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Evidences of turbulent cascading atomization mechanism in the bag-breakup regime

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Abstract

The question whether liquid atomization (or pulverization) resorts to instability dynamics (through refinements of Rayleigh-Plateau, Rayleigh-Taylor or Kelvin-Helmholtz mechanism) or to turbulent cascades similar to Richardson and Kolmogorov first ideas seems to be still open. In this paper, we report experimental evidences that both mechanisms are needed to explain the spray drop PDF obtained from an industrial nozzle. Instability of Rayleigh-Taylor kind governs the size of the largest droplets while the smallest ones obey a PDF given by a turbulent cascading mechanism resulting in a log-Lévy stable law of stability parameter close to 1.68. This value, very close to the inverse of the Flory exponent, can be related to a recent model for intermittency modeling stemming from self-avoiding random vortex stretching.

1 Introduction

Lognormal probability density functions (PDF) are widely used by experimental analysts in many fields, including liquid pulverization. To explain their widespread appearance in many fields, Kolmogorov indirectly pioneered turbulent atomization modeling [1] by devising a discrete Markov process converging toward a lognormal PDF. Later Obukhov [2] used this result to describe the statistical distribution of intermittent dissipation in turbulent flows and this has later been retained by Kolmogorov [3] in his famous K62 modeling of turbulence intermittencies. Many works have been since done in this field and Frisch's book [4] is an excellent review. Maybe the most influential scientist still to be cited is Mandelbrot [5] who coined the word fractal, developed the multifractal formalism and made many contributions to economics where he widely used log-stable distribution (for a definition of Lévy stable laws many textbooks do exist now but [6] is still a good reference). Unfortunately, concerning turbulence or atomization modeling, Mandelbrot merely developed general ideas but no reproducible laws. While Schertzer and Lovejoy [ref] emphasized their role in geophysics as universal multifractals (thanks to a generalized central limit theorem), Kida [7,8] explained empirically the statistical laws of turbulence intermittencies with a log-stable law of stability parameter 1.65. Though this is still debated nowadays, a recent advance in this field can be found in [9,10] where Kida's results are discussed and proved thanks to a self-avoiding random vortex stretching process. In this modeling, the topological constraint of non intersection, applied to a vortex tube, enforces the value of the stability parameter to be the inverse of Flory's exponent (i.e. $1/588$ or 1.70), a scaling exponent well known in polymer physics [11]. The stable law is proved to be fully asymmetric to the left and its scale parameter can be related to the important scale of turbulent flows: Kolmogorov's and Taylor's scales.

As for turbulent atomization modeling, few improvements have been made in this field since Kolmogorov's work: while experimentalists still resort to a variety of empirical laws [12], some of them close to the lognormal law (such as the upper limit lognormal Evans law or the log-Weibull law), theoreticians widened their view to either log-infinitely divisible distributions [13], multifractal analysis [14], or refinements of Kolmogorov first modeling [15]; all of which not very helpful for the experimentalists. In a more practical way, in [16], it has been shown that log-stable laws (which now can be easily computed thanks to FFT) are also good candidates to model some turbulent spray PDF. Atomization being a growing field of interest a more detailed review can be found for instance in [Goro].

However, though multistep cascading mechanisms are very useful to describe turbulent intermittencies, it is appropriate to question whether they pertain to atomization modeling: there are numerous records of primary atomization and secondary atomization, but almost never of ternary atomization (i.e. no third steps!). This may actually be related to the fact that after two breakup events, some air is entrapped in the neighborhood of the drops in what is often called the “added mass” so that liquid droplets are no longer sheared. However recent evidence indicates that a cascading mechanism seems to be still appropriate [17] for high-speed sprays. For lower speed, most analysis made in the non turbulent regime, resort to instability theory. It usually leads to a competition between surface tension effect (Rayleigh-Plateau mechanism), acceleration of a droplet in the ambient air (Rayleigh-Taylor mechanism) and shear instability appearing on the edge of the droplet (Kelvin-Helmholtz mechanism, for a review of these mechanisms, see [18] for instance). In a recent study of the so-called non turbulent bag breakup regime [19], the Rayleigh-Taylor mechanism is widely used to explain experimental results.

In this work, we will show firstly, that for an industrial nozzle, in the bag-breakup regime, Rayleigh-Taylor instability can explain the first stage of the breakup (as well as accepted values of the so-called bag breakup regime) It is then reported how, secondly, a turbulent cascade mechanism seems to be necessary to describe the finer droplets PDF resulting from the burst of the bag. In a way, this can be related to the late stage of the turbulent mixing in the Rayleigh-Taylor instability where recently a Kolmogorov-like cascading scenario has been put in evidence (cf. [20] for a description of the one-phase RT instability developing between a hot gas and a cold gas).

2 Experimental setup

Experimental setup was devised to test several kinds of industrial nozzles, among which two Lechler Nozzles (ref. 665-042 and 665-122) were tested. Their aperture is depicted in Fig. 1: it is made of two identical circle segments pieced together. The measured value of parameter $2a$ and $2b$ of Fig. 1 are respectively $977\ \mu\text{m}$ and $646\ \mu\text{m}$ for ref. 665-042. and 1185 and $864\ \mu\text{m}$ for ref. 665-122. Their equivalent radii r_{eq} are thus respectively $452\ \mu\text{m}$ and $488\ \mu\text{m}$. Results obtained from either nozzle are thus quite similar. Their nominal flow rate is around 80 liters of water per minutes (varying with pressure). Several pressures were tested but most measurements were made at 8 bars and 15 bars.

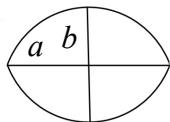


FIG. 1: sketch of the nozzle orifice and photograph of the experimental set-up (Lechler Nozzle ref. 665-042, 8 bars)

For the 665-122 nozzle, for a given pressure of 15 bars, the liquid velocity U was found to be 41 m/s and the standard deviation u' around this velocity was 2 m/s (cf. FIG. 3.). Let us notice that these velocities are independent of the droplets size. Data were collected using a Dantec Dynamics PDA (Phase Doppler Anemometer) and a green argon continuous laser (wavelength: 514.5 nm). The PDA equipped with a classic receiver, has been used in

refraction mode with a diffusion angle of 72° . According to the manufacturer droplets size can be measured with confidence in a dynamic interval ranging from $1x$ to $40x$ (or 1.6 decades). This means that bigger droplets have a tendency to saturate the photomultipliers while smaller ones may not trigger it. However data were collected over three decades. While droplet frequency measurement may not be very accurate on this wide range, caution was made to configure measurements so that small drops resulting from the bag-breakup were accurately resolved (i.e. no aperture mask and a high enough voltage amplification ratio was used in the photomultiplier). Actually the measuring volume was an ellipsoid of dimension $600\mu m \times 600\mu m \times 4000\mu m$ so that only droplets of diameter inferior to $600\mu m$ could be fully contained in the measuring volume. This setup resulted in a detected maximum droplet size of $1754\mu m$. Moreover droplets bigger than $600\mu m$ were mostly detected when they cut the volume measure on the side of it, i.e. when the amount of light they emitted was small enough not to saturate the photomultipliers. This can be seen by comparing the measured transit time to an idealized transit time given by:

$$TT_{ideal} = \frac{(600) \cdot 10^{-6}}{U} \quad (1)$$

Here the Gaussian characteristic of the laser beam is neglected and the droplets are considered as point particle. Results of such a comparison can be found in FIG. 2 which depicts the evolution of the ratio of the measured transit time (given by the PDA) to the idealized transit time as a function of droplets' diameters. It can be seen that larger droplets have a very short measured transit time indicating that they were mainly cutting the measuring volume on its borders. Accordingly, their number cannot be assessed with certainty. Following, this analysis, the center of the measurement range where the statistics of the droplets is adequately reported can be estimated to be around $50\mu m$ resulting in a PDF fully resolved somewhere between 10 and $400\mu m$ (but this is an harsh estimate). Note finally that since the three detectors are in line, what is truly measured by the PDA, are two inline radii of curvature of the interface between water and air; when these radii share a common value, the latter is then assimilated to the radius of the measured drop. This (common) approximation will be naturally made in this paper.

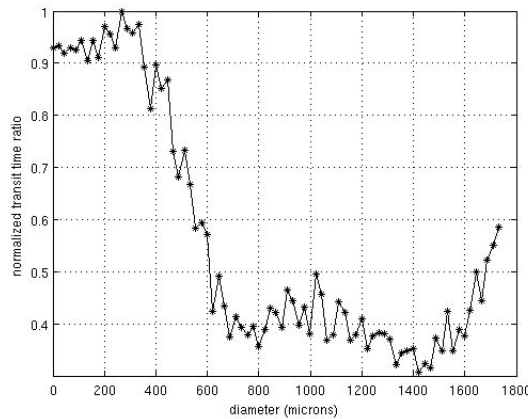


FIG. 2: comparison between the measured transit time and the ideal transit time of formula () showing that big droplets are mostly detected on the edge of the measurement volume. The overall intensity

3 Experimental results

As PDA collects both size and velocity of droplets, results can be given in the form of joint PDF. Figure 3 depicts such kind of representation. Middle picture is the joint velocity-diameter PDF and lower picture is the joint velocity-magnitude PDF (magnitude is here

defined as the decimal logarithm of the diameter; origin is chosen such that magnitude 0 stands for $1 \mu m$).

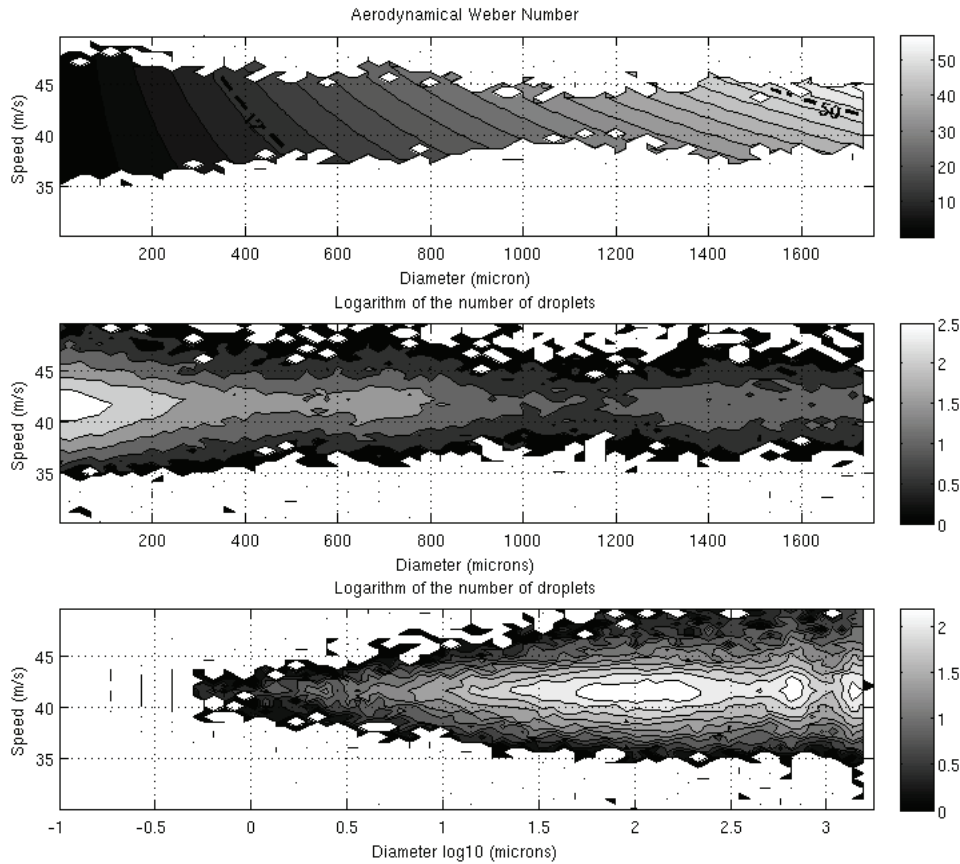


FIG. 3: experimental size-velocity PDF. (Lechler Nozzle ref. 665-122, 15 bars, 4cm below the exit) On top the domain of the bag-breakup ($12 < We < 50$) is delimited. Comparison between the frequency distribution of the droplet velocity and diameter (in the middle) and the distribution of the velocity and magnitude ($\log_{10}(d)$, on the bottom). It can be seen that a large amount of droplets are located under $100 \mu m$, that is in the aerodynamic stability domain.

The non dimensional parameters governing the stability of a droplet in an air stream are the (aerodynamic) Weber number and the Ohnesorge number defined by:

$$We_{aero} = \frac{\rho_G U^2 d}{\gamma} \quad \text{and} \quad Oh = \frac{\mu_L}{\sqrt{\gamma \rho_L d}} \quad (2)$$

where ρ_G and ρ_L stand for, respectively, air (gas) density and water (liquid) density; γ is the air-water surface tension, μ_L is the liquid dynamic viscosity while d stands for the droplet diameter. According to [21] at low Ohnesorge number (i.e. mainly for liquids with low viscosity), a Weber number inferior to 12 indicates that the droplet is aerodynamically stable whereas a Weber number located in the interval $[12, 50]$ indicates that the droplet shall break in what is called a bag-breakup (cf. Fig. 4). On top of figure 3, iso-contour of Weber number are given in the velocity-diameter plane. From it, it can be seen that most droplets on the millimeter scale are located in the bag-breakup regime whereas droplet whose radii are located under $350 \mu m$ are aerodynamically stable. Actually the fact that droplets of every size are flowing at the same speed and that the all droplets size-velocity PDF have roughly the same shape on the vertical axis seems to indicate that apart from initial bag breakups, droplets are subsequently not submitted to any particular shear from the ambient air (the opposite would result in smaller droplets going slower, being more decelerated). This suggests that the air surrounding the spray is entrapped in the water flow. This induced a slight wind in the surrounding which could actually be felt during experimental work. Therefore values of the

Weber number given by (1) is an overestimation since the velocity of water U shall be replaced by the drift velocity U_{dr} between water and air. If the preceding remark is true this means that when not submitted to primary or secondary breakup, droplets of any size can be stable in the air flow. This may also explain the observed stability of the PDF on the vertical axis.

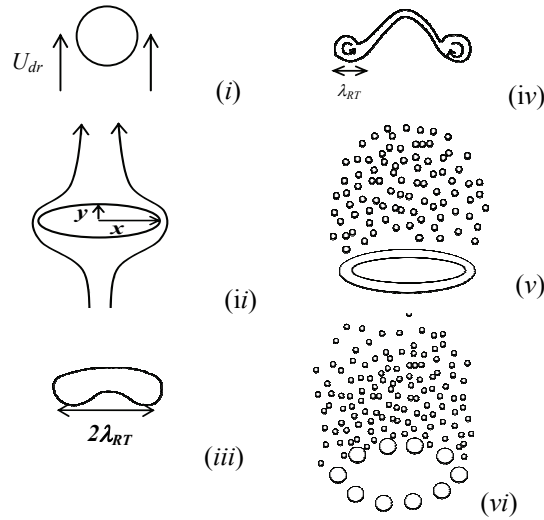


FIG. 4: the six stages of the bag breakup (cf. [23]): (i) the droplets result from the Rayleigh-Plateau mechanism, (ii) they elongate in the relative air flow, (iii) this results in a Rayleigh-Taylor wave and (iv) in the formation of a ring and a bag, (v) the blow up of the bag and (vi) of the ring leads to a bimodal distribution.

In these PDF, three peaks do appear (especially when considering the magnitude scale) each corresponding to a peculiar physical mechanism. Position of the three peaks in the distribution are found to be $1355 \mu\text{m}$ (magnitude 3.13), $661 \mu\text{m}$ (magnitude 2,82) and roughly $100 \mu\text{m}$ (and more precisely $200 \mu\text{m}$ considering the Sauter Mean Diameter of the bag). Statistics were made over 50000 droplets. Since all droplets have the same velocity, this number was large enough to obtain converged results. Note that sizing techniques based on image analysis (usually obtained by high-speed imaging) would lead to a very tedious analysis protocol to obtain such a high number (moreover for a smaller dynamic range).

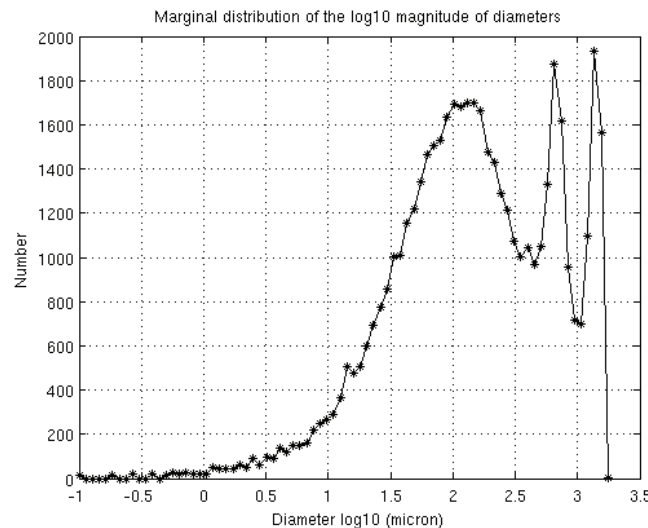


FIG. 5: marginal size distribution. (Lechler Nozzle ref. 665-122, 15 bars, 4cm below the exit) From right to left, the first droplet is given by the Rayleigh-Plateau theory, the first peak is given by the Rayleigh-Taylor theory while the second peak is another harmonic Rayleigh-Taylor peak. The third peak is the bag breakup peak.

Figure 4 depicts the classical way of describing the bag-breakup mechanism (cf. [19] for more details). Five stages are usually described: the drop is first deformed by the air stream, this leads to the onset of a Rayleigh-Taylor instability which leads to the formation of a rim and a bag. The bag bursts before the rim which follows soon afterwards (characteristic times can be found in [21,22,23]). Figure 5 shows the marginal magnitude PDF obtained from the joint velocity magnitude PDF. Small size PDF are very similar whereas large size peaks are located at roughly the same place but differs in intensity. Since they mainly corresponds to off-center droplets there is clearly a reproducibility issue concerning their intensity. Since air is entrapped in the water curtain, it can be supposed that the first peak corresponds to unburst droplets while the second peak corresponds to droplets resulting from the fragmentation of the rim and the third peak to the cloud of droplets created by the bursting of the bag. This can be confirmed by the fact that the Sauter Mean Diameter of the bag droplets has been found to be 200 μm so that the ratio:

$$\frac{SMD_{bag}}{d_{mit}} = \frac{200}{1350} \approx 0.148 \quad (3)$$

is also very close to the value 0.14 reported in [19] and [21]. An explanation of the ratio (close to 2) between the diameter of the mother droplet and the diameter of the rim droplets can be developed as in [19] by adapting Rayleigh-Taylor theory. Without fully resorting to droplets deformation and breakup (DDB) modeling [DDB], it can be assumed that at the onset of breakup, the droplet has deformed into an oblate ellipsoid of major semiaxis x and minor semiaxis y (cf. Fig. 4). Let us suppose that $x \ll y$ so that the final shape of the droplet can be assimilated to a disk. If the droplet's Reynolds number is high enough ($Re > 1000$), its drag coefficient C_d can be supposed constant and is given by $C_d \cong 1.5$ [Reitz]. Now, let us compute the condition upon which a Rayleigh Taylor wave of wavelength equal to the major semiaxis can grow on the drop surface (cutting the droplet in its center). Fastest growing wavelength is given by:

$$\lambda_{RT,max} = 2\pi \sqrt{\frac{3\gamma}{f\Delta\rho}} \quad (4)$$

where f is the deceleration of the disk in the air stream, which is given by:

$$f = \frac{3}{8} \frac{\rho_G}{\rho_L} \frac{x^2}{r^3} C_d U^2 \quad (5)$$

Condition $x = \lambda_{RT,max}$ leads to

$$\left(\frac{x}{r}\right)^4 = 32\pi^2 \frac{\gamma}{\rho_G C_d U^2 r} \approx 10.8 \quad (6)$$

where $\rho_G = 1.3 \text{ kg/m}^3$, $r = 675 \text{ }\mu\text{m}$, $U = 40 \text{ m/s}$ and $\gamma = 0.072 \text{ N.m}^{-1}$. Note that $Re = Ud/\nu_G = 1800$. This deformation corresponds to $x = 1.8r$ which means that when the droplet has reached this deformation, deceleration is high enough for a Rayleigh-Taylor wave of fastest growth to cut the drop in half. Considering the marginally stable mode leads to

$$\lambda_{RT,0} = 2\pi \sqrt{\frac{\gamma}{f\Delta\rho}} \quad (7)$$

And a ratio $x/r = 1.4$ so it is possible for such a wave to grow (slowly) before. However, when solving the full DDB, a deformation ratio of 1.8 is quite common [DDB,Reitz] and can be considered as a reachable state. Equation (6) Also reads

$$We_{min} = \frac{32\pi^2}{10.8 C_d} = 19 \quad (8)$$

and considering the marginally stable growing mode, one gets:

$$We_{\max} = \frac{32\pi^2}{3.6C_d} = 58 \quad (9)$$

Note that for a given droplet, when the We number is increased, droplet deceleration increase likewise and Rayleigh-Taylor wavelength decrease, so that equation (9) can be considered as an upper limit for the bag breakup: above this value the slowest growing RT wave cut the drop into more than two resulting in the so-called multimode breakup. Note that the onset (8) of the bag-breakup is slightly overestimated and this may be related to the assumption of constant drag coefficient which may not be valid on such low velocity (C_d increases when the drift velocity decreases).

4 Turbulent cascading mechanism

During the preceding events, numerous tiny droplets are produced as a result of the burst of the bag. Greatly different from the first two narrow peaks, their size distribution is very wide. This section will be devoted to the interpretation of this widespread distribution of fragments.

It is postulated thereafter that this widespread distribution is the image of the widespread distribution of vortices in turbulent flows, a phenomenon known as intermittencies. In [10] a detailed scenario of turbulent intermittencies is devised thank to a self-avoiding random vortex stretching mechanism which ultimately results in a log-stable distribution of vortices. In this work, in a very classic way, the size of the most common vortices is given by the Taylor micro scale and the size of the smallest vortices by the Kolmogorov scale. Stable distributions are defined by four parameters. These are proved to be $\alpha = 1.70$ (theoretically) and 1.68 (experimentally) for the stability index, $\beta = -1$ for the asymmetry parameter (both theoretically and experimentally, the resulting distributions are said to be totally skewed to the left), the scale parameter is given by

$$\sigma_\varepsilon^\alpha = \ln\left(\frac{\lambda}{\eta}\right), \quad (4)$$

and the shift parameter δ_ε is given by the average dissipation of turbulent kinetic energy per unit volume and is therefore related to the large scale of the flow.

It is here quite difficult to estimate *a priori* the different scales of turbulence. Setting some reference values for the turbulent kinetic energy and the large scale of is required [24], let us try to do so:

$$\varepsilon \cong \frac{u^{13}}{L_{\text{int}}} \cong \frac{2^3}{1350 \cdot 10^{-6}} \cong 5900 \text{ m}^2 / \text{s}^3 \quad (5)$$

$$\lambda = \sqrt{20\nu \frac{k}{\varepsilon}} \cong 137 \mu\text{m} \text{ (Magnitude 2.13)} \quad (6)$$

$$\eta \cong \left(\frac{\nu^3}{\varepsilon}\right)^{1/4} \cong 3 \mu\text{m} \text{ (Magnitude 0.48)} \quad (7)$$

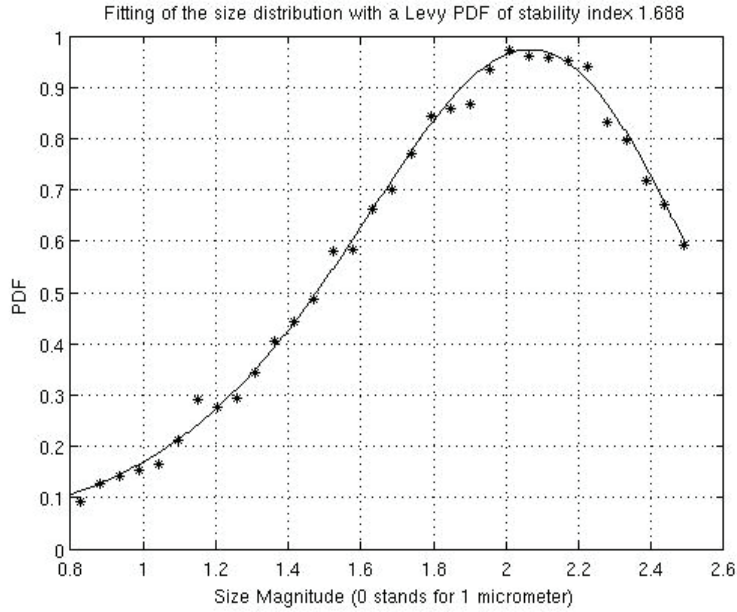


FIG. 6: The bag-breakup wide peak is fitted with a log-stable distribution of stability parameter close to 1.68

Note that the value of the turbulent fluctuating velocity inside the liquid phase has been identified with the RMS fluctuation of the droplets velocity; the value of turbulence dissipation rate ε can thus be quite underestimated. However the resulting Taylor micro scales seem to be a good order of magnitude of the most common droplets. Yet some droplets whose sizes are located under the estimated Kolmogorov scale are also observed. While this could be related to the underestimation of the turbulence dissipation, this could also be related to some 2π -phase ambiguity of the measuring device where big droplets can be interpreted as very small one (hopefully they are not very numerous). To circumvent this potential problem and to stay closer to the 1.6 decade dynamic, the bag number PDF has been fitted with a log-stable law (cf. [16] for the fitting procedure) in the magnitude range $[0.8, 2.5]$. The result is shown in figure 6. The value of the stability parameter is found to be 1.688 very close to the experimental value of 1.684 found in [10] for turbulent intermittencies (a fitting on the $[0, 2.5]$ magnitude range gives the value 1.70). Using (4) and values (5), (6), (7), the expected value of the scale parameter for the turbulent dissipation is found to be $\sigma_\varepsilon = 2.2$. Then using Kolmogorov four-fifth law,

$$|u(x+r) - u(x)|^3 = -\frac{4}{5} \varepsilon_r r, \quad (8)$$

it can be inferred[10] that the velocity gradient PDF shall extend over a range of $6\sigma_{\Delta u} = 2\sigma_\varepsilon$ or 4.4 Neperian magnitude scales (i.e. 1.9 decades). Present fitting of droplets PDF led to a scale parameter σ_d equals to 1.1 i.e. about half the scale parameter of the turbulent dissipation. It seems, at first, rather difficult to relate, theoretically, this new scale parameter to the previous turbulent intermittency scale parameter. It is well known that breakup of the bag leads to the formation of filaments which then turn into droplet [Reitz], therefore volume of the droplet shall be related in a way to the size of these filaments. However the nature of these filaments remains unknown. As they are seemingly coherent structures, a possible explanation could be that they are composed of vortex filaments resulting from a turbulent vortex cascading mechanism. Yet, this consideration is only qualitative. A possible scenario leading to more quantitative value, has been given by Hinze [Hinze, Novikov] when he devised a mechanism of droplets breakup by the turbulence of the carrier phase. It can be easily adapted to a situation where the turbulence inside the (carried) fluid leads to interface creation and to the formation of droplets. Let us equate the turbulent dynamic pressure at its surface induced by

the inner movements to the surface tension pressure of a cylindrical filament of diameter d centered in x (this leads to a breakup condition for a droplet of diameter d):

$$\frac{1}{2} \rho_L (u(x + d/2) - u(x))^2 = \frac{\gamma}{d} \quad (9)$$

Then, using Kolmogorov 4/5th law, one gets:

$$d \propto \varepsilon_d^{-2/5}, \quad (10)$$

therefore $\ln(d) = -\frac{2}{5} \ln(\varepsilon) + cst$ and the following relation between scale parameters could be expected:

$$\sigma_d = \frac{2}{5} \sigma_\varepsilon \quad (11)$$

This leads to an expected value of 0.9 for σ_d whereas 1.1 was measured. Agreement is not perfect and may be partly due to the harsh estimates of the different turbulent scales and to some uncertainty on the parameter estimation of the log-stable law.

Minus one

5 Conclusion

In this work, it has been shown that for some high flow-rate industrial spray, in the bag-breakup atomization mechanism, the drop PDF was composed of three peaks. The first two peaks are narrow and correspond respectively to the mother droplet peak and to the peak of daughter droplets created by burst of the basal ring, or rim of the bag. The bag leads to a very wide range of fragments. Average size of these fragments can be related to the Taylor scale of turbulent eddies developing inside the droplet during the formation of the bag and its bursting in numerous filaments.

No new parameter needed

6 Acknowledgement

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7 Appendix

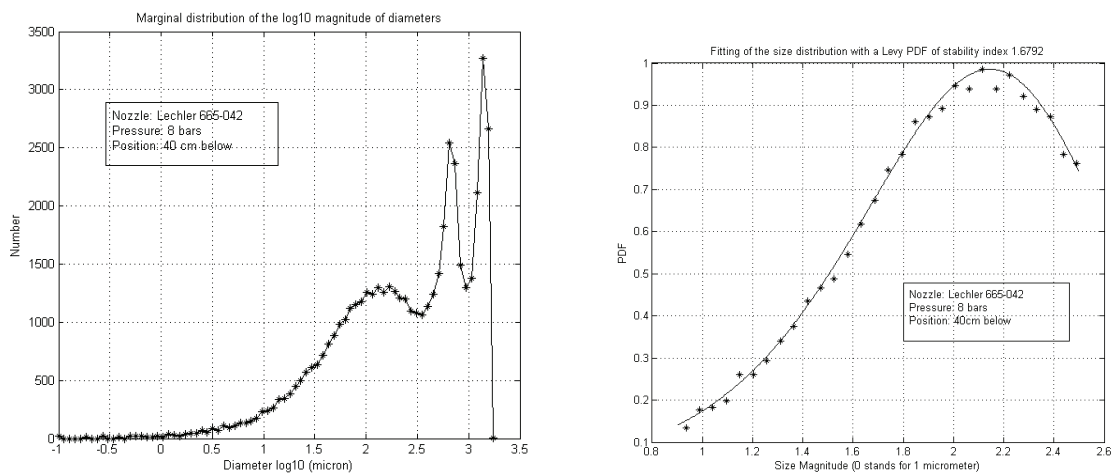


FIG. 7: Results for the 665-042 Lechler Nozzle 40 cm below the exit/

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