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## ON NEW PARALLEL-CROSS MECHANISMS

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**Abstract.** This paper addresses the new parallel-cross mechanisms, their classification and analysis. Several crossing kinematic chains of these mechanisms are located between the parallel kinematic chains. The contribution concerning to these mechanisms is their more large workspace than this of parallel mechanisms.

**Keywords:** Parallel mechanism, parallel-cross mechanism, workspace, pressure angle, singularity.

### 1. Introduction

It is well known that the workspace of the parallel manipulators can be less than this of the serial manipulators because of interference of the links of the kinematic chains or because of singularities (see for example the works of Gough [1], Stewart [2], Hunt [3], Merlet [4], Gosselin and Angeles [5], Parenti-Castelli and Innocenti [6]). Besides, if parallel mechanism contains numerous kinematic chains then the construction of the base or of the moving platform can be rather complete due to necessity to situate on these links many kinematic pairs connecting the kinematic chains with the base and with the platform. One of the solutions of this problem is to situate two or more actuators in several kinematic chains so that the number of these chains will be less than six. However, in this way the architecture of the kinematic chains containing several actuators can be complete.

In this paper, we develop other solution, which carried out by using mechanisms of parallel-cross structure (parallel-cross mechanisms), i.e. mechanisms in that some crossing kinematic chains are located between parallel kinematic chains (Glazunov et al. [7]). We represent the graphical classification of these mechanisms and their analysis comparing them with parallel mechanisms taking in account singularities and pressure angles, well known in the mechanism design (Sutherland and Roth [8], Lin and Chang [9]) but not so often applied to the parallel mechanisms (see Arakelian et al. [10]).

### 2. Synthesis of parallel-cross structures

In this part, we would like to synthesize all possible structures with a number of degrees of freedom from 1 to 6, with at least one leg attached to two others (two legs must not be linked together by two others). In the following synthesis, each leg can be composed with at most 6 joints, and we consider that a leg with 6 joints has obligatorily one pair actuated.

The results are summed up in table 1. In this table, the notation  $ijkl-mn$  (with  $i, j, k, l, m, n = 2$  to  $6$ ) corresponds to:

- left term: the legs attached to both the fixed base and the moving platform with respectively  $i, j, k$  and  $l$  joints (passive of active);
- right term: the legs attached to other legs with respectively  $m$  and  $n$  joints (passive of active).

Table 1. Representation of all possible structures with at least one leg attached to two others.

		Number of degrees of freedom					
		6	5	4	3	2	1
Number of legs	6	66666-6	56666-6	55666-6	55566-6	55556-6	55555-6
		6666-66	66666-5	56666-5	55666-5	66555-5	55556-5
		666-666	5666-66	5566-66	5556-66	5555-66	5555-56
			6666-65	5666-65	5566-65	6655-55	5556-55
			566-666	6666-55	5666-55	6555-65	555-556
			666-665	556-666	555-666	666-566	556-555
				566-665	556-665	665-555	
				666-655	566-655	655-655	
					666-555		
	5	6666-6	5666-6	4666-6	5556-6	5555-6	5555-5
		666-66	6666-5	6666-4	555-66	555-56	555-55
			566-66	5566-6	556-65	655-55	
			666-65	5666-5	566-55		
				466-66			
				666-64			
			556-66				
			566-65				
			666-55				
	4	666-6	566-6	466-6	366-6	356-6	355-6
			666-5	556-6	666-3	366-5	356-5
				566-5	456-6	656-3	556-3
				666-4	466-5	455-6	445-6
					566-4	456-5	446-5
						566-4	456-4
					555-5	455-5	
						555-4	
	3	66-6	56-6	55-6	45-6	26-6	25-6
			66-5	56-5	46-5	66-2	26-5
					65-4	35-6	65-2
					36-6	56-3	35-5
					66-5	36-5	55-3
					55-5	44-6	44-5
						46-4	45-4
						55-4	
						54-5	

Thus, 118 different kinds of structures can be synthesized. However, such a representation gives no information about the location of the attachment points of the moving legs on the legs fixed to both base and platform. Thus, the results of table 1 can be detailed in table 2 which represents structures with the moving legs located as close as possible of the fixed base. In this table, dark lines represent rigid links and circles pairs with one degree of freedom (active of passive). The notation  $i-j$  in the column in left stands for  $i$  legs linked to both base and platform and  $j$  legs attached to two other legs. 118 structures can be depicted in this table, but only the structures with six legs. It is obvious that other architectures are possible, depending on the location of the attachment of the moving legs. Moreover, these 118 possible structures can be also differentiated by the position of the actuated joints.

Table 2. Schematic representation of results of table 1 for manipulators with legs as close as possible from the base.

		Number of degrees of freedom						
		6	5	4	3	2	1	
Number of legs	6	5-1						
		4-2						
		3-3						

In the following part, we propose to analyze two structures which can be synthesized by our approach and which have interesting characteristics.

### 3. Description of the proposed structure

Let us now consider the architecture of the new proposed mechanisms with 6 DOF (Fig. 1). Two cases can be considered in function of the position of the actuators: the actuators are located upon the base (Fig. 1a); the actuators are located under the base (Fig. 1b). Both manipulators are composed of one fixed base  $A_1A_2A_3$  and one moving platform  $B_1B_2B_3$ . The platform is linked to the base by the use of three legs, each of which composed of one spherical at point  $A_i$ , one actuated prismatic pairs  $A_iB_i$  and one spherical joint  $B_i$ . Moreover these legs are linked together by the use of three other linkages composed of one spherical joint  $C_i$ , one actuated prismatic pair  $C_iD_j$  and one spherical joint  $D_j$  ( $j = i + 1$  for  $i = 1, 2$ ,  $j = 1$  for  $i = 3$ ). It is obvious that such a mechanism contains intern mobility which can be constrained by the use of cardan joints (Fig. 1c). Moreover, as shown on Fig. 1c, the spherical joints can be replaced by three revolute joints. Considering a manipulator composed of 6 pris-

matic, 6 spherical and 6 cardan joints, the Grübler formula can give the number of degrees of freedom  $m$  of the mechanism:

$$m = 6n - 5p_5 - 4p_4 - 3p_3 = 6 \quad (1)$$

where:  $n$  corresponds to the number of body without the base ( $n = 13$ );  $p_5$  corresponds to the number of pairs with only one degree of mobility ( $p_5 = 6$  prismatic joints);  $p_4$  corresponds to the number of pairs with two degrees of mobility ( $p_4 = 6$  cardan joints);  $p_3$  corresponds to the number of pairs with three degree of mobility ( $p_3 = 6$  spherical joints).

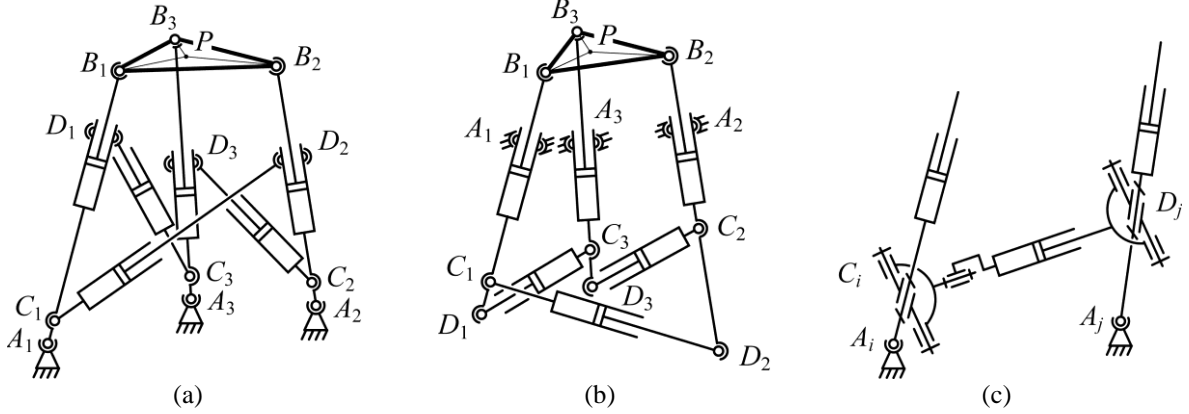


Figure. 1. Possible architectures of the new proposed mechanism with three actuators (a) upon the base and (b) under the base, and (c) one possible design for the legs  $C_iD_j$ .

Thus, the simultaneous displacements of actuated joints allow the mechanism having 3 translatory and 3 rotary DOF. We denote: the lengths of the prismatic pairs  $A_iB_i$  as  $\rho_i$ , ( $i = 1$  to 3); the lengths of the prismatic pairs  $C_iD_j$  as  $\rho_{i+3}$ , ( $j = i + 1$  for  $i = 1, 2$ ,  $j = 1$  for  $i = 3$ ). Triangles  $A_1A_2A_3$  and  $B_1B_2B_3$  are considered equilateral. We consider that the controlled point is the point  $P$  located at the centre of the triangle  $B_1B_2B_3$ . The position of  $P$  is expressed by the vector  $[x, y, z]^T$ . The base frame is centred at point  $O$  located at the centre of triangle  $A_1A_2A_3$  (point  $O$  is not represented on Fig. 1). The  $x_0$  axis of the base frame is directed along the line  $A_1A_2$  and the  $y_0$  axis along the line  $OA_3$ .

The rotations of the platform are denoted  $\alpha, \beta, \gamma$ , which can be obtained by expressing the directional cosines in terms of  $z_0$ - $x_1$ - $y_2$  Euler angles  $\alpha, \beta, \gamma$  (Fig. 2).

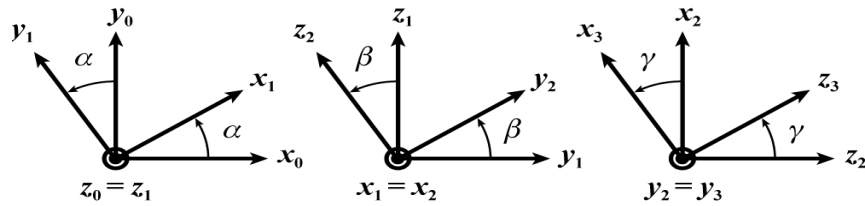


Figure. 2. Description of the chosen Euler angles.

Let us consider inverse geometric analysis. The positions of points  $A_i, B_i, C_i, D_i$  of the manipulators can be expressed as ( $i = 1, 2, 3$ ):

$$\mathbf{OA}_i = \mathbf{Rot}(\delta_i, z_0) [R_b \quad 0 \quad 0]^T \quad (2)$$

$$\mathbf{OB}_i = \begin{bmatrix} x \\ y \\ z \end{bmatrix} + \mathbf{Rot}(\alpha, z_0) \mathbf{Rot}(\beta, x_1) \mathbf{Rot}(\gamma, y_2) \begin{bmatrix} R_{pl} \cos \delta_i \\ R_{pl} \sin \delta_i \\ 0 \end{bmatrix} \quad (3)$$

$$\mathbf{OC}_i = \mathbf{OA}_i + \frac{l_1}{\rho_i} \mathbf{A}_i \mathbf{B}_i \quad (4)$$

$$\mathbf{OD}_i = \mathbf{OA}_i + \frac{l_2}{\rho_i} \mathbf{A}_i \mathbf{B}_i \quad (5)$$

with  $R_b = OA_i$ ,  $R_{pl} = PB_i$ ,  $\delta_i = [-5\pi/6, -\pi/6, \pi/2]$ , ( $i = 1, 2, 3$ ) (as said previously, triangles  $A_1A_2A_3$  and  $B_1B_2B_3$  are considered equilateral),  $l_1 = \overline{A_iC_i}$ ,  $l_2 = \overline{A_iD_i}$ , and  $\mathbf{Rot}(\zeta, \mathbf{w})$  the matrix representing the rotation of angle  $\zeta$  ( $\zeta = \alpha, \beta$  and  $\gamma$ ) around the  $\mathbf{w}$ -axis of the intermediate frame ( $\mathbf{w} = \mathbf{z}_0, \mathbf{x}_1$  and  $\mathbf{y}_2$ ).

Thus the expressions of the articular coordinates are given by:

$$\rho_i = \sqrt{(\mathbf{A}_i \mathbf{B}_i)^T (\mathbf{A}_i \mathbf{B}_i)}, \text{ for } i = 1, 2, 3, \quad (6)$$

$$\rho_{i+3} = \sqrt{(\mathbf{C}_i \mathbf{D}_j)^T (\mathbf{C}_i \mathbf{D}_j)}, (j = i + 1 \text{ for } i = 1, 2, j = 1 \text{ for } i = 3). \quad (7)$$

Let us consider kinematic analysis. The closure equations are represented by the expressions:

$$f_i = 0 = \rho_i^2 - (\mathbf{A}_i \mathbf{B}_i)^T (\mathbf{A}_i \mathbf{B}_i) \text{ for } i = 1, 2, 3, \quad (8)$$

$$f_{i+3} = 0 = \rho_{i+3}^2 - (\mathbf{C}_i \mathbf{D}_j)^T (\mathbf{C}_i \mathbf{D}_j), (j = i + 1 \text{ for } i = 1, 2, j = 1 \text{ for } i = 3). \quad (9)$$

Differentiating the closure equations with respect to time, one can obtain the following expression (Gosselin and Angeles [5]):

$$\mathbf{A} \mathbf{v} + \mathbf{B} \dot{\mathbf{q}} = \mathbf{0} \quad (10)$$

where:

$$\mathbf{A} \text{ and } \mathbf{B} \text{ are two matrices; } \mathbf{A} = \left[ \frac{\partial f_j}{\partial X_i} \right], \mathbf{B} = \left[ \frac{\partial f_j}{\partial \rho_i} \right], X_i = (x, y, z, \alpha, \beta, \gamma).$$

$\dot{\mathbf{q}}$  is the vector of the time derivatives of the coordinates of the actuators;  $\dot{\mathbf{q}} = [\dot{\rho}_1, \dot{\rho}_2, \dot{\rho}_3, \dot{\rho}_4, \dot{\rho}_5, \dot{\rho}_6]^T$ .

$\mathbf{v}$  is the vector of the time derivatives of the position of the platform;  $\mathbf{v} = [\dot{x}, \dot{y}, \dot{z}, \dot{\alpha}, \dot{\beta}, \dot{\gamma}]^T$ .

The vector  $\mathbf{v}$  can be related to the twist  $\mathbf{t}$  of the platform expressed in the base frame using a transformation matrix  $\mathbf{D}$  (Merlet [4]):

$$\mathbf{t} = \mathbf{D} \mathbf{v}, \text{ with } \mathbf{D} = \begin{bmatrix} \mathbf{I}_{3 \times 3} & \mathbf{0}_{3 \times 3} \\ \mathbf{0}_{3 \times 3} & \mathbf{L} \end{bmatrix} \quad \text{where } \mathbf{L} = \begin{bmatrix} 0 & \cos \alpha & -\sin \alpha \cos \beta \\ 0 & \sin \alpha & \cos \alpha \cos \beta \\ 1 & 0 & \sin \beta \end{bmatrix} \quad (11)$$

$\mathbf{I}_{3 \times 3}$  and  $\mathbf{0}_{3 \times 3}$  corresponds to identity and zero matrices of dimensions  $3 \times 3$  respectively.

Therefore the Jacobian matrix  $\mathbf{J}$  relating the twist  $\mathbf{t}$  of the platform to the velocities of the actuators is given by the relation:

$$\mathbf{J} = -\mathbf{D} \mathbf{A}^{-1} \mathbf{B} \quad (12)$$

The elements of the matrices  $\mathbf{A}$  and  $\mathbf{B}$  depend on the parameters of the mechanisms and on the position of the moving platform.

The next section deals with the comparison between the performances of the proposed manipulators and two other manipulators: the Stewart and Gough platforms.

#### 4. Comparative analysis of performances

In order to appreciate the performances of the new proposed mechanisms, let us compare them with two other manipulators with 6 DOF which are well known, commonly used by manufacturers and also actuated by six prismatic pairs: the Gough platform (Fig. 3a) and the Stewart platform (Fig. 3b). It is obvious that the mechanisms presented on Fig. 3 contain intern mobility which can be constrained by the use of cardan or revolute joints (Fig. 3c). Their geometric and kinematic models are well known (see for example the works of Gough [1], Stewart [2], Hunt [3], Merlet [4], Gosselin and Angeles [5], Parenti-Castelli and Innocenti [6]) and, as computing the geometrics and kinematics of these two manipulators is not the main topic of this paper, we will not present them another time.

So, we can consider the following problematic for our comparative analysis: starting from identical components (actuators, base and platform size), what are the arrangements of the actuators which will lead to the greatest workspace taking into consideration: the stroke of the actuators, the displacement of the spherical pairs and the interferences between the elements; the quality of effort transmission in the mechanism.

One industrial version of the Gough platform is the robot FANUC F200. This manipulator is well known for its high payload capacities and its ability to have little deformations under the application of a vertical force. Its approximate geometric parameters are:

- $R_{pl} = 0.2$  m and  $R_b = 0.3$  m;
- maximal stroke of the actuators  $(A_i B_i)_{max} - (A_i B_i)_{min} = 0.65$  m.

So in order to make a fair comparison, we propose to take into account the following design constraints:

- for each kind of manipulator, base and platform have to be identical to those of FANUC F200 robot;
- the stroke of the actuators has to be identical and equal to 0.65 m;
- in order to minimize the size of the manipulators, all points located on the base must be comprised on the surface defined by triangle  $A_1 A_2 A_3$ .

So now, let us analyse the size of the workspace taking into account the previous constraints.

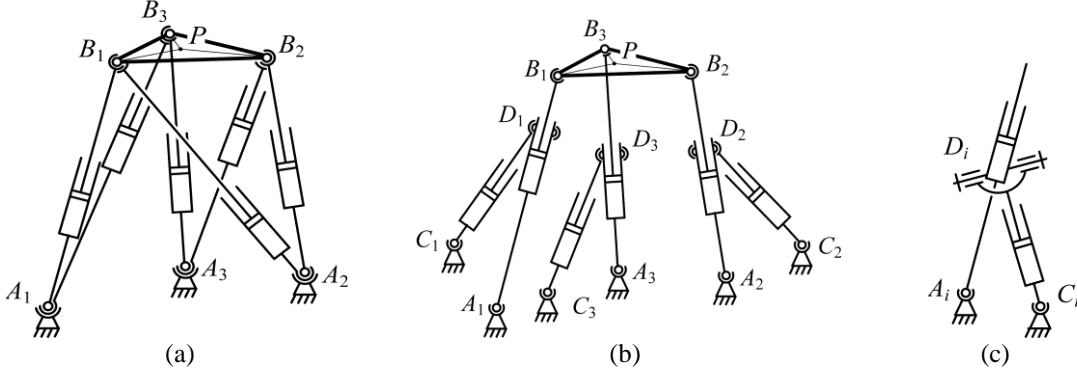


Figure 3. Schematic representation of (a) the Gough platform and (b) the Stewart platform, and (c) one other possible design for the legs of the Stewart platform.

Let us consider workspace analysis taking into account geometric limitations. The corresponding procedure is very simple. First of all, we discretize the Cartesian space into  $n$  points and the orientation space into  $p$  configurations. The step for the Cartesian space is equal to 0.05 m and for the orientation space to 10 deg for angles comprised between -60 deg and +60 deg for each angle  $\alpha$ ,  $\beta$  and  $\gamma$ . For a point  $Q$  to belong to the workspace of the considered manipulator, it should have at least one of the  $p$  configurations of the orientation space that valid the geometric limitations. Therefore, this workspace is defined as the maximal workspace (Merlet [4]).

The geometric limitations under study are, for each manipulator:

- the maximal stroke of the actuators has to be identical and equal to 0.65 m;
- in order to avoid interferences between the different elements of the manipulators, the minimal distance between to non secant links has to be equal to 0.1 m, and we consider that the minimal angle between two secant links is equal to 15°. For example, on figure 3c, the angle between links  $A_i D_i$  and  $C_i D_i$  cannot be inferior to 15°;
- the interval of rotation for the spherical joints has to be comprised between -60 deg and +60 deg.

The geometric parameters for the Gough platform are already fixed. For the other manipulators, we chose the geometric parameters that are as close as possible to the parameters of the Gough platform:

- Stewart platform: points  $C_1$ ,  $C_2$  and  $C_3$  coincide with  $A_2$ ,  $A_3$  and  $A_1$  respectively, and  $A_i D_i = 0.2$  m;
- new proposed manipulator, case one (actuators upon the base):  $l_1 = 0.15$ ,  $l_2 = 0.65$ ;
- new proposed manipulator, case two (actuators under the base):  $l_1 = -0.3$ ,  $l_2 = -0.75$ .

Fig. 4 presents the obtained workspaces. The sizes of the workspace are ( $m^3$ ): Gough platform: 0.719; Stewart platform: 1.749; new proposed manipulator, case one: 1.959; new proposed manipulator, case two: 2.168.

The next step of our analysis is the comparison the quality of effort transmission determined by pressure angles existing in the studied mechanisms. These angles can be defined as angles between vectors of forces and translational velocities of points at which the forces are applied. Note, that in Type 2 singular configurations (Gosselin and Angeles [5]) at least one of the pressure angles is equal to 90°.

For any parallel mechanism with passive joints, it is well known that the  $i$ -th line of the inverse Jacobian matrix  $\mathbf{J}^{-1}$  relating the twist of the platform expressed at point  $P$  to the particular velocities ( $\dot{\mathbf{q}} = \mathbf{J}^{-1} \dot{\mathbf{t}}$ ) corresponds to the wrench  $\mathbf{w}_i$  applied by the  $i$ -th actuator on the platform. For a 6 DOF manipulator, the inverse Jacobian matrix  $\mathbf{J}^{-1}$  can be written as:

$$\mathbf{J}^{-1} = (\mathbf{w}_1, \mathbf{w}_2, \mathbf{w}_3, \mathbf{w}_4, \mathbf{w}_5, \mathbf{w}_6)^T \quad (13)$$

Without loss of generality, let us compute the expression of the twist  $\mathbf{t}_1$  of the platform when actuator 1 is dis-

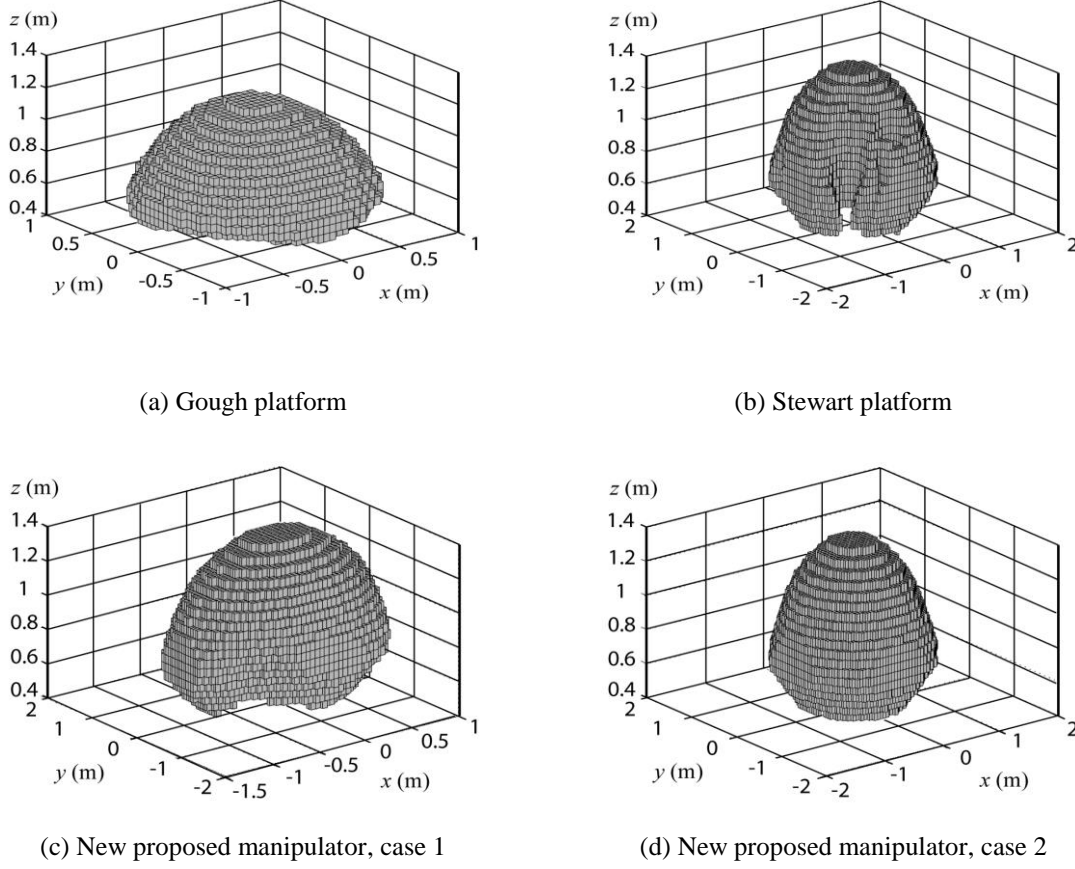


Figure 4. Workspaces of the manipulators under study.

connected and the other actuators are fixed.  $\mathbf{t}_1$  is the twist reciprocal to the wrenches  $\mathbf{w}_2$  to  $\mathbf{w}_6$  applied on the platform by the actuators 2 to 6. Therefore we can have the five following relations:

$$0 = v_{1x} F_{ix} + v_{1y} F_{iy} + v_{1z} F_{iz} + \omega_{1x} M_{ix} + \omega_{1y} M_{iy} + \omega_{1z} M_{iz}, \quad (i = 2 \text{ to } 6) \quad (14)$$

where:

$\mathbf{V}_P = [v_{1x}, v_{1y}, v_{1z}]^T$  represent the translational velocities of point  $P$  of the platform along the axes  $x$ ,  $y$  and  $z$  respectively;

$\mathbf{\Omega}_1 = [\omega_{1x}, \omega_{1y}, \omega_{1z}]^T$  represents the rotational velocities of the platform around the axes  $x$ ,  $y$  and  $z$  respectively;

$F_{ix}, F_{iy}, F_{iz}$  represent the forces applied on the platform by the  $i$ -th actuator along the axes  $x$ ,  $y$  and  $z$  respectively;

$M_{ix}, M_{iy}, M_{iz}$  represent the moments applied at point  $P$  on the platform by the  $i$ -th actuator around the axes  $x$ ,  $y$  and  $z$  respectively.

Fixing the value of  $v_{1x}$  to 1, one can determinate the expressions of the other terms of the twist  $\mathbf{t}_1$ :

$$\mathbf{t}_1 = \begin{bmatrix} 1 \\ -\mathbf{N}^{-1}\mathbf{F}_x \end{bmatrix}, \quad \mathbf{N} = \begin{bmatrix} F_{2y} & F_{2z} & M_{2x} & M_{2y} & M_{2z} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ F_{6y} & F_{6z} & M_{6x} & M_{6y} & M_{6z} \end{bmatrix}, \quad \mathbf{F}_x = [F_{2x}, F_{3x}, F_{4x}, F_{5x}, F_{6x}]^T \quad (15)$$

For each of the four manipulators, the wrench applied by actuator 1 on the point  $B_1$  of the platform is a pure force  $\mathbf{F}_1$ . The translational velocity  $\mathbf{V}_{B1}$  of point  $B_1$  can be deduced from the expression  $\mathbf{V}_{B1} = \mathbf{V}_P + \mathbf{B}_1\mathbf{P} \times \mathbf{\Omega}_1$ .

The terms of this expression are detailed above. Therefore, the pressure angle  $\alpha_1$  can be determined as:

$$\alpha_1 = \cos^{-1} \left( \frac{\mathbf{F}_1^T \mathbf{V}_{B1}}{\|\mathbf{F}_1\| \|\mathbf{V}_{B1}\|} \right) \quad (16)$$



Please note that the pressure angle is an indicator of the power transmitted by the actuator 1 to the platform, which can be written as:  $\mathcal{P} = \mathbf{F}_1^T \mathbf{V}_{B1} = \|\mathbf{F}_1\| \|\mathbf{V}_{B1}\| \cos \alpha_1$ .

These pressure angles can be calculated in a similar way for the Gough and Stewart platforms and for actuators 1 to 3 of the new mechanisms. However, for actuators 4 to 6 of the new manipulators, the way to compute the pressure angles is a bit different. Without loss of generality, let us consider the link  $C_1D_2$  (Fig. 1a). Basing on relation, equivalent to (15), it is possible to compute the twist  $\mathbf{t}_4$  of the platform when actuator 4 is disconnected and the others are fixed, and as a result, the translational velocities  $\mathbf{V}_{C1}$  and  $\mathbf{V}_{D2}$ :  $\mathbf{V}_{C1}=(l_1/\rho_1)\mathbf{V}_{B1}$ ,  $\mathbf{V}_{D2}=(l_2/\rho_2)\mathbf{V}_{B2}$ . The power transmitted by the actuator to the mechanism can be written under the form:  $\mathcal{P} = \mathbf{F}_4^T (\mathbf{V}_{D2} - \mathbf{V}_{C1}) = \mathbf{F}_4^T \mathbf{V}_{D2C1}$ , where  $\mathbf{F}_4$  represents the effort applied by the actuators on links  $A_1B_1$  and  $A_2B_2$ ,  $\mathbf{V}_{D2C1}$  represents the relative velocity of the point  $D_2$  from the point  $C_1$ . Therefore, one can define a pressure angle  $\alpha_4$ :

$$\alpha_4 = \cos^{-1} \left( \left| \frac{\mathbf{F}_4^T \mathbf{V}_{D2C1}}{\|\mathbf{F}_4\| \|\mathbf{V}_{D2C1}\|} \right| \right) \quad (17)$$

The same approach can be used in order to determine the pressure angles of actuators 5 and 6.

Thus, we can now analyse the workspace of the studied manipulators taking into account the maximal value of the pressure angles of the mechanisms. We consider that the maximal admissible value for the pressure angle is  $82^\circ$ . For one point of the workspace, if there exists at least one configuration of the orientation space for which the pressure angle is inferior to  $82^\circ$ , so this point belongs to the workspace of the robot. Therefore, we can obtain new results about the workspace of our manipulators. The sizes of the workspace are ( $m^3$ ): Gough platform: 0.719; Stewart platform: 1.727; new proposed manipulator, case one: 1.952; new proposed manipulator, case two: 2.154. The new results are similar to those obtained previously.

## 7. Conclusion.

Thus, in this article, the parallel-cross manipulators are represented as well as their classification. The comparison analysis executed for the new manipulators and well known parallel manipulators taking into account geometric limitations and admissible pressure angles allows to conclude, that the workspace of the new manipulators is not less than this of well known parallel manipulators.

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