing tasks of the same type in several contexts. We consistently observe the resources involved in the task and in particular if the technique is instrumented by MEP and how. Examples of such analyzes are developed in sections 4.2 and 4.3.

In addition, as we describe teacher's activities from an institutional point of view, we also take into account the system of conditions and constraints teachers are subject to. These conditions can be generic, or more specific to a given mathematical content. For example, at a general scale, teachers' choices are made within a system of conditions and constraints they cannot modify such as the imposed length of a session. This concept of system of conditions and constraints classified from a generic level to a specific level has been introduced as “an interpretative framework of the various subjections to institutions” (Wozniak 2008).

We already referred above to Ruthven (2008). Our approach is strongly connected with Ruthven's work, which is not surprising because we share the same goal of developing "a better understanding of the appropriation of new technologies by classroom teachers". Ruthven introduces "*the structuring context of the classroom practice*", which comprises five key structuring features of the classroom practice with ICT: *Working environment*, *Resource system*, *Activity format*, *Curriculum script* (a "loosely ordered model of relevant goals and actions which serves to guide the teaching of a topic"), and *Time economy*.

The *resources system* is obviously at the core of our approach. *Activity format* is close from the *activity structure* in Saxe's model, and from Monaghan's use of it. As said above, it is taken into account in our approach through the concept of didactical task. The conditions and constraints systems we consider are linked to the *working environment*, but also to the *time economy*. The *working environment* (computer facilities, change of room location...) corresponds to generic conditions and constraints, while time economy has generic (a mathematics lesson is 55 minutes long) and specific aspects (the introduction of sine can not require more than two hours).

The articulation with the *curriculum script* is more complex. The aspects pointed by Ruthven in his use of this concept will appear in our work within both the didactical techniques and the associated technologies. We consider here as technologies (and thus didactical technologies) the discourse of the teachers justifying his/her choices.

Before presenting the examples, let us briefly recall here the principles we retain for our analyses.

We select a given mathematical topic. We observe the corresponding teaching in class and its preparation out of class as much as possible. Concretely, for the data collection, we observe and film the sessions. We also interview the teacher before and after each EEB session. All the teacher and students' materials are collected during the sequence. We determine the conditions and constraints influencing the teacher's choices, from a general to a more specific level. For each of the types of tasks: choosing the subgroups, choosing the mathematical organization, choosing the didactical organization, we observe how the teacher does it, which are the resources involved in it, and in particular which is the role of the EEB. This leads to identify didactical techniques, instrumented or not by the EEB. The teacher can justify or not the reasons for these choices by a technological discourse; we do not focus on such a discourse anyway; we rather try to understand the reasons for the choices through the analysis of the institutional conditions and constraints.

## 4. A case study: an instrumented trigonometry course

### 4.1 Presentation of the teaching

#### *French curriculum of trigonometry*

In France, secondary education is organized in two parts: the “collège” which is compulsory from grade 6 to grade 9, and the “Lycée” from grade 10 to grade 12. At the end of grade 9, students have to take an exam, the DNB (which holds for, in French: Secondary National Degree). The duration of the weekly work in mathematics is around 4h.

In grade 9, the official mathematical curriculum comprises four main areas: 1) data proceeding and functions, 2) number and operations, 3) geometry and 4) measurement. In this curriculum, trigonometry is not related with functions. It belongs to a theme of the geometry area entitled “right-angled triangle, trigonometric relations”. The contents of trigonometry in grade 9 are detailed in appendix 1. They include the introduction of the two ratios *sin* and *tan* and the formulas  $\cos^2 + \sin^2 = 1$  and  $\tan = \sin / \cos$ . The trigonometric ratio *cos* is known since grade 8. This order of introduction of the three ratios is a result of successive reforms of the “collège” curriculum. As the trigonometric ratios are introduced in a right-angled triangle, the decision to spread this teaching over two years has no mathematical reason.

In March 2007, we have observed two trigonometry teachings in grade 9. The following analysis is focused on one of two sequences observed. It had been realized by a teacher we named Carmen. Carmen chose to use MEP for this teaching. She started using MEP four years ago. She is also registered as a MEP's user. Carmen has a strong degree of integration of MEP (Assude 2008), MEP intervenes as an instrument for Carmen in several types of tasks; we will see it in what follows. She has dedicated a sequence of 9 sessions for her trigonometry teaching. Before analyzing her choices, we present the general conditions and constraints she is subject to when designing this teaching.

### ***General conditions and constraints for Carmen's trigonometry teaching***

The setting up of a trigonometry sequence incorporating MEP is subject to several kinds of conditions and constraints. First of all, some of these constraints depend on the scholar institution: between 7 to 10 hours is the allocated time for a trigonometry sequence in grade 9 (this is thus linked with time economy). Furthermore, some social conditions and constraints are characteristic of Carmen's school. Indeed, the students there are mostly from a poor socio economic background. Teachers notice a lot of unacceptable behaviors and they cannot imagine letting some students, even a few, working on their own on a paper and pencil exercise.

For our study, it is also necessary to take into account the specificities of the use of ICT in classroom. The working environment defined by Ruthven (2008) and the available ICT facilities strongly constrain the possible uses of MEP. Carmen's school is equipped with two computer laboratories (about fifteen computers connected to the internet in each room) and with a video projector for all the school. There are also several tables in the middle of each computer laboratory. Thanks to this spatial organization, it is possible to have half of the class working individually on computers while the other half is working on paper and pencil in the same room. However the constraint concerning students' behavior in Carmen's school makes this organization difficult to manage for the teachers. For practical purposes, it is essential for a teacher working with two half-classes to have an access to a room contiguous to the computer laboratory. The teacher can this way separate the groups and simultaneously oversee their work.

### ***Carmen's choices for her trigonometry sequence***

Carmen chooses to dedicate 9 sessions (S1 to S9) to trigonometry. MEP is used in 5 sessions: S1, S2, S4, S5 and S8. Initially, Carmen had planned to use MEP in S6 but a network breakdown made it impossible. During S6, S8 and S9 (assessment session), Carmen divides the students in two groups she calls the "High Level" group and the "Low Level" group. She proposes a different work to each group of students. This mathematical and didactical organization is quite rare in France.

We will not detail here Carmen's choices of mathematical content. Some of these choices will be analyzed in the following section. A detailed presentation of the sequence is provided in

appendix 1, and compared with the class textbook, and with MEP's content (the use of MEP requires first and foremost an important choice of contents for a teacher. MEP's trigonometry chapter comprises 335 questions).

The full sequence is presented in appendix 1, in relation with the official curriculum and the content of some textbooks.

### *Examples of didactic tasks and techniques at the sequence level*

We do not intend to present a complete description and analysis of all the techniques involved, but to display our approach, and the observations it permits, through significant examples. We will focus for the sequence level on the type of tasks "*choosing sub-groups of students*", and associated subtasks. This type of task is very important in Carmen's case. Her class comprises indeed 24 students, 5 of them with a very low level in mathematics, 3 of them showing on the opposite good mathematical abilities, and the others in between. This situation is one major reason for the use of MEP by Carmen. She develops several kinds of techniques to manage this heterogeneity, involving several sets of resources, some including MEP and others not.

#### *Choice of the sub-groups*

Even before the beginning of the trigonometry teaching, Carmen intended to split the class into two sub-groups at least for a part of the teaching. Two main reasons governed this choice: the low level in mathematics of a significant number of students and the organizational constraints of her working environment. It is naturally difficult for the teacher to intervene with more than two sub-groups. And the size of each sub-group is constrained by the number of computers in the computer laboratory because Carmen planned for each sub-group a MEP session, with only individual students on the computers. Thus at the beginning of the teaching, the decision of working at some point with two half-classes was taken; and Carmen needed to decide the composition of the two halves. She used MEP for this decision. The two halves were determined after an individual work on MEP in sessions 4 and 5. During these sessions, the MEP's exercises proposed were direct applications of the course. Carmen decided *a priori* to create a "Low Level" group containing all the students who only get 3 out of 10 as maximum mark for at least one of the exercises. She used for this purpose the session's sheet, and her personal paper session's sheet (which registers the best mark reached by each student if several tries on the same exercise were made; see section 2). After the S4 and S5 sessions, Carmen filled the table in her session's sheet. She retained the 7 students



with one mark lower than 3 for one the exercises. Then she complemented the group to reach a number of 12 students, choosing amongst the ones with a mark 4 or 5. Carmen develops a technique instrumented by MEP to decide the composition of the sub-groups. This composition could have been decided even before the session, according to an average mark in mathematics. Carmen considered trigonometry as a new topic, and did not want to introduce for it a splitting of the class based on previous results. And she chose to organize her diagnostic on MEP, instead of proposing exercises on a paper to avoid a long correction. MEP session's sheet allowed her to compose the two groups within one day, starting with the two half-classes the day after session 5. This technique can be described as “making an introductory course, then direct application exercises and constitute, with MEP help, a sub-group with the students who don't succeed”. During Carmen four years of MEP use, we observed this technique several times, not only in grade 9 for trigonometry teaching. Carmen can be considered as a MEP “expert” and she has developed stable uses of MEP.

#### *Choice of the common mathematical organization*

Carmen has then to determine the mathematical organizations dedicated to the whole class, the “Low Level” group and to the “High Level” group during the entire sequence. To understand Carmen's choices, it is essential to notice that Grade 9 is in France an important step for further orientation. Indeed, some students will attend a “general grade 10”, while others will turn towards more professional studies. Carmen explains her technique for choosing the common mathematical organization by a discourse grounded on the following technological argument: “the students who will turn to professional sections do not need...”. This argument applies, according to her, to: *Particular values of sin and cos*; formulas:  $\cos^2 + \sin^2 = 1$  and  $\tan = \sin/\cos$ ; *Calculations in complex configurations* and *Discovery of the unit circle* (for details see lines *g*, *h* and *i* of table 1 in Appendix 1). But she does not use the official curriculum of the professional sections involved; and some of these, like for example mechanics sections curriculum, include indeed the mathematical contents she removes this way. The main resource involved in this technique is the official grade 9 curriculum. While the textbooks always propose tasks which are not explicitly mentioned in the curriculum, for the common mathematical organization, Carmen's technique leads to systematically remove these tasks. Moreover, she even suppresses a part of the official curriculum: the formulas  $\cos^2 + \sin^2 = 1$  and  $\tan = \sin/\cos$ , because she considers these formula involving two trigonometric ratios as too difficult for the students with a low mathematical level. This choice corresponds to a personal belief or experience. It is certainly not the one expected by the institution, at

least about the choice of letting down some contents of the official curriculum. We also notice that the resources involved here do not include MEP, in spite of the presence, in MEP, of an exercises' set entitled "go further" clearly indicated as being outside of the common mathematical organization. Here, Carmen's technique can be described as "suppressing the mathematical contents which are too difficult for low level students". It seems here that the technological discourse which justifies Carmen's technique can be described as "mathematical contents which are considered as too difficult for "Low Level" students, according to the teacher's experience, must be suppressed". This technique allows Carmen to spend more time on the central content of her sequence with "Low Level" students.

#### *Choice of the didactical organizations for each subgroup*

The two half classes worked on different contents during the sessions 6, 8 and 9 (the last one being the final assessment). The "Low Level" group only worked on exercises corresponding to the common mathematical organization decided from the beginning of the teaching. Carmen planned to make them work on MEP for session 6, on paper for session 8. But a network breakdown happened during session 6, and they only worked on paper. The "High Level" group worked on exercises chosen in the class textbook during session 6, and on MEP during session 8. The exercises proposed to the "High Level" group were chosen outside of the common mathematical organization. We want to emphasize here the didactical organizations retained during sessions 6 and 8. During both sessions, Carmen remained most of the time with the "Low Level" group, providing help, watching the work being done. This was more difficult to manage during session 6, because the "High Level" group called for Carmen several times while working on the textbook. The "High Level" students were organized in three subgroups of four students each. They were discussing aloud, and sometimes disturbing the rest of the class. There was no comparable disturbance during session 8. The "High Level" students worked individually on MEP. Carmen prepared for them a MEP session with many exercises. Some students encountered difficulties, but all these difficulties were overcome with MEP feedback and help. The didactical technique consists here of keeping a "High Level" group busy on contents outside of the common mathematical organization in order to be more present with low level students. We observed this technique in many the MEP's sequences designed by Carmen. In Carmen's trigonometry teaching, this technique is instrumented one time by the class textbook, and one time by MEP. But MEP turns out to be more efficient, thanks to the help and feedback it proposes.

### 4.3 Didactic tasks and techniques in a particular session

We will focus on the analysis of session 2, which aims at introducing the sine and tangent ratios.

#### *Carmen's choices for session 2*

Carmen used MEP's exercise "discovery of sine" and a video projector to introduce sine and tangent to the whole class. The analysis of Carmen's scenarios has proven that this use of MEP had become typical. Indeed, each time she starts her lesson on a new content she uses MEP and a video projector with the whole class.

At first sight, it seems that Carmen's use of ICT in this session does not disrupt a routine structure. Indeed, there is no radical change in the activity cycle (Monaghan 2004) of her 'discovery and lesson' session. Students do not investigate by themselves with MEP or other ICT resources before Carmen makes her lesson. They have to look at the projected MEP's screen and answer Carmen's questions concerning sine. However, this use of MEP involves important changes in the working environment and the resource system used by Carmen.

Carmen has changed the spatial organization of her class. MEP's screen is projected on the whiteboard, in front of the students. The computer is placed on the left side of the class. Carmen stays next to it, thus shifted from her usual central position. She writes nevertheless sometimes on the whiteboard, over or beside the projected MEP's screen.

She has created a sheet for the students inspired by the MEP's exercise. But the way this sheet should be used with the students raised a real didactical problem for Carmen. She was indeed wondering whether giving the sheet at the beginning of the session (but the students will have all the reasoning stages under their eyes) or waiting till the end of the MEP exercise before distributing the sheet (but this necessitates to go once more through the whole MEP's exercise). Eventually, she decided that the students will have to fill in the sheet as she gets along the questions. Two main constraints can explain her didactical choice: the time needed to do twice in a row a MEP exercise and MEP technical feature that does not allow an access to a particular question of an exercise (an exercise has to be started by question 1). The design and the use of this paper and pencil resource are clearly instrumented by MEP. Carmen is creating for this session a coherent instrument from ICT and paper and pencil resources.

For sake of brevity, we won't detail here the analysis of the whole session. We will focus on questions 4 and 5 of the MEP's exercise 'discovery of sine'.

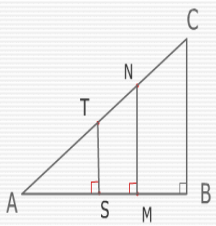
*Proving that the ratio does not depend on the sides lengths: MEP's choices, classroom discourse*

We present below a copy of MEP's screens for questions 4 and 5, with the mathematical task at stake and the expected mathematical technique to achieve it.

Exercice n°1 : Découverte

Question N°4 : Valider

Complète en utilisant les points M et N :

$$\frac{BC}{AC} = \frac{ST}{AT} = \frac{\quad}{\quad}$$


Translation of question 4:

Use points M and N to fill in:

Task: « Complete a length ratio equality (3<sup>rd</sup> ratio) corresponding to the proportional segments theorem for three parallel lines »

Tech: « Identify on the figure the segments « playing » the same role and transfer the points ».

**Figure 2.** Copies of MEP's screens; questions 4 and 5 of the exercise 'discovery of sine'.

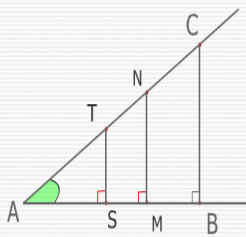
We interviewed (by e-mail) the designer of this MEP exercise. This interview allows us to understand and analyze the mathematical and didactical choices underlying the design of this exercise.

This exercise has been created in order to be used with a strong scaffolding of the teacher or with a video projector, like Carmen has chosen to use it. The designer of this MEP's exercise expect a quick answer to question 4 and a long time for the manipulation and this observation in question 5.

Exercice n°1 : Découverte

Question N°5 : ? Valider

On cherche à obtenir 0,45 comme valeur commune des trois rapports. Essaie de déplacer les trois segments [MN] ou [RS] ou [BC] et observe si la valeur évolue. Tu pourras ensuite essayer de faire varier la valeur de l'angle aigu en A.

$$\frac{BC}{AC} = \frac{ST}{AT} = \frac{MN}{AN} = 0,53 \dots$$


Translation of question 5:

We try to reach 0.45 as a common value of the three ratios. Try to move the three segments [MN] or [RS] or [BC] and observe if the value changes. Then, you will be able to try to change the value of the acute angle A.

Task: « Observe if the value of equal length ratios changes when moving the parallels segments »

Tech: « Move the segments' extremities on the half-lines by using the computer mouse ».

The fact that the value of the ratio will not change whatever the length of the segments is established in question 3: question 3 asks indeed to prove the equality  $BC/AC=ST/AT$  with the proportional segments theorem. Thus the sine could be defined in question 3 or 4. But the choice of the designer was different. A third ratio is introduced in question 4. Moreover, question 5 starts with the handling of the segments  $[BC]$  or  $[ST]$ , and one more observation of the invariance of the ratio. The succession of questions 3, 4 and 5 is not guided by mathematical reasons but didactical ones. According to the designer of this exercise, the role of the third ratio's introduction is to be sure that students memorize this new ratio (corresponding to opposite length/hypotenuse). This questions the status of a generic case in relation to an accumulation of examples.

Let us provide elements of analysis of Carmen's mathematical and didactical choices during the projection of questions 4 and 5. We focus on two specific tasks: "*introduction of new contents*" (subtask of "*choice of the mathematical organization*"), and "*time management*" (subtask of "*choice of the didactical organization*"). The analysis corresponds to the following transcribed extract.

1. S: MN above AN?
2. C: MN above AN right? OK. So actually, here we have MN above AN. In other words, whatever the triangle is, whatever the length of the sides of the triangle is; we could imagine like that as many triangles as we want, of the moment they are right-angled, well then the ratio... provided that, what is in common, really what do they have in common these triangles?
3. S: they are right-angled.
4. C: they are right-angled, and they have...
5. S: a side...they have a common side, because here... they have the same (vertex).
6. C: they have the same (vertex), do they have the same sides?
7. S: No no... angles.
8. C: Do they have the same angles? Yes, angle A is the same for everybody [Carmen shows angle A on the projected MEP's screen]. Are you OK? And these angles here, Are they equals? Well yes, they are.... OK it is not the length of the triangles which changes, whatever the length of the sides is, le ratio here remains the same.
9. C: So we continue. [Beginning of question 5] So we try to reach 0,44 as a common value for the three ratios. We have three equal ratios here; in this case, we have 0,53. The question asks to try to move the segments, to see if it depends on the segments. So I hope I'll move them correctly... If I do that with M, OK. Does it change?
10. S: No...

11. C: No, Well come on...S...no either ... T ... no either. In other words, if... euh... the ratio in question, it does not depend, it's what I've already said just before, on the length of the sides. So on the ... on the triangle itself. O.K. [Carmen presses the "submit" button to reach the second part of question 5]

**Figure 3.** Extract of session 2: a student is asked to provide the answer to question 4.

*“Choice of the mathematical organization”: Carmen’s techniques to introduce new contents.*

During this session, Carmen often adds some mathematical technological explanations to justify the students’ answers to MEP’s questions. She also formulates some properties which remain implicit in MEP questions. For example, in question 4, she asks students what will be the ratio corresponding to the triangle ANM. Once students give the answer (line 1), the MEP’s task is finished. Carmen fills in the answer but does not press the “submit” button. She takes advantage of this question to try to make explicit for the students the underlying property of this question: “in this configuration, if the angles of the triangles are equal, the ratio does not change whatever the length of the sides of the triangles is.” (Lines 2 to 8). We can identify here a didactical instrumented technique concerning the division of the responsibilities between MEP and Carmen for the introduction of news mathematical contents. Indeed, MEP proposes the tasks and techniques of the mathematical organization taught while Carmen’s contribution is to supply with mathematical technological elements. In addition, from line 2 to line 8, we can identify Carmen’s technique to organize her responsibilities and students’ ones towards the mathematical contents when she elaborates the technological dimension of the mathematical organization she puts in place: she questions the class. The students’ role is thus to answer Carmen’s tightly guided questions, with the help of the projected MEP’s screens. This technique does not seem to be instrumented by MEP. Asking the class tightly guided questions to make the students formulating a property is a regular didactical technique.

*“Choice of the didactical organization”: Carmen’s time management techniques.*

As we have already mentioned it, it is not necessary to formulate the property: “in this configuration, if the angles of the triangles are equal, the ratio does not change whatever the length of the sides of the triangles is.” to answer MEP’s question 4. Moreover, question 5 allows the investigation of this property through the move of the segments’ extremities on the half-lines and the observation of the value of the ratio. MEP entails a particular succession of mathematical organizations; articulating for example old and new contents. When using MEP, the change of mathematical organization coincides with the change of questions. So it is

instrumented by the fact of pressing the “submit” button. In the same way as MEP entails a particular succession of mathematical organizations, it also entails a particular succession of didactical organizations. Indeed, the exploration of the property at stake starts in question 4 and is the core of question 5. But Carmen makes in question 4 a synthesis on a property which lies at the heart of the “research” activity of question 5. So when she starts with question 5, she has to go back to a “research” activity about this property whereas it would have been natural for her to carry on the lesson (by answering the second part of question 5: variation of the value of acute angle A) or to do some exercises related to this property. Carmen’s succession of mathematical and didactical organizations is different from MEP’s one. So, the structure and the design conceptions of the exercise lead Carmen to guide quickly students’ observations in question 5. As Carmen moves the points with the computer mouse, she tells the students what to look at: the value of the ratio. She clearly accelerates the rhythm of her course during the first part of question 5. This can be interpreted as an example of an instrumentation phenomenon. Indeed, as the content of question 5 is not new in relation to the mathematical organization she has built at question 4, there is no need to spend too much time on it.

Moreover, Carmen has decided to be in charge on the computer mouse. This means here that she is in charge of the decision to press the “submit” button and of the manipulation of the segment. Thus she is free to add some information concerning a MEP exercise: a student would have press the “submit” button after the answer to the MEP’s question (line 1) to validate it. This also gives her the possibility to accelerate the rhythm of the lesson when necessary (line 11). This choice can be interpreted as an instrumented technique of time management.

## Conclusion

We studied in this article the consequences of the integration of an EEB in a mathematics teaching. We have shown that using an EEB influences the teacher’s choices, his/her activity, in class and outside the class.

Teachers' activity is likely to be influenced by any kind of resource. However, we observed some specific consequences of the features of an EEB. We especially emphasised its

intervention for differentiation purposes; the teacher's choice to draw on an EEB for differentiation is a direct consequence of these features.

However, the scope of the theoretical principles exposed here goes behind the specific case of EEBs, and even of online resources.

We consider, as Ruthven (2008) does that a given resource is part of a wider resources system, and that several intertwined features of the classroom context must be considered to study the integration of a resource. In our theoretical approach, we devote a specific attention to institutional features.

Furthermore, we have chosen to propose a generalisation of the instrumental approach (Guin et al. 2005) for the study of any kind of teachers' resources. Adopting this perspective leads to identify the influence of the resource on the teacher's activity as an instrumentation phenomenon, while the elaboration of a personal construct by the teacher drawing on the resource is interpreted as instrumentalization.

The instrumental approach is grounded on concepts of cognitive ergonomy (Rabardel 1995), and of anthropological theory (Chevallard 1992). We proposed an evolution of this approach, aiming at illuminating genesis processes for teachers, and incorporating recent developments of anthropological theory, in terms of institutional conditions and constraints in particular. Gueudet and Trouche (to appear) developed a documentary approach, encompassing the use of any kind of resources by teachers for their out of class work. In this approach the genesis processes are studied in terms of schemes: rules of actions and operational invariants in particular.

According to Drijvers et al. (to appear): "One of the future challenges for the further development of instrumentation theory is to fine-tune the balance- including both the similarities and the differences- between the cognitive ergonomics frame and the anthropological theory of didactics". We fully agree with this claim; balancing both perspectives is certainly also necessary to study teachers' genesis.

## Bibliography

Artigue, M. (2002). Learning mathematics in a CAS environment: the genesis of a reflection about instrumentation and the dialectics between technical and conceptual work, *International Journal of Computers for Mathematical Learning* 7(3), 245-274.



Bueno-Ravel, L., & Gueudet, G. (2009). Online resources in mathematics : Teachers' geneses and didactical techniques. *International Journal of Computers for Mathematical Learning*, 14(1), 1-20. DOI 10.1007/s10758-009-9143-0

Assude, T. (2008). Teacher's practices and degree of ICT integration, in Pitta-Pantazi, D. and Philippou, G. *Proceedings of the fifth congress of the European Society for Research in Mathematics Education*, CERME 5, Larnaca, Chypre.

Bueno-Ravel, L., Gueudet, G. (2008). Online resources in mathematics: teacher's genesis of use, in Pitta-Pantazi, D. and Philippou, G. *Proceedings of the fifth congress of the European Society for Research in Mathematics Education*, CERME 5, Larnaca, Chypre.

Cazes, C., Gueudet, G., Hersant, M., Vandebrouck, F. (2007). Using e-Exercise Bases in mathematics: case studies at university, *International Journal of Computers for Mathematical Learning* 11(3), 327-350.

Chevallard, Y. (1992). Concepts fondamentaux de la didactique : perspectives apportées par une approche anthropologique. *Recherches en Didactique des Mathématiques* 12(1), 77-111, La Pensée Sauvage, Grenoble.

Chevallard, Y. (2002). Ecologie et régulation, in J.-L. Dorier, M. Artaud, M. Artigue, R. Berthelot, R. Floris (dir.) *Actes de la XIème Ecole d'été de didactique des mathématiques, Corps* (pp.41-56). Grenoble : La Pensée Sauvage.

Gueudet, G. (2006). Scénarios d'usage de base d'exercices en ligne, in H. Godinet, J.-P. Pernin (dir.), *Scénariser l'enseignement et l'apprentissage : une nouvelle compétence pour le praticien ?* (pp. 39-44). Lyon : INRP. En ligne : <http://www.inrp.fr/publications/edition-electronique/> Accessed April 15, 2008.

Gueudet, G., Trouche, L. (to appear). Vers de nouveaux systèmes documentaires des professeurs de mathématiques ? (*Towards new mathematics teachers' documentary systems?*) In Bloch, I. et Conne, F. (eds) *Actes de la quatorzième école d'été de didactique des mathématiques*, La Pensée Sauvage, Grenoble.

Guin, D., Ruthven, K., Trouche, L. (eds.) (2005). *The didactical challenge of symbolic calculators: turning a computational device into a mathematical instrument*, Springer, New York.

Haspekian, M. (2005). An "instrumental approach" to study the integration of a computer tool into mathematics teaching: the case of spreadsheets. *International Journal of Computers for Mathematical Learning* 10(2), 109-141.

Lagrange, J.-B. (2000). L'intégration d'instruments informatiques dans l'enseignement : une approche par les techniques. *Educational Studies in Mathematics* 43(1), 1-30.

Lagrange, J.-B., Bessières, D., Blanchard, M., Loisy, C., Vandebrouck, F. (dir.) (2007). *Genèses d'usages professionnels des technologies chez les enseignants*, rapport intermédiaire de l'ACI GUPTEN, <http://gupten.free.fr> Accessed April 15, 2008.

Monaghan, J. (2004). 'Teachers' activities in technology-based mathematics', *International Journal of Computers for Mathematical Learning* 9(3), 327-357.

Rabardel, P. (1995). *Les hommes et les technologies, approche cognitive des instruments contemporains*. Paris : Armand Colin. (english version at [http://ergoserv.psy.univ-paris8.fr/Site/default.asp?Act\\_group=1](http://ergoserv.psy.univ-paris8.fr/Site/default.asp?Act_group=1) Accessed April 15, 2008.

Remillard, J.T. (2005). Examining key concepts in research on teachers' use of mathematics curricula, *Review of Educational Research* 75(2), 211-246.

Ruthven, K., Henessy, S., Deane, R. (2005). Incorporating Internet resources into classroom practice: pedagogical perspectives and strategies of secondary-school subject teachers, *Computers & Education* 44, 1-34.

Bueno-Ravel, L., & Gueudet, G. (2009). Online resources in mathematics : Teachers' geneses and didactical techniques. *International Journal of Computers for Mathematical Learning*, 14(1), 1-20. DOI 10.1007/s10758-009-9143-0

Ruthven, K. (2008). Teachers, technologies and the structures of schooling, in Pitta-Pantazi, D. and Philippou, G. *Proceedings of the fifth congress of the European Society for Research in Mathematics Education*, CERME 5, Larnaca, Chypre.

Trouche, L., Guin, D. (2006). Des scénarios pour et par les usages, in H. Godinet, J.-P. Pernin (dir.), *Scénariser l'enseignement et l'apprentissage : une nouvelle compétence pour le praticien ?* (pp. 77-82). Lyon : INRP. En ligne : <http://www.inrp.fr/publications/edition-electronique/> Accessed April 15, 2008

Vergnaud, G. (1998). Toward a cognitive theory of practice, in A. Sierpiska & J. Kilpatrick (Eds.), *Mathematics education as a research domain: a search for identity*, Dordrecht: Kluwer Academic Publisher, pp.227-241.

Wozniak, F. (2008). Conditions and constraints in the teaching of statistics: the scale of levels of determination, in Pitta-Pantazi, D. and Philippou, G. *Proceedings of the fifth congress of the European Society for Research in Mathematics Education*, CERME 5, Larnaca, Chypre.

## APPENDIX 1

Carmen's trigonometry sequence. The second column presents MEP's contents ; the third column of the table presents Carmen's choices, corresponding to each entry of the curriculum.

	Curriculum and textbooks	MEP	Carmen's organization
a	Right-angled triangles' properties and associated vocabulary. Cosine (pre requisite)	Chapter "cos" in grade 8. In grade 9: "for a good start" series.	<b>S1:</b> Work in pairs with MEP. Grade 9 exercises: "cosines" and "(configurations)". Write on a paper the solutions of 8 MEP questions.
b	Sine and tangent definition	Two exercises named "discovery": one for sine and one for tangent.	<b>S2:</b> Whole class, video projector. Carmen leans on MEP exercise "discovery" from the series "sine". The exercise is transformed to introduce also tangent. Students have a sheet built from MEP to fill in.
c	Calculations of cos, sin, tan for a given angle. Calculation of an angle with a calculator for a given value of cos, sin or tan	Three exercises on the use of calculator: sin, tan and synthesis.	<b>S3:</b> Whole class, paper and pencil work, calculator. Assessed in <b>S9</b> .
d	Simple uses, direct calculations of cos, sin, tan; writing the appropriate ratios in a right-angle triangle. Choose the appropriate formula: cos, sin, tan	Simple calculations of sin, tan and synthesis.  A synthesis series named "Sin, cos or tan?"	Exercises' sheet for homework distributed at the end of <b>S2</b> . <b>S4/S5:</b> two half-class (split according to the alphabetical order). Simple exercises on MEP (synthesis) and on paper. Exercises' sheet for homework distributed at the end of <b>S5</b> . Assessed in <b>S9</b> for the " <u>Low Level</u> " group.
e	Use of cos, sin, tan to determine a missing length, a missing angle.	Seven exercises on sin, tan and synthesis.	On MEP and paper in <b>S4/S5</b> . (Support) for the " <u>Low Level</u> " group in <b>S6</b> and <b>S8</b> on paper. Assessed for the whole class in <b>S9</b>
f	Concrete problems	One exercise	On MEP in <b>S4/S5</b> . For the " <u>High Level</u> " group in <b>S6</b> with paper and <b>S8</b> with MEP. For the " <u>Low Level</u> " group in <b>S8</b> with paper. Assessed in <b>S9</b> with detailed questions for the " <u>Low Level</u> " group, with no

			detailed questions for the " <u>High Level</u> " group.
g	Particular values of sin and cos. Formulas: $\cos^2 + \sin^2 = 1$ and $\tan = \sin/\cos$	One exercise on exact values  Three exercises on the formulas	<b>S7</b> : Whole class, paper and pencil work. For the " <u>High Level</u> " group on MEP in <b>S8</b> . Exact values assessed in <b>S9</b> for the " <u>High Level</u> " group.
h	Calculations in complex configurations.	One exercise.	For the " <u>High Level</u> " group on MEP in <b>S8</b> . Assessed in <b>S9</b> for the " <u>High Level</u> " group.
i	Complementary angles. Angles in 3-D. Discovery of the unit circle.	One exercise Two exercises Four exercises.	For the " <u>High Level</u> " group on MEP in <b>S8</b> (3 exercises).

Table 1. Trigonometry in grade 9: mathematical organization in the curriculum, in MEP and in Carmen's sequence.