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# Minimal parameterization of Fundamental Matrices using motion and camera properties.

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## Abstract

This paper addresses the optimal recovery of the displacement and projection parameters from uncalibrated monocular video sequences. We study the particular cases of camera and objects displacements and camera projection in order to extract an optimized parameterization of the problem of parameters recovery for each cases.

This work follows previous studies on particular cases of displacement, scene geometry and camera analysis and focuses on the particular forms of fundamental matrices. This paper introduces the idea of using not all particular cases as individual cases but grouping these cases into a tractable number of sets, using properties on fundamental matrices.

Some experiments were performed in order to demonstrate that if several models are correct, the model with the least parameters gives the best estimate, corresponding to the true case.

*Key words:* Fundamental Matrix, Particular displacement, Parameters estimation

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## 1 Introduction

This paper deals with video sequences taken by an uncalibrated camera in an unknown environment. Our interest is to estimate as many parameters as possible on the camera and objects motion and the camera projection using a strategy of hypothesis testing.

Many efforts have been made in the Computer Vision community for determining motion and camera parameters from video sequences. Relations between

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2D views exist, Faugeras (1993), as the fundamental matrix  $\mathbf{F}$ , but, in the general case, we cannot extract all the unknown parameters from this  $\mathbf{F}$  matrix. It is however possible in some particular situations.

This work follows previous work on particular cases of displacement, scene geometry and camera analysis Viéville and Lingrand (1999); Lingrand (1999, 2000). It focuses on the particular forms of fundamental matrices.

Several authors have already been interested in particular cases of projection: Aloimonos (1990), Dementhon and Davis (1989), Horaud et al. (97), Soatto and Perona (1995), Ma et al. (1999), Quan (1996), or displacement: Hartley (1994), de Agapito et al. (1998), Viéville (1994), Armstrong et al. (1994). Some of them consider several cases and compare each result, in order to automatically determine which case was performed.

We call by general case the situation where we don't know anything about motion or camera projection. A particular case is when we know (or make the hypothesis) that a parameter is null, constant or known, or related to other parameters. A particular case has fewer parameters and/or simpler equations than the general one.

The motivations for these studies are threefold:

- to eliminate singularities of general equations by considering each case that may conduct to singularity,
- to estimate the parameters with more robustness using a simplified model (an adapted model gives more accuracy than the general one as shown in Viéville and Lingrand (1999)), and
- to retrieve parameters that cannot be retrieved in the general case because we eliminate some unknowns that are meaningless in the particular case studied.

It is already known that the large number of particular cases prevent examining all the cases linearly. In this paper, we introduce a new way to deal with this amount of cases in three steps. (1) We eliminate, with some simple rules, some redundant cases and some physically impossible cases. (2) We divide the set of cases into two sets, each corresponding to homographic or fundamental relations. (3) We divide again the fundamental cases into sets corresponding to particular forms. We will provide details for each of these steps in the following sections.

## 2 Stereo framework

In this section, we present the stereo framework and the notations we will use in this paper.

**Rigid displacements:** We consider a rigid or piecewise rigid scene. A 3D-point  $\mathbf{M} = [X \ Y \ Z \ 1]^T$  is moving onto  $\mathbf{M}' = [X' \ Y' \ Z' \ 1]^T$  by a rotation  $\mathbf{R}$  followed by a translation  $\mathbf{t} = [t_0 \ t_1 \ t_2]^T$  :

$$\mathbf{M}' = \mathbf{R} \mathbf{M} + \mathbf{t}$$

A rotation matrix  $\mathbf{R}$  depends only on 3 parameters  $\mathbf{r} = [r_0 \ r_1 \ r_2]^T$  related to the rotation angle  $\theta$  and axis  $\mathbf{u}$  by:

$$\mathbf{r} = 2 \tan(\theta/2) \mathbf{u} \Leftrightarrow \theta = 2 \arctan(\|\mathbf{r}\|/2)$$

A rigid displacement us then parameterized by 6 parameters.

We note by  $\tilde{\mathbf{r}}$  the antisymmetric matrix representing the cross-product  $\mathbf{r} \wedge \cdot$ :

$$\forall \mathbf{x} \quad \tilde{\mathbf{r}} \mathbf{x} = \mathbf{r} \wedge \mathbf{x}$$

The rotation matrix  $\mathbf{R} = e^{\mathbf{r} \wedge \cdot} = e^{\tilde{\mathbf{r}}}$  can be developed as a rational Rodrigues formula, Rodrigues (1840) :

$$\mathbf{R} = \mathbf{I} + \begin{bmatrix} \tilde{\mathbf{r}} + \frac{1}{2}\tilde{\mathbf{r}}^2 \\ 1 + \frac{\mathbf{r}^T \cdot \mathbf{r}}{4} \end{bmatrix}$$

**Camera projection:** The most commonly camera model states that a 3D-point  $\mathbf{M} = [X \ Y \ Z \ 1]^T$  is projected with a perspective projection onto an image plane on a 2D-point  $\mathbf{m} = [u \ v \ 1]^T$ . In the reference frame attached to the camera, the projection equation is :

$$Z \mathbf{m} = \underbrace{\begin{pmatrix} \alpha_u & \gamma & u_0 & 0 \\ 0 & \alpha_v & v_0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}}_{\mathbf{A}} \mathbf{M} \quad (1)$$

where  $\alpha_u$  and  $\alpha_v$  represent the horizontal and vertical lengths,  $u_0$  and  $v_0$  correspond to the image of the optical center and  $\gamma$  is the skew factor. Those parameters are the intrinsic parameters and are collected in the projection matrix  $\mathbf{A}$ .

**Considering two frames:** Let  $I_1$  and  $I_2$  denote two images. In the general case, there exists a fundamental relation, Faugeras (1993), between points  $\mathbf{m}_2$  in  $I_2$  and points  $\mathbf{m}_1$  in  $I_1$  :

$$\mathbf{m}_2^T \mathbf{F} \mathbf{m}_1 = \mathbf{0}$$

where  $\mathbf{F}$  is called the fundamental matrix and is related to the intrinsic and extrinsic parameters by :

$$\mathbf{F} = (\tilde{\mathbf{A}_2} \mathbf{t}) \mathbf{A}_2 \mathbf{R} \mathbf{A}_1^{-1}$$

where  $\mathbf{A}_1$  and  $\mathbf{A}_2$  are the projection matrix for the first and second frames, see (1).

This kind of relationship vanishes if the displacement is a pure rotation or if the scene is planar. The relation between points is homographic :

$$\mathbf{m}_2 = \mathbf{H} \mathbf{m}_1$$

where  $\mathbf{H}$  is called the homographic matrix. Another study on homographic matrices can be found in Lingrand (2000).

### 3 Deriving all particular cases

In order to study all particular cases of cameras and objects displacements and camera projection, we will examine each particular value, considering each parameter at a time. A particular model is obtained by combining several particular values.

#### 3.1 Particular cases of intrinsic parameters

Authors generally make several hypotheses regarding intrinsic parameters. For example, the most general auto-calibration hypothesis states that the intrinsic parameters are constant. They can be known or unknown. However, usually, some parameters are constant while others are not.

- The principal point of coordinates  $(u_0, v_0)$  can be fixed and/or known in some cases (for example, in the image center), thus changing the reference frame, regarding the principal point position.
- The  $\gamma$  parameter is usually assumed to be null or, at least, considered to be a constant value.

|           |   |  |
|-----------|---|--|
| <b>g1</b> | $\gamma = 0$                            | $\gamma$ constant and null                     |
| <b>g2</b> | $\gamma = \gamma_0$                     | $\gamma$ constant                              |
| <b>g3</b> | $\gamma = \gamma(\tau)$                 | $\gamma$ free                                  |
| <b>s1</b> | $\alpha_u = \alpha_v(\tau)$             | $\frac{\alpha_u}{\alpha_v}$ constant and known |
| <b>s2</b> | $\alpha_u = \alpha_u(\tau)$             | $\alpha_u$ free                                |
| <b>f1</b> | $\alpha_v = 1$                          | $\alpha_v$ constant and known                  |
| <b>f2</b> | $\alpha_v = f_0$                        | $\alpha_v$ constant                            |
| <b>f3</b> | $\alpha_v = \alpha_v(\tau)$             | $\alpha_v$ free                                |
| <b>c1</b> | $u_0 = v_0 = 0$                         | $u_0$ and $v_0$ constant and known             |
| <b>c2</b> | $u_0 = u_{00}$ and $v_0 = v_{00}$       | $u_0$ and $v_0$ constant                       |
| <b>c3</b> | $u_0 = u_0(\tau)$ and $v_0 = v_0(\tau)$ | $u_0$ and $v_0$ free                           |

Table 1  
Table of particular cases of intrinsic parameters for 2 frames

- Enciso (1995) has experimentally proven that for a large number of cameras  $\alpha_u/\alpha_v$  can be considered to be constant even if other intrinsic parameters change. We express this as  $f = \alpha_u = \alpha_v$ .

The table 1 summarizes, for each intrinsic parameter, the particular cases of interest (constant values are indexed by zero). Subsequently, we will refer to each case by the label given in the first column. For example, g1 means that the  $\gamma$  parameter is null.

### 3.2 Particular cases of displacement

**Discrete motion - continuous motion:** In an image sequence, if the displacement between two frames is small, we can approximate the rotation equations by their first order :

$$\mathbf{R} = e^{\tilde{\mathbf{r}}} = \mathbf{I} + \tilde{\mathbf{r}} + o(\tilde{\mathbf{r}})$$

which occurs frequently in images sequences except with high speed objects.

If the motion is larger, we can also consider the second order expansion

$$\mathbf{R} = \mathbf{I} + \tilde{\mathbf{r}} + \frac{\tilde{\mathbf{r}}^2}{2} + o(\tilde{\mathbf{r}}^2)$$

**About extrinsic parameters:** The rotation parameters are related to the rotation axis and the rotation angle by :  $\mathbf{r} = 2 \tan \frac{\theta}{2} \mathbf{u}$  where  $\mathbf{u}$  is a unitary

vector giving the direction of the rotation axis.

Some components of  $\mathbf{u}$  can be known or null. Some value of  $\theta$  may yield singularities;  $\theta = \frac{\pi}{4}$  and the rotation axis is parallel to the translation vector for a screw displacement.

Some robotic systems give precise values of the robot displacements (angle, axis, translation). Some values may be known (we denote by  $\underline{\theta}_0$  a constant and known value of a parameter  $\theta$ ). Other informations regarding parallelism or orthogonality to a known direction or to an other vector may also be available:

- The rotation axis is orthogonal to the translation plane (e.g. planar motion) :

$$\mathbf{r} \perp \mathbf{t} \Leftrightarrow \mathbf{r} \cdot \mathbf{t} = 0$$

- screw displacement :

$$\mathbf{r} \parallel \mathbf{t} \Leftrightarrow \exists \kappa / \mathbf{r} = \kappa \mathbf{t}$$

**All constraints on motion:** All these constraints, also called “atomic particular cases”, have simple expressions that can be easily combined. In this purpose, we use the fact that  $\mathbf{u}$  is a unary vector and that, for monocular systems, the norm of translation cannot be recovered. To parameterize these vectors with only 2 parameters, we divide each component by a non-zero component. Then, the dot-product and scalar product induce linear relations. For example,  $t_2 = 1$  and  $\mathbf{t} \perp \mathbf{r}$  are equivalent to  $t_0 u_0 + t_1 u_1 + u_2 = 0 \Rightarrow u_2 = -t_0 u_0 - t_1 u_1$

All cases are collected in the table 2.

**Generating all cases:** All particular cases, each called a “molecular case”, are generated by combining the atomic cases and solving the constraints by a substitution<sup>1</sup>. A molecular case is composed of one case in each family, a family being named by a letter ( $g$ ,  $s$ ,  $f$  or  $c$  for projection as seen in table 1 and  $u$ ,  $R$ ,  $a$ ,  $t$  or  $Z$  for motion as seen in table 2). Thus, a molecular case is identified by the sequence :

$g[1-3]f[1-3]s[1-3]c[1-3]R[1-4]a[1-2]u[1-24]t[1-12]Z[1-3]$

where  $g[1-3]$  means “one atomic case among  $g1$ ,  $g2$  and  $g3$ ”.

**How many cases do we have?** If we look at the expression of the particular above-mentioned cases, we obtain  $6.10^6$  particular cases. However, this is not the real number because of the incompatibility of some atomic cases and the redundancy of some constraints. Two different sets of atomic constraints can generate the same simplified model.

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<sup>1</sup> This was done using Maple software for symbolic computations.

|            |                           |                              |            |  |                           |
|------------|---------------------------|------------------------------|------------|--|---------------------------|
| <b>u1</b>  | $u_0 = u_2 = 0, u_1 = 1$  | rot. axis $\parallel$ y-axis | <b>R1</b>  | $\mathbf{R} = \mathbf{I}$  | null rotation             |
| <b>u2</b>  | $u_0 = 0, u_1 = 1$        | rot. axis $\perp$ x-axis     | <b>R2</b>  | $\mathbf{R} = \mathbf{I} + \tilde{\mathbf{r}}$   | first order               |
| <b>u3</b>  | $u_2 = 0, u_1 = 1$        | rot. axis $\perp$ -axis      | <b>R3</b>  | $\mathbf{R} = \mathbf{I} + \tilde{\mathbf{r}} + \frac{1}{2}\tilde{\mathbf{r}}^2$   | second order              |
| <b>u4</b>  | $u_1 = 1$                 | general case                 | <b>R4</b>  | $\mathbf{R} = \mathbf{I} + \frac{\tilde{\mathbf{r}} + \frac{1}{2}\tilde{\mathbf{r}}^2}{1 + \frac{\mathbf{r}^T \mathbf{r}}{4}}$ | general case              |
| <b>u5</b>  | $u_0 = u_2 = 0, u_1 = -1$ | rot. axis $\parallel$ y-axis |            |  |                           |
| <b>u6</b>  | $u_0 = 0, u_1 = -1$       | rot. axis $\perp$ x-axis     | <b>a1</b>  | $\theta = \frac{\pi}{2}$   | quarter turn              |
| <b>u7</b>  | $u_2 = 0, u_1 = -1$       | rot. axis $\perp$ z-axis     | <b>a2</b>  | $\theta$   | free angle                |
| <b>u8</b>  | $u_1 = -1$                | general case                 |            |  |                           |
| <b>u9</b>  | $u_0 = u_1 = 0, u_2 = 1$  | rot. axis $\parallel$ z-axis | <b>t1</b>  | $t_1 = t_2 = 0, t_0 = 1$   | trans. $\parallel$ x-axis |
| <b>u10</b> | $u_0 = 0, u_2 = 1$        | rot. axis $\perp$ x-axis     | <b>t2</b>  | $t_1 = 0, t_0 = 1$   | trans. $\perp$ y-axis     |
| <b>u11</b> | $u_1 = 0, u_2 = 1$        | rot. axis $\perp$ y-axis     | <b>t3</b>  | $t_2 = 0, t_0 = 1$   | trans. $\perp$ z-axis     |
| <b>u12</b> | $u_2 = 1$                 | general case                 | <b>t4</b>  | $t_0 = 1$  | general trans.            |
| <b>u13</b> | $u_0 = u_1 = 0, u_2 = -1$ | rot. axis $\parallel$ z-axis | <b>t5</b>  | $t_0 = t_2 = 0, t_1 = 1$   | trans. $\parallel$ y-axis |
| <b>u14</b> | $u_0 = 0, u_2 = -1$       | rot. axis $\perp$ x-axis     | <b>t6</b>  | $t_0 = 0, t_1 = 1$   | trans. $\perp$ x-axis     |
| <b>u15</b> | $u_1 = 0, u_2 = -1$       | rot. axis $\perp$ y-axis     | <b>t7</b>  | $t_2 = 0, t_1 = 1$   | trans. $\perp$ z-axis     |
| <b>u16</b> | $u_2 = -1$                | general case                 | <b>t8</b>  | $t_1 = 1$  | general trans.            |
| <b>u17</b> | $u_1 = u_2 = 0, u_0 = 1$  | rot. axis $\parallel$ x-axis | <b>t9</b>  | $t_0 = t_1 = 0, t_2 = 1$   | trans. $\parallel$ z-axis |
| <b>u18</b> | $u_1 = 0, u_0 = 1$        | rot. axis $\perp$ y-axis     | <b>t10</b> | $t_0 = 0, t_2 = 1$   | trans. $\perp$ x-axis     |
| <b>u19</b> | $u_2 = 0, u_0 = 1$        | rot. axis $\perp$ z-axis     | <b>t11</b> | $t_1 = 0, t_2 = 1$   | trans. $\perp$ y-axis     |
| <b>u20</b> | $u_0 = 1$                 | general case                 | <b>t12</b> | $t_2 = 1$  | general trans.            |
| <b>u21</b> | $u_1 = u_2 = 0, u_0 = -1$ | rot. axis $\parallel$ x-axis |            |  |                           |
| <b>u22</b> | $u_1 = 0, u_0 = -1$       | rot. axis $\perp$ y-axis     | <b>Z1</b>  | $\mathbf{t} \cdot \mathbf{u} = 0$  | trans. $\perp$ rot. axis  |
| <b>u23</b> | $u_2 = 0, u_0 = -1$       | rot. axis $\perp$ z-axis     | <b>Z2</b>  | $\mathbf{t} \wedge \mathbf{u} = 0$   | screw displ.              |
| <b>u24</b> | $u_0 = -1$                | general case                 | <b>Z3</b>  |  | no relation               |

Table 2

Table of particular cases of displacements

It is easy to eliminate incompatible constraints. It is not possible to deal with redundant constraints, because this requires to compare each set of combined constraints with all others in order to determine the similarity. The complexity of this process is  $O(n^2)$ .

Although we cannot remove redundant cases, we propose an adapted strategy to deal with the large number of cases. The idea of this paper is : (i) to eliminate some of the redundant cases by using some considerations on the

atomic cases and (ii) to limit the number of cases by studying the particular forms of the matrices.

**Reducing the number of cases:** Some redundancy are obvious :

- In case (R1), one case of axis and angle is considered.
- In cases (R2) and (R3), we do not consider (a1) when  $\theta$  is equal to  $\frac{\pi}{2}$ .
- The case (a1) is only considered if  $\mathbf{r} \parallel \mathbf{t}$ , (Z2).

This reduces the amount of cases of fundamental relations to only 216756 cases.

## 4 Forms of fundamental matrices

We have significantly reduced the number of cases but this is not small enough to be computationally tractable. We now split fundamental relations in sets of matrices by forms. The matrix form is determined using simple rules in order to obtain a very simple parameterization. We consider  $(3 \times 3)$  matrices having 9 parameters (coefficients). If a coefficient is equal to zero, then there is one less parameter. If a coefficient has the same expression or is opposite to another, there is one less parameter again. These operations are very simple and can be rapidly computed in each case. Furthermore, we know that a fundamental matrix is defined up to a scale factor, and that its determinant is fixed to 0 (removing in most cases one parameter). This process reduces the 216756 cases to only 188 subgroups.

The table in appendix A shows all the simplified forms obtained, and, for each form, an example of case that has generated it. This table will be useful for people who want to implement the algorithm.

## 5 Experiments

We have recorded several video sequences for which the camera displacement induces a fundamental relation between image points  $\mathbf{m}_1$  and  $\mathbf{m}_2$ . From each particular matrix form, we have estimated the fundamental matrix parameters with the robust least median square method in order to minimize the distance between a 2D point  $\mathbf{m}_1$  and its epipolar line  $\mathbf{F} \mathbf{m}_2$ . To deal with cases with different degrees of freedom, we use an appropriate Akaike criterion, Akaike (1972).



Fig. 1. Images for x-axis translation, small pan rotation and auto-focus

For each recorded video sequence, we have verified that the model with the minimal residual error effectively corresponds to the displacement performed by the robotic system. We present one experiment in figure 1 for which the camera has performed a small pan rotation followed by a translation parallel to the x-axis. The auto-focus was also enabled. The case with the minimal residual error corresponds to the fundamental matrix form number **59** in the table given in appendix A :

$$\mathbf{F} = \begin{pmatrix} 0 & 0 & 0 \\ x_0 & x_1 & x_2 \\ 0 & -x_2 & x_3 \end{pmatrix}$$

This particular form was obtained from cases where the rotation was approximated to its first and second order, the translation is parallel to the x-axis, the rotation axis is orthogonal to the optical axis and the intrinsic parameters are free.

## 6 Conclusion

In an earlier study on homographic matrices Lingrand (2000), we have shown that it is possible to reduce the amount of particular cases in order to make the case selection computationally feasible. In this paper, we have shown that a similar result can be obtained with fundamental matrices using redundancies. We have experimentally confirmed that our system is able to automatically select the case corresponding to the performed displacement.

The applications are twofold: (i) an incremental reconstruction of the scene and (ii) the segmentation of objects moving with different displacements or with different geometric properties in video sequences.

This work has also been extended to motion estimation of human head inside MRI scanner, improving the registration of fMRI volumes, Lingrand et al. (2001).

## Appendix A : Table of particular forms of fundamental matrices.

We denote by  $n^o$  the form number, by  $p$  the number of parameters (we have not taken into account the fact that the fundamental matrix is defined up to a scale factor and that  $\det \mathbf{F} = 0$  but we do so in our implementation) and by  $n$  the number of molecular cases that have generated a form.

| $n^o$ | $p$ | simplified form of fundamental matrix        | for example generated by: | $n$ |
|-------|-----|--|---------------------------|-----|
| 1     | 1   | [0 0 0 0 0 $x_6$ 0 - $x_6$ 0]                | g1f1sic1t1Riu24Z3a2       | 24  |
| 2     | 1   | [0 0 $x_3$ 0 0 0 - $x_3$ 0 0]                | g1f1sic1t5Riu24Z3a2       | 4   |
| 3     | 1   | [0 $x_2$ 0 - $x_2$ 0 0 0 0 0]                | g1f1sic1t9Riu24Z3a2       | 5   |
| 4     | 2   | [0 0 0 0 0 $x_6$ 0 - $x_6$ $x_9$ ]           | g1f1sic3t1Riu24Z3a2       | 12  |
| 5     | 2   | [0 0 0 0 0 $x_6$ 0 $x_8$ 0]                  | g1f3sic1t1Riu24Z3a2       | 6   |
| 6     | 2   | [0 0 0 0 0 $x_6$ $x_7$ - $x_6$ 0]            | g1f1sic1t1R2u13Z2a2       | 16  |
| 7     | 2   | [0 0 0 0 $x_5$ $x_6$ 0 - $x_6$ $x_5$ ]       | g1f1sic1t1R2u17Z1a2       | 396 |
| 8     | 2   | [0 0 0 $x_4$ 0 $x_6$ 0 - $x_6$ 0]            | g1f1sic1t1R2u1Z2a2        | 16  |
| 9     | 2   | [0 0 $x_3$ 0 0 0 - $x_3$ 0 $x_9$ ]           | g1f1sic3t5Riu24Z3a2       | 2   |
| 10    | 2   | [0 0 $x_3$ 0 0 0 - $x_3$ $x_8$ 0]            | g1f1sic1t5R2u13Z2a2       | 8   |
| 11    | 2   | [0 0 $x_3$ 0 0 0 $x_7$ 0 0]                  | g1f1sic2t5Riu24Z3a2       | 4   |
| 12    | 2   | [0 0 $x_3$ 0 0 $x_6$ - $x_3$ - $x_6$ 0]      | g1f1sic1t3Riu24Z3a2       | 17  |
| 13    | 2   | [0 $x_2$ 0 - $x_2$ 0 $x_6$ 0 - $x_6$ 0]      | g1f1sic1t11Riu24Z3a2      | 8   |
| 14    | 2   | [0 $x_2$ 0 - $x_2$ 0 $x_6$ 0 0 0]            | g1f1sic1t9R2u1Z2a2        | 24  |
| 15    | 2   | [0 $x_2$ 0 - $x_2$ $x_5$ 0 0 0 0]            | g2f3sic1t9Riu24Z3a2       | 4   |
| 16    | 2   | [0 $x_2$ 0 $x_4$ 0 0 0 0 0]                  | g1f1sic2t9Riu24Z3a2       | 3   |
| 17    | 2   | [0 $x_2$ $x_3$ - $x_2$ 0 0 0 - $x_3$ 0 0]    | g1f1sic1t10Riu24Z3a2      | 4   |
| 18    | 2   | [0 $x_2$ $x_3$ - $x_2$ 0 0 0 0 0 0]          | g1f1sic1t9R2u17Z2a2       | 12  |
| 19    | 2   | [0 $x_2$ $x_3$ 0 0 0 - $x_3$ 0 0 0]          | g1f1sic1t5R2u17Z2a2       | 8   |
| 20    | 2   | [ $x_1$ 0 $x_3$ 0 0 0 - $x_3$ 0 $x_1$ ]      | g1f1sic1t5R2u1Z1a2        | 66  |
| 21    | 2   | [ $x_1$ $x_2$ 0 - $x_2$ $x_1$ 0 0 0 0]       | g1f1sic1t10R2u11Z1a2      | 198 |
| 22    | 3   | [0 0 0 0 0 $x_6$ 0 $x_8$ $x_9$ ]             | g1f3sic2t1Riu24Z3a2       | 12  |
| 23    | 3   | [0 0 0 0 0 $x_6$ $x_7$ - $x_6$ $x_9$ ]       | g1f1sic2t1R2u13Z2a2       | 32  |
| 24    | 3   | [0 0 0 0 0 $x_6$ $x_7$ $x_8$ 0]              | g1f1sic1t1R3u13Z2a2       | 200 |
| 25    | 3   | [0 0 0 0 $x_5$ $x_6$ 0 - $x_6$ $x_9$ ]       | g1f2sic1t1R2u17Z1a2       | 396 |
| 26    | 3   | [0 0 0 0 $x_5$ $x_6$ $x_7$ - $x_6$ $x_5$ ]   | g1f1sic1t1R2u11Z2a2       | 16  |
| 27    | 3   | [0 0 0 $x_4$ 0 $x_6$ 0 $x_8$ 0]              | g1f1sic1t1R3u1Z2a2        | 56  |
| 28    | 3   | [0 0 0 $x_4$ 0 $x_6$ $x_7$ - $x_6$ 0]        | g1f1sic1t1R2u10Z2a2       | 32  |
| 29    | 3   | [0 0 0 $x_4$ $x_5$ $x_6$ 0 - $x_6$ 0]        | g2f1sic1t1R2u1Z2a2        | 32  |
| 30    | 3   | [0 0 0 $x_4$ $x_5$ $x_6$ 0 - $x_6$ $x_5$ ]   | g1f1sic1t1R2u19Z2a2       | 16  |
| 31    | 3   | [0 0 $x_3$ 0 0 0 - $x_3$ $x_8$ $x_9$ ]       | g1f1sic2t5R2u13Z2a2       | 16  |
| 32    | 3   | [0 0 $x_3$ 0 0 0 $x_7$ 0 $x_9$ ]             | g1f1sic2t5Riu24Z3a2       | 8   |
| 33    | 3   | [0 0 $x_3$ 0 0 0 $x_7$ $x_8$ 0]              | g1f1sic1t5R3u13Z2a2       | 64  |
| 34    | 3   | [0 0 $x_3$ 0 0 $x_6$ - $x_3$ - $x_6$ $x_9$ ] | g1f1sic3t3Riu24Z3a2       | 13  |
| 35    | 3   | [0 0 $x_3$ 0 0 $x_6$ - $x_3$ $x_8$ 0]        | g2f1sic1t5R2u13Z2a2       | 22  |
| 36    | 3   | [0 0 $x_3$ 0 0 $x_6$ $x_7$ - $x_6$ 0]        | g1f1sic2t3Riu24Z3a2       | 4   |

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|    |   |  |                      |      |
|----|---|--|----------------------|------|
| 37 | 3 | [0 x <sub>2</sub> 0 -x <sub>2</sub> 0 x <sub>6</sub> 0 x <sub>8</sub> 0 ]  | g1f3s1c1t11Riu24Z3a2 | 2    |
| 38 | 3 | [0 x <sub>2</sub> 0 -x <sub>2</sub> x <sub>5</sub> x <sub>6</sub> 0 -x <sub>6</sub> 0 ]                            | g3f1s1c1t11Riu24Z3a2 | 4    |
| 39 | 3 | [0 x <sub>2</sub> 0 -x <sub>2</sub> x <sub>5</sub> x <sub>6</sub> 0 0 0 ]  | g2f3s1c1t9R2u1Z2a2   | 12   |
| 40 | 3 | [0 x <sub>2</sub> 0 x <sub>4</sub> 0 x <sub>6</sub> 0 -x <sub>6</sub> 0 ]  | g1f1s2c1t11Riu24Z3a2 | 4    |
| 41 | 3 | [0 x <sub>2</sub> 0 x <sub>4</sub> 0 x <sub>6</sub> 0 0 0 ]  | g1f1s1c1t9R3u1Z2a2   | 60   |
| 42 | 3 | [0 x <sub>2</sub> 0 x <sub>4</sub> x <sub>5</sub> 0 0 0 0 ]  | g2f1s2c1t9Riu24Z3a2  | 6    |
| 43 | 3 | [0 x <sub>2</sub> x <sub>3</sub> -x <sub>2</sub> 0 0 x <sub>7</sub> 0 0 0 ]  | g1f3s1c1t0Riu24Z3a2  | 2    |
| 44 | 3 | [0 x <sub>2</sub> x <sub>3</sub> -x <sub>2</sub> 0 x <sub>6</sub> -x <sub>3</sub> -x <sub>6</sub> 0 ]              | g1f1s1c1t2Riu24Z3a2  | 40   |
| 45 | 3 | [0 x <sub>2</sub> x <sub>3</sub> -x <sub>2</sub> 0 x <sub>6</sub> 0 0 0 ]  | g1f1s1c1t9R2u19Z2a2  | 60   |
| 46 | 3 | [0 x <sub>2</sub> x <sub>3</sub> 0 0 0 -x <sub>3</sub> x <sub>8</sub> 0 ]  | g1f1s1c1t5R2u11Z2a2  | 16   |
| 47 | 3 | [0 x <sub>2</sub> x <sub>3</sub> 0 0 0 x <sub>7</sub> 0 0 0 ]  | g1f1s1c1t5R3u17Z2a2  | 64   |
| 48 | 3 | [0 x <sub>2</sub> x <sub>3</sub> x <sub>4</sub> 0 0 0 0 0 0 ]  | g1f1s1c1t9R3u17Z2a2  | 60   |
| 49 | 3 | [x <sub>1</sub> 0 x <sub>3</sub> 0 0 0 -x <sub>3</sub> 0 x <sub>9</sub> ]  | g1f2s1c1t5R2u1Z1a2   | 66   |
| 50 | 3 | [x <sub>1</sub> 0 x <sub>3</sub> 0 0 0 -x <sub>3</sub> x <sub>8</sub> x <sub>1</sub> ]                             | g1f1s1c1t5R2u10Z2a2  | 8    |
| 51 | 3 | [x <sub>1</sub> x <sub>2</sub> 0 -x <sub>2</sub> x <sub>1</sub> x <sub>6</sub> 0 0 0 ]                             | g1f1s1c1t9R2u10Z2a2  | 24   |
| 52 | 3 | [x <sub>1</sub> x <sub>2</sub> x <sub>3</sub> -x <sub>2</sub> x <sub>1</sub> 0 0 0 0 ]                             | g1f1s1c1t9R2u11Z2a2  | 24   |
| 53 | 3 | [x <sub>1</sub> x <sub>2</sub> x <sub>3</sub> 0 0 0 -x <sub>3</sub> 0 x <sub>1</sub> ]                             | g1f1s1c1t5R2u19Z2a2  | 8    |
| 54 | 4 | [0 0 0 0 0 x <sub>6</sub> x <sub>7</sub> x <sub>8</sub> x <sub>9</sub> ]   | g1f1s1c2t1R3u13Z2a2  | 400  |
| 55 | 4 | [0 0 0 0 0 x <sub>5</sub> x <sub>6</sub> 0 x <sub>8</sub> x <sub>9</sub> ]   | g1f1s1c2t1R2u17Z1a2  | 2772 |
| 56 | 4 | [0 0 0 0 0 x <sub>5</sub> x <sub>6</sub> x <sub>7</sub> -x <sub>6</sub> x <sub>9</sub> ]                           | g1f2s1c1t1R2u11Z2a2  | 16   |
| 57 | 4 | [0 0 0 0 0 x <sub>5</sub> x <sub>6</sub> x <sub>7</sub> x <sub>8</sub> x <sub>5</sub> ]                            | g2f1s1c1t1R2u11Z2a2  | 32   |
| 58 | 4 | [0 0 0 x <sub>4</sub> 0 x <sub>6</sub> x <sub>7</sub> x <sub>8</sub> 0 ]   | g1f3s1c1t1R2u10Z2a2  | 16   |
| 59 | 4 | [0 0 0 x <sub>4</sub> x <sub>5</sub> x <sub>6</sub> 0 -x <sub>6</sub> x <sub>9</sub> ]                             | g1f2s1c1t1R2u19Z2a2  | 80   |
| 60 | 4 | [0 0 0 x <sub>4</sub> x <sub>5</sub> x <sub>6</sub> 0 x <sub>8</sub> 0 ]   | g2f1s1c1t1R3u1Z2a2   | 112  |
| 61 | 4 | [0 0 0 x <sub>4</sub> x <sub>5</sub> x <sub>6</sub> x <sub>7</sub> -x <sub>6</sub> x <sub>5</sub> ]                | g1f1s1c1t1R2u12Z2a2  | 24   |
| 62 | 4 | [0 0 x <sub>3</sub> 0 0 0 x <sub>7</sub> x <sub>8</sub> x <sub>9</sub> ]   | g1f1s1c2t5R3u13Z2a2  | 128  |
| 63 | 4 | [0 0 x <sub>3</sub> 0 0 x <sub>6</sub> -x <sub>3</sub> x <sub>8</sub> x <sub>9</sub> ]                             | g2f1s1c2t5R2u13Z2a2  | 44   |
| 64 | 4 | [0 0 x <sub>3</sub> 0 0 x <sub>6</sub> x <sub>7</sub> -x <sub>6</sub> x <sub>9</sub> ]                             | g1f1s2c2t3Riu24Z3a2  | 8    |
| 65 | 4 | [0 0 x <sub>3</sub> 0 0 x <sub>6</sub> x <sub>7</sub> x <sub>8</sub> 0 ]   | g1f1s1c1t3R2u13Z2a2  | 588  |
| 66 | 4 | [0 x <sub>2</sub> 0 -x <sub>2</sub> x <sub>5</sub> x <sub>6</sub> 0 x <sub>8</sub> 0 ]                             | g2f3s1c1t11Riu24Z3a2 | 4    |
| 67 | 4 | [0 x <sub>2</sub> 0 x <sub>4</sub> 0 x <sub>6</sub> 0 x <sub>8</sub> 0 ]   | g1f1s1c1t11R2u1Z2a2  | 146  |
| 68 | 4 | [0 x <sub>2</sub> 0 x <sub>4</sub> x <sub>5</sub> x <sub>6</sub> 0 -x <sub>6</sub> 0 ]                             | g2f1s2c1t11Riu24Z3a2 | 8    |
| 69 | 4 | [0 x <sub>2</sub> 0 x <sub>4</sub> x <sub>5</sub> x <sub>6</sub> 0 0 0 ]   | g2f1s1c1t9R3u1Z2a2   | 120  |
| 70 | 4 | [0 x <sub>2</sub> x <sub>3</sub> -x <sub>2</sub> 0 x <sub>6</sub> -x <sub>3</sub> -x <sub>6</sub> x <sub>9</sub> ] | g1f3s1c2t9Riu24Z3a2  | 9    |
| 71 | 4 | [0 x <sub>2</sub> x <sub>3</sub> -x <sub>2</sub> x <sub>5</sub> x <sub>6</sub> 0 -x <sub>6</sub> x <sub>5</sub> ]  | g1f1s1c1t11R2u17Z2a2 | 8    |
| 72 | 4 | [0 x <sub>2</sub> x <sub>3</sub> -x <sub>2</sub> x <sub>5</sub> x <sub>6</sub> 0 0 0 ]                             | g2f3s1c1t9R2u17Z2a2  | 36   |
| 73 | 4 | [0 x <sub>2</sub> x <sub>3</sub> 0 0 0 x <sub>7</sub> x <sub>8</sub> 0 ]   | g1f1s2c1t5R2u11Z2a2  | 32   |
| 74 | 4 | [0 x <sub>2</sub> x <sub>3</sub> 0 x <sub>5</sub> x <sub>6</sub> -x <sub>3</sub> -x <sub>6</sub> 0 ]               | g2f1s1c1t5R2u17Z2a2  | 12   |
| 75 | 4 | [0 x <sub>2</sub> x <sub>3</sub> 0 x <sub>5</sub> x <sub>6</sub> -x <sub>3</sub> -x <sub>6</sub> x <sub>5</sub> ]  | g1f1s1c1t3R2u17Z2a2  | 8    |
| 76 | 4 | [0 x <sub>2</sub> x <sub>3</sub> x <sub>4</sub> 0 0 x <sub>7</sub> 0 0 0 ]   | g1f1s1c1t10R2u17Z2a2 | 150  |
| 77 | 4 | [0 x <sub>2</sub> x <sub>3</sub> x <sub>4</sub> 0 x <sub>6</sub> 0 0 0 0 ]   | g1f1s2c1t9R2u19Z2a2  | 24   |
| 78 | 4 | [x <sub>1</sub> 0 x <sub>3</sub> 0 0 0 -x <sub>3</sub> x <sub>8</sub> x <sub>9</sub> ]                             | g1f2s1c1t5R2u10Z2a2  | 8    |
| 79 | 4 | [x <sub>1</sub> 0 x <sub>3</sub> 0 0 0 x <sub>7</sub> 0 x <sub>9</sub> ]   | g1f1s1c2t5R2u1Z1a2   | 1056 |
| 80 | 4 | [x <sub>1</sub> 0 x <sub>3</sub> x <sub>4</sub> 0 x <sub>6</sub> -x <sub>3</sub> -x <sub>6</sub> x <sub>1</sub> ]  | g1f1s1c1t3R2u1Z2a2   | 8    |
| 81 | 4 | [x <sub>1</sub> x <sub>2</sub> 0 -x <sub>2</sub> x <sub>1</sub> x <sub>6</sub> x <sub>7</sub> -x <sub>6</sub> 0 ]  | g1f1s1c1t11R2u13Z2a2 | 16   |
| 82 | 4 | [x <sub>1</sub> x <sub>2</sub> 0 x <sub>4</sub> x <sub>5</sub> 0 0 0 0 ]   | g1f1s2c1t10R2u11Z1a2 | 990  |
| 83 | 4 | [x <sub>1</sub> x <sub>2</sub> x <sub>3</sub> -x <sub>2</sub> 0 x <sub>6</sub> -x <sub>3</sub> 0 x <sub>1</sub> ]  | g1f1s1c1t10R2u1Z2a2  | 8    |
| 84 | 4 | [x <sub>1</sub> x <sub>2</sub> x <sub>3</sub> -x <sub>2</sub> x <sub>1</sub> 0 -x <sub>3</sub> x <sub>8</sub> 0 ]  | g1f1s1c1t10R2u13Z2a2 | 16   |
| 85 | 4 | [x <sub>1</sub> x <sub>2</sub> x <sub>3</sub> -x <sub>2</sub> x <sub>1</sub> x <sub>6</sub> 0 0 0 ]                | g1f1s1c1t9R2u12Z2a2  | 36   |
| 86 | 4 | [x <sub>1</sub> x <sub>2</sub> x <sub>3</sub> 0 0 0 -x <sub>3</sub> 0 x <sub>9</sub> ]                             | g1f2s1c1t5R2u19Z2a2  | 8    |
| 87 | 4 | [x <sub>1</sub> x <sub>2</sub> x <sub>3</sub> 0 0 0 -x <sub>3</sub> x <sub>8</sub> x <sub>1</sub> ]                | g1f1s1c1t5R2u12Z2a2  | 12   |
| 88 | 5 | [0 0 0 0 x <sub>5</sub> x <sub>6</sub> x <sub>7</sub> x <sub>8</sub> x <sub>9</sub> ]                              | g1f1s1c2t1R2u11Z2a2  | 368  |
| 89 | 5 | [0 0 0 x <sub>4</sub> 0 x <sub>6</sub> x <sub>7</sub> x <sub>8</sub> x <sub>9</sub> ]                              | g1f1s1c2t1R2u10Z2a2  | 240  |
| 90 | 5 | [0 0 0 x <sub>4</sub> x <sub>5</sub> x <sub>6</sub> 0 x <sub>8</sub> x <sub>9</sub> ]                              | g1f3s1c1t1R2u19Z2a2  | 48   |
| 91 | 5 | [0 0 0 x <sub>4</sub> x <sub>5</sub> x <sub>6</sub> x <sub>7</sub> -x <sub>6</sub> x <sub>9</sub> ]                | g1f2s1c1t1R2u12Z2a2  | 24   |
| 92 | 5 | [0 0 0 x <sub>4</sub> x <sub>5</sub> x <sub>6</sub> x <sub>7</sub> x <sub>8</sub> -x <sub>5</sub> ]                | g1f1s1c1t1R3u10Z2a2  | 32   |
| 93 | 5 | [0 0 0 x <sub>4</sub> x <sub>5</sub> x <sub>6</sub> x <sub>7</sub> x <sub>8</sub> 0 ]                              | g2f1s1c1t1R2u10Z2a2  | 96   |
| 94 | 5 | [0 0 0 x <sub>4</sub> x <sub>5</sub> x <sub>6</sub> x <sub>7</sub> x <sub>8</sub> x <sub>5</sub> ]                 | g1f1s1c1t1R3u11Z2a2  | 64   |
| 95 | 5 | [0 0 x <sub>3</sub> 0 0 x <sub>6</sub> x <sub>7</sub> x <sub>8</sub> x <sub>9</sub> ]                              | g1f1s1c2t3R2u13Z2a2  | 1176 |

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|     |   |   |                      |      |
|-----|---|---|----------------------|------|
| 96  | 5 | [0 x <sub>2</sub> 0 x <sub>4</sub> x <sub>5</sub> x <sub>6</sub> 0 x <sub>8</sub> 0 ]   | g2f1s1c1t11R2u1Z2a2  | 292  |
| 97  | 5 | [0 x <sub>2</sub> x <sub>3</sub> -x <sub>2</sub> 0 x <sub>6</sub> -x <sub>3</sub> x <sub>8</sub> x <sub>9</sub> ]                           | g1f1s1c2t9R2u1Z2a2   | 26   |
| 98  | 5 | [0 x <sub>2</sub> x <sub>3</sub> -x <sub>2</sub> 0 x <sub>6</sub> x <sub>7</sub> -x <sub>6</sub> x <sub>9</sub> ]                           | g1f1s1c2t9R2u17Z2a2  | 14   |
| 99  | 5 | [0 x <sub>2</sub> x <sub>3</sub> -x <sub>2</sub> 0 x <sub>6</sub> x <sub>7</sub> x <sub>8</sub> 0 ]   | g1f3s1c1t12Riu24Z3a2 | 3    |
| 100 | 5 | [0 x <sub>2</sub> x <sub>3</sub> -x <sub>2</sub> x <sub>5</sub> x <sub>6</sub> -x <sub>3</sub> x <sub>8</sub> 0 ]                           | g3f1s1c1t10Riu24Z3a2 | 10   |
| 101 | 5 | [0 x <sub>2</sub> x <sub>3</sub> -x <sub>2</sub> x <sub>5</sub> x <sub>6</sub> 0 -x <sub>6</sub> x <sub>9</sub> ]                           | g1f2s1c1t11R2u17Z2a2 | 8    |
| 102 | 5 | [0 x <sub>2</sub> x <sub>3</sub> -x <sub>2</sub> x <sub>5</sub> x <sub>6</sub> 0 x <sub>8</sub> x <sub>5</sub> ]                            | g2f1s1c1t11R2u17Z2a2 | 12   |
| 103 | 5 | [0 x <sub>2</sub> x <sub>3</sub> 0 0 0 x <sub>7</sub> x <sub>8</sub> x <sub>9</sub> ]   | g1f1s1c2t5R2u11Z2a2  | 240  |
| 104 | 5 | [0 x <sub>2</sub> x <sub>3</sub> 0 x <sub>5</sub> x <sub>6</sub> -x <sub>3</sub> -x <sub>6</sub> x <sub>9</sub> ]                           | g1f2s1c1t3R2u17Z2a2  | 32   |
| 105 | 5 | [0 x <sub>2</sub> x <sub>3</sub> 0 x <sub>5</sub> x <sub>6</sub> -x <sub>3</sub> x <sub>8</sub> 0 ]   | g2f1s1c1t5R2u11Z2a2  | 36   |
| 106 | 5 | [0 x <sub>2</sub> x <sub>3</sub> 0 x <sub>5</sub> x <sub>6</sub> x <sub>7</sub> -x <sub>6</sub> x <sub>5</sub> ]                            | g1f1s1c1t3R3u17Z2a2  | 40   |
| 107 | 5 | [0 x <sub>2</sub> x <sub>3</sub> x <sub>4</sub> 0 x <sub>6</sub> x <sub>7</sub> -x <sub>6</sub> 0 ]   | g1f1s2c1t12Riu24Z3a2 | 6    |
| 108 | 5 | [0 x <sub>2</sub> x <sub>3</sub> x <sub>4</sub> x <sub>5</sub> x <sub>6</sub> 0 -x <sub>6</sub> x <sub>5</sub> ]                            | g1f1s1c1t11R3u17Z2a2 | 40   |
| 109 | 5 | [0 x <sub>2</sub> x <sub>3</sub> x <sub>4</sub> x <sub>5</sub> x <sub>6</sub> 0 0 0 ]   | g2f1s1c1t9R3u17Z2a2  | 168  |
| 110 | 5 | [x <sub>1</sub> 0 x <sub>3</sub> 0 0 0 x <sub>7</sub> x <sub>8</sub> x <sub>9</sub> ]   | g1f1s1c2t5R2u10Z2a2  | 128  |
| 111 | 5 | [x <sub>1</sub> 0 x <sub>3</sub> x <sub>4</sub> 0 x <sub>6</sub> -x <sub>3</sub> -x <sub>6</sub> x <sub>9</sub> ]                           | g1f2s1c1t3R2u1Z2a2   | 8    |
| 112 | 5 | [x <sub>1</sub> 0 x <sub>3</sub> x <sub>4</sub> 0 x <sub>6</sub> -x <sub>3</sub> x <sub>8</sub> x <sub>1</sub> ]                            | g1f1s1c1t3R3u1Z2a2   | 16   |
| 113 | 5 | [x <sub>1</sub> x <sub>2</sub> 0 -x <sub>2</sub> x <sub>1</sub> x <sub>6</sub> x <sub>7</sub> x <sub>8</sub> 0 ]                            | g1f1s1c1t11R3u13Z2a2 | 56   |
| 114 | 5 | [x <sub>1</sub> x <sub>2</sub> 0 x <sub>4</sub> x <sub>5</sub> x <sub>6</sub> 0 0 0 ]   | g1f1s2c1t9R2u10Z2a2  | 120  |
| 115 | 5 | [x <sub>1</sub> x <sub>2</sub> x <sub>3</sub> -x <sub>2</sub> 0 x <sub>6</sub> -x <sub>3</sub> 0 x <sub>9</sub> ]                           | g1f2s1c1t10R2u1Z2a2  | 8    |
| 116 | 5 | [x <sub>1</sub> x <sub>2</sub> x <sub>3</sub> -x <sub>2</sub> x <sub>1</sub> 0 x <sub>7</sub> x <sub>8</sub> 0 ]                            | g1f1s1c1t10R3u13Z2a2 | 56   |
| 117 | 5 | [x <sub>1</sub> x <sub>2</sub> x <sub>3</sub> 0 0 0 -x <sub>3</sub> x <sub>8</sub> x <sub>9</sub> ]   | g1f2s1c1t5R2u12Z2a2  | 12   |
| 118 | 5 | [x <sub>1</sub> x <sub>2</sub> x <sub>3</sub> 0 0 0 x <sub>7</sub> 0 x <sub>9</sub> ]   | g1f1s2c1t5R2u19Z2a2  | 32   |
| 119 | 5 | [x <sub>1</sub> x <sub>2</sub> x <sub>3</sub> 0 0 0 x <sub>7</sub> x <sub>8</sub> -x <sub>1</sub> ]   | g1f1s1c1t5R3u11Z2a2  | 16   |
| 120 | 5 | [x <sub>1</sub> x <sub>2</sub> x <sub>3</sub> 0 0 0 x <sub>7</sub> x <sub>8</sub> x <sub>1</sub> ]  | g1f1s1c1t5R3u10Z2a2  | 32   |
| 121 | 5 | [x <sub>1</sub> x <sub>2</sub> x <sub>3</sub> x <sub>2</sub> x <sub>5</sub> x <sub>6</sub> -x <sub>3</sub> -x <sub>6</sub> x <sub>1</sub> ] | g2f1s1c1t9R2u1Z1a2   | 70   |
| 122 | 5 | [x <sub>1</sub> x <sub>2</sub> x <sub>3</sub> x <sub>4</sub> -x <sub>1</sub> x <sub>6</sub> 0 0 0 ]   | g1f1s1c1t9R3u19Z2a2  | 48   |
| 123 | 5 | [x <sub>1</sub> x <sub>2</sub> x <sub>3</sub> x <sub>4</sub> 0 x <sub>6</sub> -x <sub>3</sub> 0 x <sub>1</sub> ]                            | g1f1s1c1t10R3u1Z2a2  | 16   |
| 124 | 5 | [x <sub>1</sub> x <sub>2</sub> x <sub>3</sub> x <sub>4</sub> x <sub>1</sub> x <sub>6</sub> 0 0 0 ]  | g1f1s1c1t9R3u10Z2a2  | 96   |
| 125 | 5 | [x <sub>1</sub> x <sub>2</sub> x <sub>3</sub> x <sub>4</sub> x <sub>5</sub> 0 0 0 0 ]   | g1f1s2c1t9R2u11Z2a2  | 24   |
| 126 | 6 | [0 0 0 x <sub>4</sub> x <sub>5</sub> x <sub>6</sub> x <sub>7</sub> x <sub>8</sub> x <sub>9</sub> ]  | g1f1s1c1t1R3u12Z2a2  | 5160 |
| 127 | 6 | [0 x <sub>2</sub> x <sub>3</sub> -x <sub>2</sub> 0 x <sub>6</sub> x <sub>7</sub> x <sub>8</sub> x <sub>9</sub> ]                            | g1f1s1c2t9R2u19Z2a2  | 199  |
| 128 | 6 | [0 x <sub>2</sub> x <sub>3</sub> -x <sub>2</sub> x <sub>5</sub> x <sub>6</sub> -x <sub>3</sub> x <sub>8</sub> x <sub>9</sub> ]              | g2f3s1c2t11Riu24Z3a2 | 34   |
| 129 | 6 | [0 x <sub>2</sub> x <sub>3</sub> -x <sub>2</sub> x <sub>5</sub> x <sub>6</sub> 0 x <sub>8</sub> x <sub>9</sub> ]                            | g1f3s1c1t11R2u17Z2a2 | 44   |
| 130 | 6 | [0 x <sub>2</sub> x <sub>3</sub> -x <sub>2</sub> x <sub>5</sub> x <sub>6</sub> x <sub>7</sub> x <sub>8</sub> 0 ]                            | g2f3s1c1t10Riu24Z3a2 | 10   |
| 131 | 6 | [0 x <sub>2</sub> x <sub>3</sub> 0 x <sub>5</sub> x <sub>6</sub> -x <sub>3</sub> x <sub>8</sub> x <sub>9</sub> ]                            | g3f1s1c1t3R2u17Z2a2  | 8    |
| 132 | 6 | [0 x <sub>2</sub> x <sub>3</sub> 0 x <sub>5</sub> x <sub>6</sub> x <sub>7</sub> -x <sub>6</sub> x <sub>9</sub> ]                            | g1f2s1c1t3R3u17Z2a2  | 40   |
| 133 | 6 | [0 x <sub>2</sub> x <sub>3</sub> 0 x <sub>5</sub> x <sub>6</sub> x <sub>7</sub> x <sub>8</sub> 0 ]  | g2f1s1c1t5R3u17Z2a2  | 192  |
| 134 | 6 | [0 x <sub>2</sub> x <sub>3</sub> 0 x <sub>5</sub> x <sub>6</sub> x <sub>7</sub> x <sub>8</sub> x <sub>5</sub> ]                             | g1f1s1c1t3R2u11Z2a2  | 32   |
| 135 | 6 | [0 x <sub>2</sub> x <sub>3</sub> x <sub>4</sub> 0 x <sub>6</sub> x <sub>7</sub> x <sub>8</sub> 0 ]  | g1f3s2c1t12Riu24Z3a2 | 3    |
| 136 | 6 | [0 x <sub>2</sub> x <sub>3</sub> x <sub>4</sub> x <sub>5</sub> x <sub>6</sub> 0 0 -x <sub>6</sub> x <sub>9</sub> ]                          | g1f2s1c1t11R3u17Z2a2 | 40   |
| 137 | 6 | [0 x <sub>2</sub> x <sub>3</sub> x <sub>4</sub> x <sub>5</sub> x <sub>6</sub> 0 x <sub>8</sub> x <sub>5</sub> ]                             | g1f1s1c1t11R2u19Z2a2 | 32   |
| 138 | 6 | [0 x <sub>2</sub> x <sub>3</sub> x <sub>4</sub> x <sub>5</sub> x <sub>6</sub> x <sub>7</sub> -x <sub>6</sub> x <sub>5</sub> ]               | g1f1s1c1t12R2u17Z2a2 | 84   |
| 139 | 6 | [x <sub>1</sub> 0 x <sub>3</sub> x <sub>4</sub> 0 x <sub>6</sub> -x <sub>3</sub> x <sub>8</sub> x <sub>9</sub> ]                            | g1f2s1c1t3R3u1Z2a2   | 16   |
| 140 | 6 | [x <sub>1</sub> 0 x <sub>3</sub> x <sub>4</sub> 0 x <sub>6</sub> x <sub>7</sub> -x <sub>6</sub> x <sub>9</sub> ]                            | g1f1s2c1t3R2u1Z2a2   | 16   |
| 141 | 6 | [x <sub>1</sub> 0 x <sub>3</sub> x <sub>4</sub> 0 x <sub>6</sub> x <sub>7</sub> -x <sub>6</sub> x <sub>9</sub> ]                            | g1f2s2c1t6R2u5Z3a2   | 16   |
| 142 | 6 | [x <sub>1</sub> x <sub>2</sub> 0 x <sub>4</sub> x <sub>1</sub> x <sub>6</sub> x <sub>7</sub> x <sub>8</sub> 0 ]                             | g1f1s1c1t11R2u10Z2a2 | 48   |
| 143 | 6 | [x <sub>1</sub> x <sub>2</sub> 0 x <sub>4</sub> x <sub>5</sub> x <sub>6</sub> x <sub>7</sub> -x <sub>6</sub> 0 ]                            | g1f1s2c1t11R2u13Z2a2 | 16   |
| 144 | 6 | [x <sub>1</sub> x <sub>2</sub> x <sub>3</sub> -x <sub>2</sub> 0 x <sub>6</sub> x <sub>7</sub> 0 x <sub>9</sub> ]                            | g1f3s1c1t10R2u1Z2a2  | 8    |
| 145 | 6 | [x <sub>1</sub> x <sub>2</sub> x <sub>3</sub> -x <sub>2</sub> x <sub>1</sub> x <sub>6</sub> x <sub>7</sub> x <sub>8</sub> 0 ]               | g1f1s1c1t12R2u13Z2a2 | 126  |
| 146 | 6 | [x <sub>1</sub> x <sub>2</sub> x <sub>3</sub> -x <sub>2</sub> x <sub>5</sub> x <sub>6</sub> -x <sub>3</sub> x <sub>6</sub> x <sub>9</sub> ] | g1f1s1c1t10R2u10Z1a2 | 144  |
| 147 | 6 | [x <sub>1</sub> x <sub>2</sub> x <sub>3</sub> -x <sub>2</sub> x <sub>5</sub> x <sub>6</sub> x <sub>3</sub> -x <sub>6</sub> x <sub>9</sub> ] | g1f1s1c1t11R2u11Z1a2 | 144  |
| 148 | 6 | [x <sub>1</sub> x <sub>2</sub> x <sub>3</sub> 0 0 0 x <sub>7</sub> x <sub>8</sub> x <sub>9</sub> ]  | g1f1s1c1t5R3u12Z2a2  | 1536 |
| 149 | 6 | [x <sub>1</sub> x <sub>2</sub> x <sub>3</sub> x <sub>2</sub> x <sub>5</sub> x <sub>6</sub> -x <sub>3</sub> -x <sub>6</sub> x <sub>9</sub> ] | g1f1s1c1t3R2u19Z1a2  | 358  |
| 150 | 6 | [x <sub>1</sub> x <sub>2</sub> x <sub>3</sub> x <sub>2</sub> x <sub>5</sub> x <sub>6</sub> -x <sub>3</sub> x <sub>8</sub> x <sub>1</sub> ]  | g2f1s1c1t5R2u10Z2a2  | 12   |
| 151 | 6 | [x <sub>1</sub> x <sub>2</sub> x <sub>3</sub> x <sub>4</sub> 0 x <sub>6</sub> -x <sub>3</sub> 0 x <sub>9</sub> ]                            | g1f2s1c1t10R3u1Z2a2  | 16   |
| 152 | 6 | [x <sub>1</sub> x <sub>2</sub> x <sub>3</sub> x <sub>4</sub> 0 x <sub>6</sub> -x <sub>3</sub> x <sub>8</sub> x <sub>1</sub> ]               | g1f1s1c1t12R2u1Z2a2  | 42   |
| 153 | 6 | [x <sub>1</sub> x <sub>2</sub> x <sub>3</sub> x <sub>4</sub> 0 x <sub>6</sub> x <sub>7</sub> 0 x <sub>1</sub> ]                             | g1f1s1c1t10R2u19Z2a2 | 16   |
| 154 | 6 | [x <sub>1</sub> x <sub>2</sub> x <sub>3</sub> x <sub>4</sub> x <sub>1</sub> 0 x <sub>7</sub> x <sub>8</sub> 0 ]                             | g1f1s1c1t10R2u11Z2a2 | 48   |

followed on next page

| from previous page |   |   |       |       |        |        |       |        |        |        |                      |      |
|--------------------|---|---|-------|-------|--------|--------|-------|--------|--------|--------|----------------------|------|
| 155                | 6 | $[x_1 \ x_2 \ x_3 \ x_4 \ x_5 \ x_6 \ -x_3 \ -x_6 \ x_1]$ |       |       |        |        |       |        |        |        | g2f1s1c1t3R2u1Z2a2   | 24   |
| 156                | 6 | $[x_1 \ x_2 \ x_3 \ x_4 \ x_5 \ x_6 \ 0 \ 0 \ 0]$         |       |       |        |        |       |        |        |        | g1f1s1c1t9R3u1Z2a2   | 1428 |
| 157                | 7 | [0  | $x_2$ | $x_3$ | $-x_2$ | $x_5$  | $x_6$ | $x_7$  | $x_8$  | $x_9$  | g1f1s1c2t11R2u17Z2a2 | 270  |
| 158                | 7 | [0  | $x_2$ | $x_3$ | 0      | $x_5$  | $x_6$ | $x_7$  | $x_8$  | $x_9$  | g1f1s1c2t3R2u11Z2a2  | 2480 |
| 159                | 7 | [0  | $x_2$ | $x_3$ | $x_4$  | 0      | $x_6$ | $x_7$  | $x_8$  | $x_9$  | g1f1s1c2t10R2u17Z2a2 | 912  |
| 160                | 7 | [0  | $x_2$ | $x_3$ | $x_4$  | $x_5$  | $x_6$ | 0      | $x_8$  | $x_9$  | g1f2s1c1t11R2u19Z2a2 | 536  |
| 161                | 7 | [0  | $x_2$ | $x_3$ | $x_4$  | $x_5$  | $x_6$ | $x_7$  | $-x_6$ | $x_9$  | g1f2s1c1t12R2u17Z2a2 | 84   |
| 162                | 7 | [0  | $x_2$ | $x_3$ | $x_4$  | $x_5$  | $x_6$ | $x_7$  | $x_8$  | 0      | g2f1s1c1t10R2u17Z2a2 | 318  |
| 163                | 7 | [ $x_1$   | 0     | $x_3$ | $x_4$  | 0      | $x_6$ | $x_7$  | $x_8$  | $x_9$  | g1f1s1c2t3R2u10Z2a2  | 640  |
| 164                | 7 | [ $x_1$   | $x_2$ | 0     | $x_4$  | $x_5$  | $x_6$ | $x_7$  | $x_8$  | 0      | g1f1s2c1t11R2u10Z2a2 | 584  |
| 165                | 7 | [ $x_1$   | $x_2$ | $x_3$ | $-x_2$ | 0      | $x_6$ | $x_7$  | $x_8$  | $x_9$  | g1f1s1c2t10R2u1Z2a2  | 48   |
| 166                | 7 | [ $x_1$   | $x_2$ | $x_3$ | $-x_2$ | $x_1$  | $x_6$ | $x_7$  | $x_8$  | $x_9$  | g1f1s1c2t10R2u11Z1a2 | 1104 |
| 167                | 7 | [ $x_1$   | $x_2$ | $x_3$ | $-x_2$ | $x_5$  | $x_6$ | $-x_3$ | $x_8$  | $x_9$  | g1f1s1c1t10R2u10Z2a2 | 32   |
| 168                | 7 | [ $x_1$   | $x_2$ | $x_3$ | $-x_2$ | $x_5$  | $x_6$ | $x_7$  | $-x_6$ | $x_9$  | g1f1s1c1t11R2u11Z2a2 | 32   |
| 169                | 7 | [ $x_1$   | $x_2$ | $x_3$ | $x_2$  | $x_5$  | $x_6$ | $-x_3$ | $x_8$  | $x_9$  | g2f2s1c1t5R2u10Z2a2  | 12   |
| 170                | 7 | [ $x_1$   | $x_2$ | $x_3$ | $x_4$  | 0      | $x_6$ | $-x_3$ | $x_8$  | $x_9$  | g1f2s1c1t12R2u1Z2a2  | 42   |
| 171                | 7 | [ $x_1$   | $x_2$ | $x_3$ | $x_4$  | 0      | $x_6$ | $x_7$  | 0      | $x_9$  | g1f1s2c1t10R2u19Z2a2 | 168  |
| 172                | 7 | [ $x_1$   | $x_2$ | $x_3$ | $x_4$  | $x_5$  | 0     | $x_7$  | $x_8$  | 0      | g1f1s2c1t10R2u11Z2a2 | 120  |
| 173                | 7 | [ $x_1$   | $x_2$ | $x_3$ | $x_4$  | $x_5$  | $x_6$ | $-x_3$ | $-x_6$ | $x_9$  | g1f1s1c1t3R2u19Z2a2  | 104  |
| 174                | 7 | [ $x_1$   | $x_2$ | $x_3$ | $x_4$  | $x_5$  | $x_6$ | $-x_3$ | $x_8$  | 0      | g2f1s1c1t10R2u13Z2a2 | 32   |
| 175                | 7 | [ $x_1$   | $x_2$ | $x_3$ | $x_4$  | $x_5$  | $x_6$ | $-x_3$ | $x_8$  | $x_1$  | g2f1s1c1t10R2u1Z2a2  | 262  |
| 176                | 8 | [0  | $x_2$ | $x_3$ | $x_4$  | $x_5$  | $x_6$ | $x_7$  | $x_8$  | $x_9$  | g1f1s1c2t11R2u19Z2a2 | 5220 |
| 177                | 8 | [ $x_1$   | $x_2$ | $x_3$ | $-x_2$ | $x_5$  | $x_6$ | $x_7$  | $x_8$  | $x_9$  | g1f1s1c2t10R2u10Z1a2 | 1232 |
| 178                | 8 | [ $x_1$   | $x_2$ | $x_3$ | $x_2$  | $x_5$  | $x_6$ | $x_7$  | $x_8$  | $x_9$  | g1f1s1c2t3R2u19Z1a2  | 1564 |
| 179                | 8 | [ $x_1$   | $x_2$ | $x_3$ | $x_4$  | $-x_1$ | $x_6$ | $x_7$  | $x_8$  | $x_9$  | g1f1s1c2t9R3u19Z2a2  | 96   |
| 180                | 8 | [ $x_1$   | $x_2$ | $x_3$ | $x_4$  | 0      | $x_6$ | $x_7$  | $x_8$  | $x_9$  | g1f1s1c2t10R2u19Z2a2 | 1104 |
| 181                | 8 | [ $x_1$   | $x_2$ | $x_3$ | $x_4$  | $x_1$  | $x_6$ | $x_7$  | $x_8$  | $x_9$  | g1f1s1c2t10R2u11Z2a2 | 384  |
| 182                | 8 | [ $x_1$   | $x_2$ | $x_3$ | $x_4$  | $x_5$  | $x_6$ | $-x_3$ | $x_8$  | $x_9$  | g2f1s1c1t10R2u10Z1a2 | 774  |
| 183                | 8 | [ $x_1$   | $x_2$ | $x_3$ | $x_4$  | $x_5$  | $x_6$ | $x_3$  | $x_8$  | $x_9$  | g2f1s1c1t11R2u11Z1a2 | 288  |
| 184                | 8 | [ $x_1$   | $x_2$ | $x_3$ | $x_4$  | $x_5$  | $x_6$ | $x_7$  | $-x_6$ | $x_9$  | g1f1s2c1t11R2u11Z1a2 | 352  |
| 185                | 8 | [ $x_1$   | $x_2$ | $x_3$ | $x_4$  | $x_5$  | $x_6$ | $x_7$  | $x_6$  | $x_9$  | g1f1s2c1t10R2u10Z1a2 | 144  |
| 186                | 8 | [ $x_1$   | $x_2$ | $x_3$ | $x_4$  | $x_5$  | $x_6$ | $x_7$  | $x_8$  | $-x_1$ | g2f1s1c1t5R3u11Z2a2  | 32   |
| 187                | 8 | [ $x_1$   | $x_2$ | $x_3$ | $x_4$  | $x_5$  | $x_6$ | $x_7$  | $x_8$  | 0      | g1f1s2c1t12R2u13Z2a2 | 1078 |
| 188                | 8 | [ $x_1$   | $x_2$ | $x_3$ | $x_4$  | $x_5$  | $x_6$ | $x_7$  | $x_8$  | $x_1$  | g2f1s1c1t10R2u19Z2a2 | 128  |

Table of particular forms of fundamental matrices.

## References

- Akaike, H., 1972. Use of an information theoretic quantity for statistical model identification. In: 5th Hawai'i Int. Conf. System Sciences. pp. 249–250.
- Aloimonos, J., Aug. 1990. Perspective approximations. Image and Vision Computing 8 (3), 179–192.
- Armstrong, M., Zisserman, A., Beardsley, P., Sep. 1994. Euclidean structure from uncalibrated images. In: Hancock, E. (Ed.), Proceedings of the 5th British Machine Vision Conference. BMVA Press, York, UK, pp. 508–518.
- de Agapito, L., Hayman, E., Reid, I. L., Sep. 1998. Self-calibration of a rotating camera with varying intrinsic parameters. In: British Machine Vision Conference. BMVA Press, Southampton, UK.
- Dementhon, D., Davis, L. S., 1989. Exact and approximate solutions to the three-point perspective problem. Tech. Rep. CAR-TR-471, Computer Vision Laboratory, University of Maryland.

- Enciso, R., Dec. 1995. Auto-calibration des capteurs visuels actifs. reconstruction 3D active. Ph.D. thesis, Université Paris XI Orsay.
- Faugeras, O., 1993. Three-Dimensional Computer Vision: a Geometric Viewpoint. MIT Press.
- Hartley, R., May 1994. Self-calibration from multiple views with a rotating camera. In: Eklundh, J.-O. (Ed.), Proceedings of the 3rd European Conference on Computer Vision. Vol. 800-801 of Lecture Notes in Computer Science. Springer-Verlag, Stockholm, Sweden, pp. 471–478.
- Horaud, R., Dornaika, F., Lamiroy, B., Christy, S., 97. Object pose: The link between weak perspective, paraperspective, and full perspective. *The International Journal of Computer Vision* 22 (2).
- Lingrand, D., Jul. 1999. Analyse adaptative du mouvement dans des séquences monoculaires non calibrées. Ph.D. thesis, Université de Nice - Sophia Antipolis, INRIA, Sophia Antipolis, France.
- Lingrand, D., Sep. 2000. Particular forms of homography matrices. In: Proceedings of the 11th British Machine Vision Conference. Vol. 2. British Machine Vision Association, BMVA Press, The University of Bristol, pp. 596–605.
- Lingrand, D., Montagnat, J., Collins, L., Gotman, J., Jun. 2001. Compensating small head displacements for an accurate fmri registration. In: Austvoll, I. (Ed.), Proceedings of the 10th Scandinavian Conference on Image Analysis. Bergen, Norway, pp. 10–16.
- Ma, Soatto, Kosecka, Sastry, 1999. Euclidean Reconstruction and Reprojection Up to Subgroups. In: Proceedings of the 7th International Conference on Computer Vision. IEEE Computer Society, IEEE Computer Society Press, Kerkyra, Greece.
- Quan, L., May 1996. Self-calibration of an affine camera from multiple views. *The International Journal of Computer Vision* 19 (1), 93–105.
- Rodrigues, O., 1840. Des lois géométriques qui régissent les déplacements d'un système solide dans l'espace, et de la variation des coordonnées provenant de ces déplacements considérés indépendamment des causes qui peuvent les produire. *Journal de Mathématiques Pures et Appliquées* 5, pp. 380–440.
- Soatto, S., Perona, P., Jun. 1995. Dynamic rigid motion estimation from weak perspective. In: Proceedings of the 5th International Conference on Computer Vision. IEEE Computer Society Press, Boston, MA, pp. 321–328.
- Viéville, T., 1994. Autocalibration of visual sensor parameters on a robotic head. *Image and Vision Computing* 12.
- Viéville, T., Lingrand, D., 1999. Using specific displacements to analyze motion without calibration. *The International Journal of Computer Vision* 31 (1), 5–29.