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# Discussion of “Uncertainty of excess density and settling velocity of mud derived from in situ measurements” by M. Fettweis

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The author discussed an important issue in mud floc knowledge, which is the uncertainties in the measurements of excess density and settling velocity. He pointed out the lack of accuracy in measuring instruments, and the statistical nature of the variables. However, the author did not discuss the limitations of the theories and semi-empirical formula he used. Some assumptions in the calculations need also to be discussed in more details; and some calculations are not mathematically correct.

## The link between excess density and settling velocity

The fall velocity equation suggested by Winterwerp (1998, cf. Eq.5 in the original paper) relies on the Stokes law, *i.e.* it corresponds to the balance between the gravitational and drag force for a spherical non cohesive particle, and on an additional empirical equation, which includes the effect of the Reynolds number on the settling velocity (coefficient  $f(Re) = (1 + 0.15Re^{0.687})^{-1}$ , see Graf, 1971). The fall velocity may be written as follows :

$$w_s = \frac{c_f}{18} \frac{\Delta\rho}{\eta} g D_f^2 f(Re) \quad (1)$$

where  $c_f$  is a coefficient depending on the shape, roughness and porosity of the particles ( $c_f \approx \alpha/\beta$  following Winterwerp developments). According to Eq.1, the standard deviation for the settling velocity is directly related to the standard deviation for the excess density  $\Delta\rho = \rho_f - \rho_w$ . And there is no real need to introduce the fractal theory to explain the uncertainty of settling velocity. It only yields a new uncertainty as it introduces a new unknown parameter  $n_f$ ... On the other hand, there are several additional terms that are missing in Eq.9 (original paper) such as the errors due to the coefficient  $c_f$ , to the molecular viscosity of water, which is function of temperature and SPM concentration, and to the Reynolds number  $Re = w_s D_f / \nu$  ( $\nu$  is the kinematic viscosity of water), or more exactly to the function  $f(Re)$ . The standard deviation  $\sigma_{w_s}$  should be written as follows :

$$\sigma_{w_s} = \sqrt{\left(\frac{\partial w_s}{\partial \Delta\rho}\right)^2 \sigma_{\Delta\rho}^2 + \left(\frac{\partial w_s}{\partial D_f}\right)^2 \sigma_{D_f}^2 + \left(\frac{\partial w_s}{\partial c_f}\right)^2 \sigma_{c_f}^2 + \left(\frac{\partial w_s}{\partial \eta}\right)^2 \sigma_{\eta}^2 + \left(\frac{\partial w_s}{\partial f(Re)}\right)^2 \sigma_{f(Re)}^2} \quad (2)$$

If we follow the author assumptions, the three last terms are negligible, and so :

$$\frac{\sigma_{w_s}}{w_s} \approx \sqrt{\left(\frac{\sigma_{\Delta\rho}}{\Delta\rho}\right)^2 + 4 \left(\frac{\sigma_{D_f}}{D_f}\right)^2} \quad (3)$$

The author finally found as an average the relative  $\sigma_{D_f}/D_f \approx 30\%$ ,  $\sigma_{\Delta\rho}/\Delta\rho \approx 10\%$ , and then  $\sigma_{w_s}/w_s \approx 100\%$ . If these results are coherent with Eq.3 (although overestimated, from Eq.3  $\sigma_{w_s}/w_s \approx 60\%$ ), it clearly appears that the uncertainties on  $D_f$  has a much larger influence on the uncertainties on  $w_s$  than the uncertainties on  $\Delta\rho$ . It should also be noted that Eq.1 (or Eq.5

in the original paper) is actually an implicit formula as  $Re$  is a function of  $w_s$ ; thus, Eq.2 is not strictly correct (so are Eqs.9 and A4 to A8 in the original paper).

Following the same idea, from a mathematical point of view, Eq.6 should be written :

$$\sigma_{\Delta\rho} = \sqrt{\left(\frac{\partial\Delta\rho}{\partial M_p}\right)^2 \sigma_{M_p}^2 + \left(\frac{\partial\Delta\rho}{\partial V_f}\right)^2 \sigma_{V_f}^2 + \left(\frac{\partial\Delta\rho}{\partial \rho_p}\right)^2 \sigma_{\rho_p}^2 + \left(\frac{\partial\Delta\rho}{\partial \rho_w}\right)^2 \sigma_{\rho_w}^2} \quad (4)$$

as there is an uncertainty on  $\rho_w$  (even if this latter is negligible compared to the other parameters).

### On the estimation of the uncertainty

When one wants to discuss about uncertainties, it seems obvious that all uncertainties should be discussed. One cannot exclude systematic errors due to the lack of accuracy of the measuring instruments. Some attempt to estimate this error is needed; otherwise, the results are biased. For example, the volume concentration  $V_f$  may typically be estimated as :

$$V_f = c_v \frac{\pi D_f^3}{6} \quad (5)$$

where  $c_v$  is a shape factor taking into account the fact that a floc is not spherical. Then, the standard deviation  $\sigma_{V_f}$  reduces to :

$$\sigma_{V_f} = \sqrt{\left(\frac{\partial V_f}{\partial D_f}\right)^2 \sigma_{D_f}^2 + \left(\frac{\partial V_f}{\partial c_v}\right)^2 \sigma_{c_v}^2} \quad (6)$$

As the uncertainty on the floc size  $D_f$  is much larger than the uncertainty on the floc sphericity and  $\partial V_f / \partial D_f \gg \partial V_f / \partial c_v$ ,  $\sigma_{c_v}$  may be neglected in Eq. 6.

For the settling velocity, an important uncertainty that needs to be discussed is  $\sigma_{c_f}$ . Based on a study on non-cohesive particle of various shape, Camenen (2007) showed that  $w_s$  could be underestimated by 20% approximately. On the other hand, Johnson *et al.* (1996) showed that the settling velocity of fractal floc may be underestimated by a factor 5 because of the porosity of the floc. In the same way, the uncertainty on  $w_s$  due to the empirical function of the Reynolds number  $\sigma_{f(Re)}$  should be estimated :

$$\sigma_{f(Re)} = \sqrt{\left(\frac{\partial f(Re)}{\partial Re}\right)^2 \sigma_{Re}^2 + \sigma_f^2} \quad (7)$$

where  $\sigma_f$  is the standard deviation on the empirical equation  $f$ .

In term of the presentation of the error, the author should clarify some points where results are getting unphysical. Defining the error on the primary particle size such as  $D_p = 1.5 \pm 3.5 \mu\text{m}$  (cf. Tab.2) seems awkward ( $D_p > 0$ ). It may be better in that case to estimate an error on  $\log D_p$ . Similarly, it is incorrect to write  $nf = 3.23 \pm 0.06$  knowing that  $nf \leq 3$  by definition. However, it indicates that there is an error in the estimation of the uncertainties. Based on Eq.1 of the original paper (where  $\propto$  may be replaced by  $=$  following Winterwerp, 1998), a better estimation of  $nf$  would be :

$$nf = 3 + \frac{\ln \left[ \frac{\rho_f - \rho_w}{\rho_p - \rho_w} \right]}{\ln \left[ \frac{D_f}{D_p} \right]} \quad (8)$$

Using this equation, we ensure  $nf < 3$  is satisfied as  $\rho_f < \rho_p$  and  $D_f > D_p$ . Eq.2 (original paper) yields too approximate results as  $D_p$  cannot be assumed as an independent variable. From Eq. 8, the standard deviation  $\sigma_{nf}$  is then :

$$\sigma_{nf} = \sqrt{\left(\frac{\partial nf}{\partial D_f}\right)^2 \sigma_{D_f}^2 + \left(\frac{\partial nf}{\partial D_p}\right)^2 \sigma_{D_p}^2 + \left(\frac{\partial nf}{\partial \rho_f}\right)^2 \sigma_{\rho_f}^2 + \left(\frac{\partial nf}{\partial \rho_p}\right)^2 \sigma_{\rho_p}^2 + \left(\frac{\partial nf}{\partial \rho_w}\right)^2 \sigma_{\rho_w}^2} \quad (9)$$

Moreover, one should not confuse an uncertainty on the measurements of a variable and the physical distribution of this variable. It is somehow an error to discuss uncertainties on the diameter of flocs when measuring the size of a population of flocs.

Finally, as both settling velocity and floc density are the main unknown parameters for the cohesive sediments, it would be very interesting to combine the *LISST 100C* apparatus with some optical apparatus to determine the settling velocity as the INSSEV apparatus developed at Plymouth University (Manning & Dyer, 2002). Such an experiment would yield to significant improvements on the determination of the uncertainties for each method.

## References

- Manning, A. & Dyer, K. (2002), ‘The use of optics for the in situ determination of flocculated mud characteristics’, *J. of Optics A: Pure & Applied Optics* **7**, 71–81.
- Camenen, B. (2007), ‘A simple and general formula for the settling velocity of particules in suspension’, *J. Hydraulic Eng.* **133**(2), 229–233.
- Graf, W. (1971), *Hydraulics of Sediment Transport*, McGraw-Hill, New York, USA.
- Johnson, C., Li, X. & Logan, B. (1996), ‘Settling velocity of fractal aggregates’, *Environmental Science & Technology* **30**(6), 1911–1918.
- Winterwerp, J. (1998), ‘A simple model for turbulence induced flocculation of cohesive sediment’, *J. Hydraulic Res.* **36**(3), 300–326.