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# Spin-squeezing and Dicke state preparation through single-photon heterodyne measurement

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We investigate the collapse of a coherent superposition of two-level atoms induced by a single-photon heterodyne quantum non-demolition (QND) measurement, and show that such a measurement process leads to the generation of spin-squeezed and Dicke states. We describe the stochastic process of the measurement and the associated atomic evolution in a wavevector formalism. Analytical formulas of the atomic distribution momenta are derived in the weak coupling regime for both short and long time behavior and are in good agreement with those from a Monte-Carlo simulation.

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Generation of highly entangled collective spin states such as spin-squeezed states (SSS) [1, 2] and Dicke states [3–5] is of first importance for quantum information [6] or to perform sub-shot-noise precision measurements [7–10]. To create collective entanglement, a non-linear evolution of the system is required, usual ways are: the use of interactions between the particles [11], probing with squeezed light [12] or cavity mediated interaction [9]. The strong non-linearity of a measurement (due to the state renormalization) can also lead to such states [13–16].

Measurement induced spin-squeezed atomic ensembles have been created using either a Mach-Zehnder interferometer and simultaneous interaction with several optical frequencies [17] or cavity transmission modulation [9]. Nevertheless, the experimental realization of highly engineered atomic states is still challenging and a deep understanding of the underlying stochastic process of the measurement is needed. The dynamic of the atomic state collapse can be described as a succession of partial measurements. Several methods have been used to model this collapse process, like stochastic Schrödinger equation [4, 18], master equation [19] or wavevector formalism [13]. Moreover, partial measurements allows to prepare almost deterministic states implementing a quantum feedback loop [4, 20–22], it can also be exploited for quantum error correction [23].

In this article, we show that a heterodyne detection based on frequency-modulation spectroscopy [24, 25] of an atomic sample can generate both a SSS and a Dicke state. With such a method, the frequency components can be spatially overlapped in a single probe beam making the detection strongly immune to common mode dephasing sources (*e.g.* acoustic noise and refraction index fluctuations). To model this detection scheme, we use the wavevector formalism previously introduced for homodyne detection [13]. This approach shows strong similarities

with the method used in [26, 27] to reconstruct the photonic state from the results of a QND measurement sequence. We derive analytical expressions for short and long time evolution of the atomic system and compare it to Monte-Carlo numerical simulations. In particular, we show that the evolution of the atomic distribution variance is deterministic at short time scales whereas it exhibits a stochastic behavior for long measurement times.

We consider a coherent ensemble of two levels atoms and study the evolution of the population difference between the two levels. The QND measurement is realized by estimating the dephasing induced by the atomic sample on a set of far off-resonance single-photons sent one by one through the atomic cloud. Since the atom-light interaction entangles the atoms and the photon, the detection of a photon performs a conditional measurement of the atomic distribution. The measurement induced backaction results in a collapse of the initial coherent spin state (CSS) into a SSS and a Dicke state.

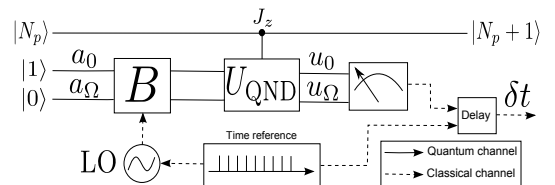


FIG. 1: Scheme of the heterodyne detection QND device. A single-photon is sent through a spectral beamsplitter  $B$  and interacts non-destructively with the atomic ensemble ( $U_{\text{QND}}$ ). The detection time of the photon is then measured.

The QND apparatus considered is shown in Fig. 1. The input optical state is composed of two spectral modes. The mode (0) at a frequency  $\omega_0/2\pi$  is close to the frequency of the atomic transition, and will therefore be strongly phase shifted when traveling through the atoms. The mode ( $\Omega$ ) is at a far off resonance frequency ( $\omega_0 + \Omega)/2\pi$  will be unaffected. A single-photon

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state  $|1\rangle$  in the mode (0) is associated to the photon annihilation operator  $a_0$ , and the vacuum state  $|0\rangle$  in the mode ( $\Omega$ ) is associated to  $a_\Omega$ . A single side-band optical phase modulator  $B$  is used as a “spectral beamsplitter” to generate spectrally mode-entangled single-photons. A local oscillator (LO) drives  $B$  with frequency  $\Omega/2\pi$ . We consider the driving signal as a noise-free classical field. After passing through the atoms, the two-channel-optical-state is sent to a single-photon detector which is assumed to be perfect. Detecting a photon in the mode (0) is associated with the operator  $u_0$ , whereas detecting a photon in the mode ( $\Omega$ ) is associated with  $u_\Omega$ . If the coherence length of the optical source is much larger than the LO wavelength, the two modes (0) and ( $\Omega$ ) are in a coherent superposition, and the spatial overlap of these two modes generates a beating at frequency  $\Omega/2\pi$ . A pulse generator, in phase with the LO, delivers regularly spaced pulses (at time interval  $\tau = 2\pi/\Omega$ ) used as a reference to measure the arrival time of the photons.

The atomic ensemble is a collection of a fixed number  $N_{\text{at}}$  of two-levels atoms  $\{|a_i\rangle, |b_i\rangle\}$ , described as angular momenta  $\{j_x^{(i)}, j_y^{(i)}, j_z^{(i)}\}$  where  $j_x^{(i)} = |b_i\rangle\langle a_i| + |a_i\rangle\langle b_i|$ ,  $j_y^{(i)} = i(|a_i\rangle\langle b_i| - |b_i\rangle\langle a_i|)$  and  $j_z^{(i)} = |b_i\rangle\langle b_i| - |a_i\rangle\langle a_i|$ . The atomic ensemble is characterized by the collective spin operators  $J_k = \sum_i j_k^{(i)}$ ,  $k = x, y, z$ . A collective state  $|n\rangle$  is an eigenstate of both  $J_z$  and  $\mathbf{J}^2 = J_x^2 + J_y^2 + J_z^2$ , and is called a Dicke state. Since  $J_z|n\rangle = n|n\rangle$ ,  $n$  is the population difference between the two atomic levels. After  $N_p$  photons detected the atomic state is  $|N_p\rangle = \sum_n c_n(N_p)|n\rangle$ . The initial atomic state is a CSS polarized along  $J_x$ , which means an average population difference  $\langle J_z \rangle = 0$ . For  $N_{\text{at}} \gg 1$ , the coefficients  $c_n(0)$  are well approximated by a gaussian distribution:

$$c_n(0) = \frac{1}{2^{N_{\text{at}}/2}} \sqrt{\binom{N_{\text{at}}}{n}} \sim \frac{1}{\sqrt{\pi N_{\text{at}}}} \exp\left(-\frac{n^2}{N_{\text{at}}}\right). \quad (1)$$

The spectral beamsplitter  $B$  creates a coherent superposition of the two input spectral modes (0) and ( $\Omega$ ). Its action is described by the following matrix acting on  $(a_0, a_\Omega e^{i\Omega t})$ :

$$B \propto \begin{pmatrix} \sqrt{T} & -\sqrt{R} \\ \sqrt{R}e^{i\Omega t} & \sqrt{T}e^{i\Omega t} \end{pmatrix}, \quad (2)$$

where, in analogy with a spatial beamsplitter,  $R$  is the probability for a photon in the carrier mode (0) to be in the side-band mode ( $\Omega$ ) at the output of  $B$ , and  $T$  is the probability for a photon to stay in the carrier mode. We neglect any absorption and dispersion in  $B$  so that  $R + T = 1$ . The phase-shift induced by a single atom ( $n = 1$ ) at  $\omega_0$  is  $\phi \ll 1$ . When  $\omega_0 + \Omega$  is far from any atomic transition, it experiences no phase-shift and the QND interaction can be written as

$$U_{\text{QND}} = \begin{pmatrix} e^{i\phi J_z} & 0 \\ 0 & 1 \end{pmatrix}. \quad (3)$$

In the present work the absorption of the probe by the atoms and the decoherence induced is neglected, the problem is treated in [13]. The scattering matrix of this system is  $S = U_{\text{QND}}B$ :

$$S \propto \begin{pmatrix} \sqrt{T} & -\sqrt{R} \\ \sqrt{R}e^{i(\Omega t - \phi J_z)} & \sqrt{T}e^{i(\Omega t - \phi J_z)} \end{pmatrix}. \quad (4)$$

A click at the detector corresponds to the annihilation of a photon either in mode (0) or ( $\Omega$ ). After the measurement the state of the system satisfies the relation  $|0_0, 0_\Omega\rangle \otimes |N_p + 1\rangle \propto (u_0 + u_\Omega)|1_0, 0_\Omega\rangle \otimes |N_p\rangle$ . The stochastic recurrence relation defining the atomic state evolution after one measurement gives

$$|N_p + 1\rangle \propto [(\sqrt{T} + \sqrt{R}) \cos((\phi J_z - \Omega \tilde{t})/2) + i(\sqrt{T} - \sqrt{R}) \sin((\phi J_z - \Omega \tilde{t})/2)] |N_p\rangle, \quad (5)$$

where  $\tilde{t}$  is the stochastic detection time of the photon. This expression contains a beatnote signal at frequency  $\Omega/2\pi$  with a contrast  $\mathcal{C} = 2\sqrt{RT}$ .

The reconstruction of this beatnote photon after photon is achieved by comparing the arrival time  $t_k$  of the  $k$ -th photon with the last time reference pulse  $p\tau$ , where  $p \in \mathbb{N}$  is the number of pulses counted during  $t_k$ . We estimate the phase-shift  $\tilde{\varphi}_k \equiv \Omega \delta t_k$  from the time delay  $\delta t_k = t_k - p\tau$  between the last pulse from the time reference and the photon detection. The atomic state after  $N_p$  photons detected becomes  $|N_p\rangle = \sum_n \mathcal{F}_{N_p}(n) c_n(0) |n\rangle$ , where

$$\mathcal{F}_{N_p}(n) \propto \prod_{k=1}^{N_p} [(\sqrt{T} + \sqrt{R}) \cos((\phi n - \tilde{\varphi}_k)/2) + i(\sqrt{T} - \sqrt{R}) \sin((\phi n - \tilde{\varphi}_k)/2)] \quad (6)$$

It is straightforward to calculate the probability  $P(\varphi)$  to measure a phase-shift  $\varphi$  when detecting the  $(N_p + 1)$ -th photon:

$$P(\varphi) = \frac{1}{2\pi} \sum_n |c_n(N_p)|^2 (1 + \mathcal{C} \cos(\phi n - \varphi)). \quad (7)$$

In the weak coupling regime the phase shift determined by the whole atomic sample is small, then  $\phi n \ll 1$  for  $n \in [-N_{\text{at}}, N_{\text{at}}]$ . We can develop Eq. (7) at first order in  $\phi n$  around a centered probability distribution  $P_0(\varphi) = (1 + \mathcal{C} \cos \varphi)/2\pi$ . We find  $P(\varphi) = P_0(\varphi) + \delta P[\delta \tilde{\varphi}](\varphi)$ , where  $\delta P[\delta \tilde{\varphi}](\varphi) = -\mathcal{C} \sin \varphi \delta \tilde{\varphi}/2\pi$  and  $\delta \tilde{\varphi} = -\phi \langle J_z \rangle$  is a stochastic parameter depending on the followed trajectory.  $P_0(\varphi)$  is the contribution for a zero phase detection, whereas  $\delta P[\delta \tilde{\varphi}](\varphi)$  describes the atomic phase-shift.

To study the evolution of the atomic state, we split the measurement into sequences of  $N_t$  photon detections, each inducing a negligible evolution of the atomic wavefunction. It is achieved if the signal-to-noise ratio over one sequence is small ( $\phi^2 N_{\text{at}} N_t \ll 1$ ). In the weak coupling limit the number of photons in each sequence can

be large since  $\phi^2 N_{\text{at}} \ll 1$ . The detection probability at the end of the  $j$ -th sequence of  $N_t$  measurements is  $P_j(\varphi) = P_0(\varphi) + \delta P[\delta\tilde{\varphi}_j](\varphi)$ . If we choose a phase measurement resolution  $\pi/m$  ( $m \in \mathbb{N}$  and  $m \gg 1$ ), we find from Eq. (6):

$$|\mathcal{F}_{N_p}(n)|^2 \propto \prod_{j=1}^{N_p/N_t} \prod_{l=-m}^m f_{N_p}^{(j,l)}(n), \quad (8)$$

where we have defined:

$$f_{N_p}^{(j,l)}(n) = [1 + \mathcal{C} \cos(\phi n - \pi l/m)]^{N_{j,l}}, \quad (9)$$

and  $N_{j,l} = \frac{\pi}{m} P_j(\frac{\pi l}{m}) N_t$  is the number of measurements giving the result  $\varphi = \pi l/m$  during the  $j$ -th sequence. For  $N_{j,l} \gg 1$  (which is verified for  $N_t \gg m^3$  [30]), the second order Taylor expansion of  $f_{N_p}^{(j,l)}(n)$  around  $\phi n = 0$  can be identified with that of a gaussian distribution:

$$f_{N_p}^{(j,l)}(n) \propto \exp[-2M_l^2 N_{j,l} (n - n_l)^2], \quad (10)$$

where

$$M_l^2 = \frac{\mathcal{C}\phi^2}{4} \frac{\mathcal{C} + \cos(\pi l/m)}{(1 + \mathcal{C} \cos(\pi l/m))^2}, \quad (11)$$

$$n_l = \frac{1}{\phi} \frac{1 + \mathcal{C} \cos(\pi l/m)}{\mathcal{C} + \cos(\pi l/m)} \sin(\pi l/m). \quad (12)$$

By converting the discrete sum over  $l$  into an integral, Eq. (8) and Eq. (10) give finally:

$$|\mathcal{F}_{N_p}(n)|^2 \propto \exp[-2M^2 N_p (n^2 + 2\overline{\delta\tilde{\varphi}}n/\phi)], \quad (13)$$

where  $\overline{\delta\tilde{\varphi}} = \frac{N_t}{N_p} \sum_{j=1}^{N_p/N_t} \delta\tilde{\varphi}_j$  is the average mean position over the followed trajectory, and  $M^2 = \phi^2(1 - \sqrt{1 - \mathcal{C}^2})/4$  is the measurement strength.

It is interesting to compare the initial atomic state distribution  $c_n(0)$  from Eq. (1) with  $\mathcal{F}_{N_p}(n)$ . Depending on their relative width, two regimes have to be considered during the atomic state evolution: the short time limit - where the width of the atomic distribution  $\Delta J_z^2$  is large and the atomic state is in a superposition of many atomic levels - and the long time limit where the distribution is very narrow and the state is split on a few levels of amplitude  $\mathcal{F}_{N_p}(n)$ . The boundary between these two regimes occurs for  $\Delta J_z^2 \sim 1$ , *i.e.* for  $N_p \sim M^{-2}$ .

In the short time limit, defined by the condition  $N_p \ll M^{-2}$ ,  $|\mathcal{F}_{N_p}(n)|^2$  is broad and  $|c_n(N_p)|^2$  is a gaussian distribution characterized by the following mean position and variance:

$$\langle J_z \rangle = -\mathcal{C}^2 \xi^2 \kappa^2 \overline{\delta\tilde{\varphi}}/\phi, \quad (14)$$

$$\Delta J_z^2 = \xi^2 N_{\text{at}}/4, \quad (15)$$

where  $\kappa^2 = M^2 N_{\text{at}} N_p$  is the signal-to-noise ratio and  $\xi^2 = 1/(1 + \kappa^2)$  is the squeezing factor. For  $N_p \geq 1$ ,

the squeezing factor drops below unity and the initial CSS collapses into a SSS as a consequence of the measurement process. The remarkable result is that  $\Delta J_z^2$  is independent of the stochastic parameter  $\overline{\delta\tilde{\varphi}}$  at first order and is thus deterministic, as shown with the numerical simulation in Fig. 3 (b).

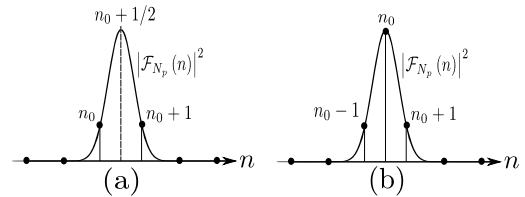


FIG. 2: Extreme cases for long time evolution. (a) The mean position of the atomic distribution is in the middle of two eigenvalues of  $J_z$ . (b) The mean position of the atomic distribution is centered on an eigenvalue of  $J_z$ .

In the long time limit ( $N_p \gg M^{-2}$ ) the atomic distribution is very narrow ( $\Delta J_z^2 \ll 1$ ) and the atomic state  $c_n(N_p) \sim \mathcal{F}_{N_p}(n)$  is spread over a few eigenstates. The evolution strongly depends on the distance between the mean value  $\langle J_z \rangle$  and the closest eigenvalue of  $J_z$ . Two extreme cases can be considered (Fig. 2): either  $\langle J_z \rangle$  is in the middle of two eigenvalues, which turns out to be an unstable equilibrium of the system, or it is an eigenvalue of  $J_z$ , corresponding to the stable equilibrium state (for a discussion on the stability of the attractors see [4, 28]). In the first case shown in Fig. 2 (a) the distribution is centered between two eigenstates of  $J_z$ , namely  $|n_0\rangle$  and  $|n_0 + 1\rangle$ . The atomic state is well approximated by a superposition of these two states  $|N_p\rangle \approx (|n_0\rangle + |n_0 + 1\rangle)/\sqrt{2}$ , since the amplitude of the other states decreases exponentially. In this case  $\langle J_z \rangle = n_0 + 1/2$  and the variance is  $\Delta J_z^2 = \langle N_p | J_z^2 | N_p \rangle - \langle J_z \rangle^2 = 1/4$ , which results to be the upper bound value in the long time limit. This limit clearly appears in the numerical calculations, as shown in Fig. 3 (b). The second case represented in Fig. 2 (b) corresponds to an atomic distribution centered on an eigenstate  $|n_0\rangle$  of  $J_z$ . The atomic state is then a superposition of three eigenstates  $|N_p\rangle \approx \sum_{k=-1,0,1} \mathcal{F}_{N_p}(n_0 + k) |n_0 + k\rangle$ . From the normalization condition, the symmetry of the state around  $n_0$ , and Eq. (13), we have the following relations:

$$\sum_{k=-1,0,1} |\mathcal{F}_{N_p}(n_0 + k)|^2 = 1 \quad (16)$$

$$|\mathcal{F}_{N_p}(n_0 \pm 1)|^2 = |\mathcal{F}_{N_p}(n_0)|^2 e^{-2M^2 N_p}. \quad (17)$$

Since  $\langle J_z \rangle = n_0$ , it follows  $\Delta J_z^2 = 2 \exp(-2M^2 N_p)$  which is the lower bound for the variance evolution, as plotted in Fig. 3 (b).

We have verified this behavior using a numerical simulation of the quantum trajectories (Fig. 3). The initial atomic state is given by Eq. (1). For each photon, the probability  $P_{N_p+1}(\varphi)$  to measure a phase  $\varphi$  for the

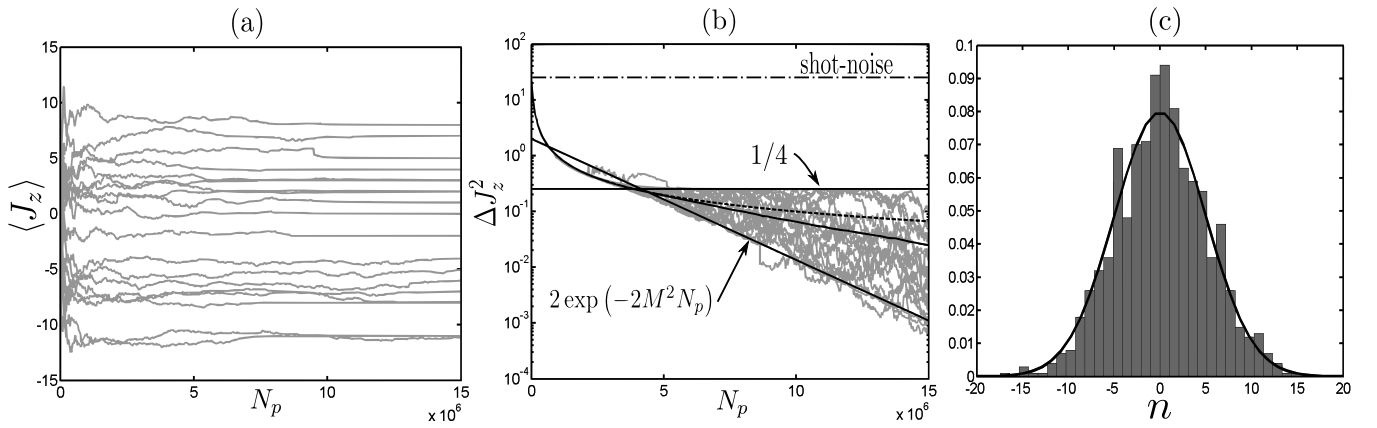


FIG. 3: Trajectories of the atomic distribution momenta obtained with a Monte-Carlo simulation, once set  $N_{\text{at}} = 100$ ,  $\phi = 10^{-3}$  rad, and  $R = T = 1/2$ . (a) Mean position of the atomic population  $\langle J_z \rangle$  as a function of the number of measured photons. For a long enough measurement the state converges to a Dicke state. (b) Variance evolution during the measurement sequence. The solid line is the average over 1000 trajectories (only 20 are plotted) while the dashed line is the result of the analytical calculation of the variance at short time, given by Eq. (15). At short time scale, the evolution is deterministic as predicted by the analytical study. At long time scale, the variances become stochastic but remain bounded. In addition, the average variance over the trajectories is below the short time behavior. (c) The histogram of the mean positions at the end of the trajectories is compared to the initial distribution (solid line). The measurement process satisfies the Born probability rule  $P_n = |\langle n | N_p = 0 \rangle|^2$ .

next detected photon is given by Eq. (7). More explicitly, we use the cumulative distribution function associated to this density of probability, which is  $F_{N_p+1}(\varphi) = \int_{-\pi}^{\varphi} P_{N_p+1}(\theta) d\theta$ . By generating a random number with an uniform distribution over  $[0, 1]$  and numerically inverting  $F_{N_p+1}$  we get the detected phase of the  $(N_p + 1)$ -th photon. The recurrence relation of Eq. (5) gives the new atomic distribution. The quantum trajectory is obtained repeating this sequence.

In conclusion, we analysed a single photon heterodyne detection where a time reference is used to extract the beatnote induced by the coherent superposition of two optical modes. This measurement strategy can permit robust atomic state preparation because of its strong immunity to technical noise and refraction index fluctuations, due to the spatial overlap of frequency components in the probe beam. Using a wavevector formalism in the weak coupling limit, we developed a model for

the stochastic detection process where analytical expressions are derived for both short and long time behavior. We described the collapse of a coherent spin state under this repeated measurement and shown it generates both spin-squeezed and Dicke states. This method of detection could be used to generate other kind of non-classical states such as Schrödinger cats or NOON states, whereas the implementation of quantum feedback would make possible a deterministic state preparation. Such states could be used to perform highly non-classical interferometry at the Heisenberg limit [29].

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- [30] The condition  $N_t \gg m^3$  is chosen to have a large number of photons in the intervals  $[-\pi, -\pi + \pi/m]$  and  $[\pi - \pi/m, \pi]$ , where  $P(\varphi)$  is close to zero. It results from the following condition when  $\mathcal{C} \sim 1$ :

$$N_t \int_{\pi-\pi/m}^{\pi} P_0(\varphi) d\varphi \sim N_t \left[ \frac{1-\mathcal{C}}{2m} + \frac{\pi^2 \mathcal{C}}{12m^3} \right] \gg 1.$$