

Minimal parameterization of Fundamental Matrices using motion and camera properties

Diane Lingrand

To cite this version:

Diane Lingrand. Minimal parameterization of Fundamental Matrices using motion and camera properties. Robotics and Autonomous Systems, 2002, 39 (3-4), pp.169–179. 10.1016/S0921-8890(02)00202- 6 . hal-00459258

HAL Id: hal-00459258 <https://hal.science/hal-00459258v1>

Submitted on 23 Feb 2010

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

Minimal parameterization of Fundamental Matri
es using motion and amera properties.

Diane LINGRAND

INRIA - Projet RobotVis 2004 , route des Lucioles – B.P. 93 06902 Sophia Antipolis Cedex, Fran
e

Abstra
t

This paper addresses the optimal recovery of the displacement and projection parameters from un
alibrated mono
ular video sequen
es. We study the parti
ular cases of camera and objects displacements and camera projection in order to extract an optimized parameterization of the problem of parameters recovery for each cases.

This work follows previous studies on particular cases of displacement, scene geometry and amera analysis and fo
uses on the parti
ular forms of fundamental matrices. This paper introduces the idea of using not all particular cases as individual ases but grouping these ases into a tra
table number of sets, using properties on fundamental matri
es.

Some experiments were performed in order to demonstrate that if several models are correct, the model with the least parameters gives the best estimate, corresponding to the true ase.

Key words: Fundamental Matrix, Parti
ular displa
ement, Parameters estimation

$\mathbf{1}$ **Introduction**

This paper deals with video sequen
es taken by an un
alibrated amera in an unknown environment. Our interest is to estimate as many parameters as possible on the camera and objects motion and the camera projection using a strategy of hypothesis testing.

Many efforts have been made in the Computer Vision community for determining motion and amera parameters from video sequen
es. Relations between

Preprint submitted to Elsevier S
ien
e 20 August 2001

 $Email \ address: \ \mathtt{Diane}.\mathtt{Lingrand@sophia.inria.fr}$ (Diane $\mathtt{LINGRAND}$).

2D views exist, Faugeras (1993), as the fundamental matrix \bf{F} , but, in the general case, we cannot extract all the unknown parameters from this **F** matrix. It is however possible in some particular situations.

This work follows previous work on particular cases of displacement, scene geometry and amera analysis Vieville and Lingrand (1999); Lingrand (1999, 2000). It fo
uses on the parti
ular forms of fundamental matri
es.

Several authors have already been interested in particular cases of projection: Aloimonos (1990), Dementhon and Davis (1989), Horaud et al. (97), Soatto and Perona (1995), Ma et al. (1999), Quan (1996), or displa
ement: Hartley (1994),de Agapito et al. (1998), Vieville (1994), Armstrong et al. (1994). Some of them onsider several ases and ompare ea
h result, in order to automati ally determine whi
h ase was performed.

We call by general case the situation where we don't know anything about motion or amera proje
tion. A parti
ular ase is when we know (or make the hypothesis) that a parameter is null, onstant or known, or related to other parameters. A particular case has fewer parameters and/or simpler equations than the general one.

The motivations for these studies are threefold:

- to eliminate singularities of general equations by considering each case that may ondu
t to singularity,
- to estimate the parameters with more robustness using a simplified model (an adapted model gives more accuracy than the general one as shown in Vieville and Lingrand (1999), and
- to retrieve parameters that cannot be retrieved in the general case because we eliminate some unknowns that are meaningless in the particular case studied.

It is already known that the large number of particular cases prevent examining all the ases linearly. In this paper, we introdu
e a new way to deal with this amount of ases in three steps. (1) We eliminate, with some simple rules, some redundant cases and some physically impossible cases. (2) We divide the set of cases into two sets, each corresponding to homographic or fundamental relations. (3) We divide again the fundamental ases into sets orresponding to parti
ular forms. We will provide details for ea
h of these steps in the following sections.

In this section, we present the stereo framework and the notations we will use in this paper.

Rigid displacements: We consider a rigid or piecewise rigid scene. A 3Dpoint $\mathbf{M} = [X \ Y \ Z \ 1]^T$ is moving onto $\mathbf{M} = [X \ Y \ Z \ 1]^T$ by a rotation \mathbf{R} followed by a translation $\mathbf{t} = [t_0 \ t_1 \ t_2]^\top$:

$$
\mathbf{M}' = \mathbf{R} \, \mathbf{M} + \mathbf{t}
$$

A rotation matrix \bf{n} depends only on 5 parameters $\bf{r} = [r_0 \, r_1 \, r_2]$ -related to the rotation angle θ and axis **u** by:

$$
\mathbf{r} = 2 \tan(\theta/2) \mathbf{u} \Leftrightarrow \theta = 2 \arctan(\|\mathbf{r}\|/2)
$$

A rigid displa
ement us then parameterized by 6 parameters.

We note by $\tilde{\mathbf{r}}$ the antisymmetric matrix representing the cross-product $\mathbf{r} \wedge \cdot$:

$$
\forall x\ \ \tilde{r}\,x = r\wedge x
$$

The rotation matrix $\mathbf{R} = e^{\mathbf{r} \wedge \cdot} = e^{\mathbf{r}}$ can be developed as a rational Rodrigues formula, Rodrigues (1840) :

$$
\mathbf{R} = \mathbf{I} + \left[\frac{\tilde{\mathbf{r}} + \frac{1}{2}\tilde{\mathbf{r}}^2}{1 + \frac{\mathbf{r}^T \cdot \mathbf{r}}{4}}\right]
$$

Camera projection: The most commonly camera model states that a 3Dpoint $\mathbf{M} = [X \, Y \, Z \, Y]$ is projected with a perspective projection onto an image plane on a 2D-point $\mathbf{m} = \lceil u \, | \, v \, | \, 1 \rceil$. In the reference frame attached to the amera, the proje
tion equation is :

$$
Z \mathbf{m} = \begin{pmatrix} \alpha_u & \gamma & u_0 & 0 \\ 0 & \alpha_v & v_0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \mathbf{M}
$$
 (1)

where α_u and α_v represent the horizontal and vertical lengths, u_0 and v_0 correspond to the image of the optical center and γ is the skew factor. Those parameters are the intrinsi parameters and are olle
ted in the proje
tion matrix A.

Considering two frames: Let I_1 and I_2 denote two images. In the general case, there exists a fundamental relation, Faugeras (1993), between points m_2 in I_2 and points m_1 in I_1 :

$$
\mathbf{m_2}^T\,\mathbf{F}\,\mathbf{m_1}=\mathbf{0}
$$

where \bf{F} is called the fundamental matrix and is related to the intrinsic and extrinsi parameters by :

$$
\mathbf{F} = (\mathbf{A}_2 \, \mathbf{t}) \, \mathbf{A}_2 \, \mathbf{R} \, \mathbf{A}_1^{-1}
$$

where A_1 and A_2 are the projection matrix for the first and second frames, see (1).

This kind of relationship vanishes if the displa
ement is a pure rotation or if the s
ene is planar. The relation between points is homographi :

$$
\mathbf{m_2}=\mathbf{H} \, \mathbf{m_1}
$$

where H is called the homographic matrix. Another study on homographic matri
es an be found in Lingrand (2000).

3 Deriving all particular cases

In order to study all particular cases of cameras and objects displacements and camera projection, we will examine each particular value, considering each parameter at a time. A particular model is obtained by combining several parti
ular values.

3.1 Particular cases of intrinsic parameters

Authors generally make several hypotheses regarding intrinsi parameters. For example, the most general autoalibration hypothesis states that the intrinsi parameters are onstant. They an be known or unknown. However, usually, some parameters are onstant while others are not.

- The principal point of coordinates (u_0, v_0) can be fixed and/or known in some cases (for example, in the image center), thus changing the reference frame, regarding the principal point position.
- The γ parameter is usually assumed to be null or, at least, considered to be a onstant value.

Table 1

Table of particular cases of intrinsic parameters for 2 frames

• Enciso (1995) has experimentally proven that for a large number of cameras α_u/α_v can be considered to be constant even if other intrinsic parameters change. We express this as $f = \alpha_u = \alpha_v$.

The table 1 summarizes, for each intrinsic parameter, the particular cases of interest (
onstant values are indexed by zero). Subsequently, we will refer to each case by the label given in the first column. For example, g1 means that the γ parameter is null.

3.2 Parti
ular ases of displa
ement

Discrete motion - continuous motion: In an image sequence, if the displa
ement between two frames is small, we an approximate the rotation equations by their first order :

$$
\mathbf{R} = e^{\tilde{\mathbf{r}}} = \mathbf{I} + \tilde{\mathbf{r}} + o(\tilde{\mathbf{r}})
$$

which occurs frequently in images sequences except with high speed objects.

If the motion is larger, we can also consider the second order expansion

$$
\mathbf{R} = \mathbf{I} + \tilde{\mathbf{r}} + \frac{\tilde{\mathbf{r}}^2}{2} + o(\tilde{\mathbf{r}}^2)
$$

About extrinsic parameters: The rotation parameters are related to the rotation axis and the rotation angle by : $\mathbf{r} =$ z tan $\frac{1}{2} \mathbf{u}$ where \mathbf{u} is a unitary vector giving the direction of the rotation axis.

Some components of **u** can be known or null. Some value of θ may yield singularities; $\theta = \frac{1}{4}$ and the rotation axis is parallel to the translation vector for a s
rew displa
ement.

Some robotic systems give precise values of the robot displacements (angle, axis, translation). Some values may be known (we denote by \mathcal{A}_0 a constant and known value of a parameter θ). Other informations regarding parallelism or orthogonality to a known direction or to an other vector may also be available:

The rotation axis is orthogonal to the translation plane (e.g. planar motion) :

$$
\mathbf{r} \perp \mathbf{t} \Leftrightarrow \mathbf{r}.\mathbf{t} = 0
$$

• screw displacement :

 $\mathbf{r} \parallel \mathbf{t} \Leftrightarrow \exists \kappa / \mathbf{r} = \kappa \mathbf{t}$

All constraints on motion: All these constraints, also called "atomic particular ases", have simple expressions that an be easily ombined. In this purpose, we use the fact that **u** is a unary vector and that, for monocular systems, the norm of translation cannot be recovered. To parameterize these vectors with only 2 parameters, we divide each component by a non-zero component. Then, the dot-produ
t and s
alar produ
t indu
e linear relations. For example, $t_2 = 1$ and $t \perp r$ are equivalent to $t_0 u_0 + t_1 u_1 + u_2 = 0 \Rightarrow u_2 = -t_0 u_0 - t_1 u_1$

All ases are olle
ted in the table 2.

Generating all cases: All particular cases, each called a "molecular case", are generated by combining the atomic cases and solving the constraints by a substitution f. A molecular case is composed of one case in each lamily, a family being named by a letter $(g, s, f \text{ or } c \text{ for projection as seen in table 1})$ and u , R , a , t or Z for motion as seen in table 2). Thus, a molecular case is identied by the sequen
e :

 $g[1-3]f[1-3]s[1-3]c[1-3]R[1-4]a[1-2]u[1-24]t[1-12]Z[1-3]$

where $g[1-3]$ means "one atomic case among $g1, g2$ and $g3$ ".

How many cases do we have? If we look at the expression of the particular above-mentioned cases, we obtain 6.10° particular cases. However, this is not the real number because of the incompatibility of some atomic cases and the redundancy of some constraints. Two different sets of atomic constraints can generate the same simplied model.

t in this was done using maple software for symbolic computations.

Table 2

Table of particular cases of displacements

It is easy to eliminate in
ompatible onstraints. It is not possible to deal with redundant constraints, because this requires to compare each set of combined onstraints with all others in order to determine the similarity. The omplexity of this process is $O(n^2)$.

Although we annot remove redundant ases, we propose an adapted strategy to deal with the large number of cases. The idea of this paper is : (i) to eliminate some of the redundant cases by using some considerations on the atomic cases and (ii) to limit the number of cases by studying the particular forms of the matri
es.

Reducing the number of cases: Some redundancy are obvious :

- In case (R1), one case of axis and angle is considered.
- In cases (R2) and (R3), we do not consider (a1) when θ is equal to $\frac{1}{2}$.
- The case (a1) is only considered if $\mathbf{r} \parallel \mathbf{t}$, (Z2).

This redu
es the amount of ases of fundamental relations to only 216756 ases.

4 Forms of fundamental matri
es

We have significantly reduced the number of cases but this is not small enough to be omputationally tra
table. We now split fundamental relations in sets of matri
es by forms. The matrix form is determined using simple rules in order to obtain a very simple parameterization. We onsider (3-3) matri
es having 9 parameters (
oeÆ
ients). If a oeÆ
ient is equal to zero, then there is one less parameter. If a coefficient has the same expression or is opposite to another, there is one less parameter again. These operations are very simple and an be rapidly computed in each case. Furthermore, we know that a fundamental matrix is defined up to a scale factor, and that its determinant is fixed to 0 (removing in most ases one parameter). This pro
ess redu
es the 216756 ases to only 188 subgroups.

The table in appendix A shows all the simplified forms obtained, and, for each form, an example of ase that has generated it. This table will be useful for people who want to implement the algorithm.

5 Experiments

We have recorded several video sequences for which the camera displacement induces a fundamental relation between image points m_1 and m_2 . From each parti
ular matrix form, we have estimated the fundamental matrix parameters with the robust least median square method in order to minimize the distance between a 2D point m_1 and its epipolar line $F m_2$. To deal with cases with different degrees of freedom, we use an appropriate Akaike criterion, Akaike (1972).

Fig. 1. Images for x-axis translation, small pan rotation and auto-fo
us

For each recorded video sequence, we have verified that the model with the minimal residual error effectively corresponds to the displacement performed by the robotic system. We present one experiment in figure 1 for which the amera has performed a small pan rotation followed by a translation parallel to the x-axis. The auto-fo
us was also enabled. The ase with the minimal residual error orresponds to the fundamental matrix form number 59 in the table given in appendix A :

$$
\mathbf{F} = \begin{pmatrix} 0 & 0 & 0 \\ x_0 & x_1 & x_2 \\ 0 & -x_2 & x_3 \end{pmatrix}
$$

This parti
ular form was obtained from ases where the rotation was approximated to its first and second order, the translation is parallel to the x-axis, the rotation axis is orthogonal to the optical axis and the intrinsic parameters are free.

6 Con
lusion

In an earlier study on homographic matrices Lingrand (2000), we have shown that it is possible to redu
e the amount of parti
ular ases in order to make the ase sele
tion omputationally feasible. In this paper, we have shown that a similar result can be obtained with fundamental matrices using redundancies. We have experimentally confirmed that our system is able to automatically select the case corresponding to the performed displacement.

The applications are twofold: (i) an incremental reconstruction of the scene and (ii) the segmentation of objects moving with different displacements or with different geometric properties in video sequences.

This work has also been extended to motion estimation of human head inside MRI scanner, improving the registration of fMRI volumes, Lingrand et al. $(2001).$

Appendix A : Table of particular forms of fundamental matrices.

We denote by \mathbf{n}° the form number, by p the number of parameters (we have not taken into account the fact that the fundamental matrix is defined up to a scale factor and that $\det \mathbf{F} = 0$ but we do so in our implementation) and by **n** the number of molecular cases that have generated a form.

	from previous page											
155	6	$\lfloor x_1 \rfloor$	x_2	x_3	x_4	x_{5}	x_6	$-x_3$	$-x_6$	x_1	g2f1s1c1t3R2u1Z2a2	24
156	6	$\lceil x_1 \rceil$	x_2	x_{3}	x_4	x_{5}	x_{6}	Ω	$\mathbf{0}$	0 1	g1f1s1c1t9R3u12Z2a2	1428
157	$\overline{7}$	$\overline{0}$	x_2	x_3	$-x_2$	x_{5}	x_{6}	x_7	x_8	x_9	g1f1s1c2t11R2u17Z2a2	270
158	$\overline{7}$	$\overline{0}$	x_2	x_3	Ω	x_{5}	x_{6}	x_7	x_{8}	x_9	g1f1s1c2t3R2u11Z2a2	2480
159	$\overline{7}$	$\overline{0}$	x_2	x_3	x_4	0	x_{6}	x_7	x_{8}	x_9	g1f1s1c2t10R2u17Z2a2	912
160	$\overline{7}$	$\overline{10}$	x_2	x_3	x_4	x_{5}	x ₆	0	$x_{\,8}$	x_9	g1f2s1c1t11R2u19Z2a2	536
161	$\overline{7}$	$\overline{0}$	x_2	x_3	x_4	x_{5}	x ₆	x_7	$-x_6$	x_9	g1f2s1c1t12R2u17Z2a2	84
162	$\overline{7}$	$\overline{10}$	x_2	x_3	x_4	x_{5}	x ₆	x_7	$x_{\,8}$	0 ¹	g2f1s1c1t10R2u17Z2a2	318
163	$\overline{7}$	$\left[x_1\right]$	$\mathbf{0}$	x_3	x_4	0	x_6	x_{7}	x_{8}	x_9	g1f1s1c2t3R2u10Z2a2	640
164	$\overline{7}$	$\left[x_{1}\right]$	x_2	0	x_4	x_{5}	x_6	x_7	x_{8}	0 ¹	g1f1s2c1t11R2u10Z2a2	584
165	$\overline{7}$	$ x_1$	x_2	x_3	$-x_2$	0	x_{6}	x_7	x_{8}	x_9	g1f1s1c2t10R2u1Z2a2	48
166	$\overline{7}$	$ x_1 $	x_2	x_{3}	$-x_2$	x_1	x_{6}	x_7	x_{8}	x_9	g1f1s1c2t10R2u11Z1a2	1104
167	$\overline{7}$	$\lfloor x_1 \rfloor$	x_2	x_3	$-x_2$	x_{5}	x_6	$-x_3$	x_{8}	x_9	g1f1s1c1t10R2u10Z2a2	32
168	$\overline{7}$	$\left x_1 \right $	x_2	x_3	$-x_2$	x_{5}	x_6	x_7	$-x_6$	x_9	g1f1s1c1t11R2u11Z2a2	32
169	$\overline{7}$	$\left x_1 \right $	x_2	x_3	x_2	x_{5}	x_6	$-x_3$	$x_{\,8}$	x_9	g2f2s1c1t5R2u10Z2a2	12
170	$\overline{7}$	$\left[x_1\right]$	x_2	x_3	x_4	0	x ₆	$-x_3$	x_{8}	x_9	g1f2s1c1t12R2u1Z2a2	42
171	$\overline{7}$	$ x_1 $	x_2	x_3	x_4	0	x ₆	x_7	Ω	x_9	g1f1s2c1t10R2u19Z2a2	168
172	$\overline{7}$	$\left[x_{1}\right]$	x_2	x_3	x_4	x_{5}	$\bf{0}$	x_7	x_{8}	0 ¹	g1f1s2c1t10R2u11Z2a2	120
173	$\overline{7}$	$ x_1$	x_2	x_3	x_4	x_{5}	x ₆	$-x_3$	$-x_6$	x_9	g1f1s1c1t3R2u19Z2a2	104
174	$\overline{7}$	$ x_1$	x_2	x_{3}	x_4	x_{5}	x_6	$-x_3$	x_{8}	0 ¹	g2f1s1c1t10R2u13Z2a2	32
175	$\overline{7}$	$ x_1$	x_2	x_3	x_4	x_{5}	x ₆	$\sim x_3$	x_{8}	x_1	g2f1sic1t10R2u1Z2a2	262
176	8	$ 0\rangle$	x_2	x_3	x_4	x_{5}	x_{6}	x_7	x_{8}	x_9	g1f1s1c2t11R2u19Z2a2	5220
177	8	$\left[x_1\right]$	x_2	x_3	$-x_2$	x_{5}	x_6	x_7	x_{8}	x_{9}	g1f1s1c2t10R2u10Z1a2	1232
178	8	$\left[x_1\right]$	x_2	x_{3}	x_2	x_{5}	x_6	x_7	x_{8}	x_9	g1f1s1c2t3R2u19Z1a2	1564
179	8	$ x_1$	x_2	x_3	x_4	$-x_1$	x_6	x_7	x_{8}	x_9	g1f1s1c2t9R3u19Z2a2	96
180	8	$ x_1$	x_2	x_3	x_4	0	x ₆	x_7	x_{8}	x_9	g1f1s1c2t10R2u19Z2a2	1104
181	8	$ x_1$	x_2	x_3	x_4	x_1	x ₆	x_7	x_8	x_9	g1f1s1c2t10R2u11Z2a2	384
182	8	$\left[x_{1}\right]$	x_2	x_3	x_4	x_{5}	x ₆	$-x_3$	x_{8}	x_9	g2f1s1c1t10R2u10Z1a2	774
183	8	$ x_1 $	x_2	x_3	x_4	x_{5}	x_6	x_3	x_{8}	x_9	g2f1s1c1t11R2u11Z1a2	288
184	8	$ x_1$	x_2	x_{3}	x_4	x_{5}	x_{6}	x_7	$-x_6$	x_9	g1f1s2c1t11R2u11Z1a2	352
185	8	$\left[x_1\right]$	x_2	x_3	x_4	x_{5}	x_6	x_7	x_6	x_9	g1f1s2c1t10R2u10Z1a2	144
186	8	$ x_1$	x_2	x_{3}	x_4	x_{5}	x_{6}	x_7	x_{8}	$-x_1$	g2f1s1c1t5R3u11Z2a2	32
187	8	$ x_1$	x_2	x_3	x_4	x_{5}	x ₆	x_7	x_{8}	0 ¹	g1f1s2c1t12R2u13Z2a2	1078
188	8	$ x_1$	x_2	x_3	x_4	x_{5}	x ₆	x_7	x_{8}	x_1	g2f1s1c1t10R2u19Z2a2	128

Table of particular forms of fundamental matrices.

References

- Akaike, H., 1972. Use of an information theoretic quantity for statistical model identification. In: 5th Hawaï Int. Conf. System Sciences. pp. 249-250.
- Aloimonos, J., Aug. 1990. Perspective approximations. Image and Vision Computing $8(3)$, 179-192.
- Armstrong, M., Zisserman, A., Beardsley, P., Sep. 1994. Euclidean structure from uncalibrated images. In: Hancock, E. (Ed.), Proceedings of the 5th British Machine Vision Conference. BMVA Press, York, UK, pp. 508-518.
- de Agapito, L., Hayman, E., Reid, I. L., Sep. 1998. Self-calibration of a rotating camera with varying intrinsic parameters. In: British Machine Vision Conference. BMVA Press, Southampton, UK.
- Dementhon, D., Davis, L. S., 1989. Exact and approximate solutions to the threepoint perspective problem. Tech. Rep. CAR-TR-471, Computer Vision Laboratory, University of Maryland.
- Enciso, R., Dec. 1995. Auto-calibration des capteurs visuels actifs. reconstruction 3D active. Ph.D. thesis, Université Paris XI Orsay.
- Faugeras, O., 1993. Three-Dimensional Computer Vision: a Geometric Viewpoint. MIT Press.
- Hartley, R., May 1994. Self-calibration from multiple views with a rotating camera. In: Eklundh, J.-O. (Ed.), Proceedings of the 3rd European Conference on Computer Vision. Vol. 800-801 of Lecture Notes in Computer Science. Springer-Verlag, Stockholm, Sweden, pp. 471–478.
- Horaud, R., Dornaika, F., Lamiroy, B., Christy, S., 97. Object pose: The link between weak perspective, paraperspective, and full perspective. The International Journal of Computer Vision 22 (2).
- Lingrand, D., Jul. 1999. Analyse adaptative du mouvement dans des séquences monoculaires non calibrées. Ph.D. thesis, Université de Nice - Sophia Antipolis, INRIA, Sophia Antipolis, France.
- Lingrand, D., Sep. 2000. Particular forms of homography matrices. In: Proceedings of the 11th British Machine Vision Conference. Vol. 2. British Machine Vision Association, BMVA Press, The University of Bristol, pp. 596–605.
- Lingrand, D., Montagnat, J., Collins, L., Gotman, J., Jun. 2001. Compensating small head displacements for an accurate fmri registration. In: Austroll, I. (Ed.), Proceedings of the 10th Scandinavian Conference on Image Analysis. Bergen, Norway, pp. $10-16$.
- Ma, Soatto, Kosecka, Sastry, 1999. Euclidean Reconstruction and Reprojection Up to Subgroups. In: Proceedings of the 7th International Conference on Computer Vision. IEEE Computer Society, IEEE Computer Society Press, Kerkyra, Greece.
- Quan, L., May 1996. Self-calibration of an affine camera from multiple views. The International Journal of Computer Vision 19 (1), 93-105.
- Rodrigues, O., 1840. Des lois géométriques qui régissent les déplacements d'un système solide dans l'espace, et de la variation des coordonnées provenant de ces déplacements considérés indépendamment des causes qui peuvent les produire. Journal de Mathématiques Pures et Appliquées 5, pp. 380–440.
- Soatto, S., Perona, P., Jun. 1995. Dynamic rigid motion estimation from weak perspective. In: Proceedings of the 5th International Conference on Computer Vision. IEEE Computer Society Press, Boston, MA, pp. 321-328.
- Viéville, T., 1994. Autocalibration of visual sensor parameters on a robotic head. Image and Vision Computing 12.
- Viéville, T., Lingrand, D., 1999. Using specific displacements to analyze motion without calibration. The International Journal of Computer Vision 31 (1) , 5–29.