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Complete shaking force and shaking moment balancing of planar parallel manipulators with prismatic pairs

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ABSTRACT

This paper deals with the complete shaking force and shaking moment balancing of planar parallel manipulators with prismatic pairs. The cancellation of the dynamic loads transmitted to the ground is a challenge for these types of manipulators.

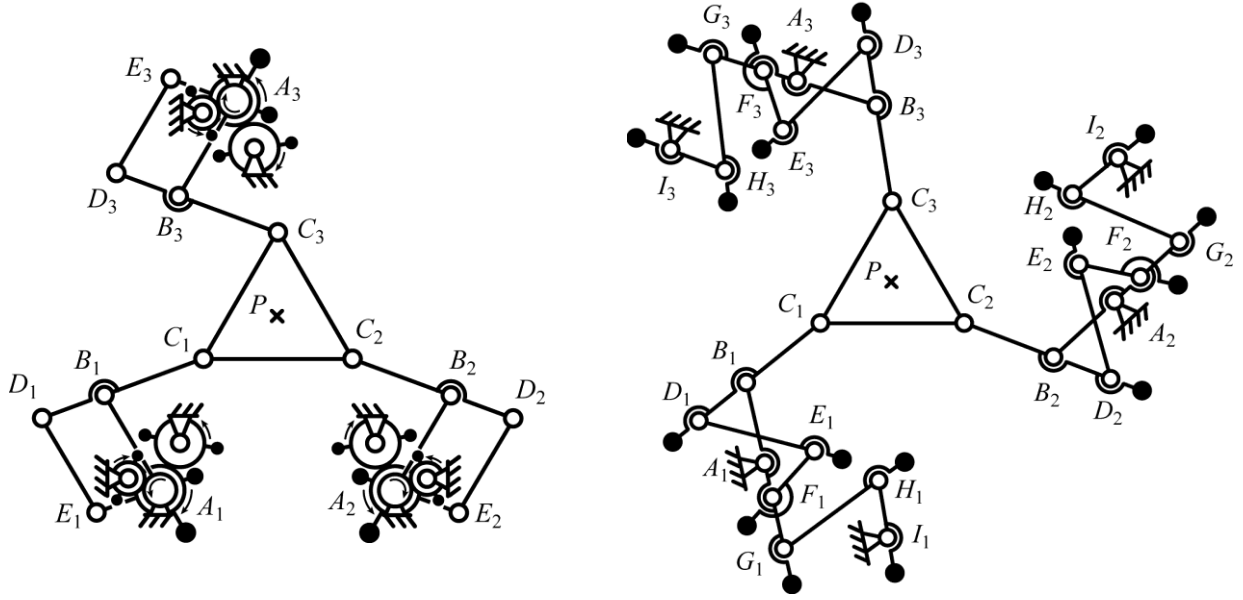
It is obvious that the classical methods based on the optimal redistribution of movable masses and additional counter-rotations can be used to cancel shaking force and shaking moment. However, such a balancing of parallel manipulators with prismatic pairs is only attained via a considerably complicated design. This paper shows that it is possible to balance planar parallel mechanisms using Scott-Russell mechanisms. Such an approach enables a division of the number of counter-rotations by two. Numerical simulations carried out using ADAMS software validate the obtained results and illustrate that the suggested balancing enables the creation of a parallel manipulator transmitting no inertia load to its base.

Index terms – *Shaking force, shaking moment, balancing, planar parallel robots with prismatic pairs.*

1. INTRODUCTION

Shaking force balancing is mostly obtained via an optimal redistribution of movable masses [1-10] or adjustment of kinematic parameters [11]. The cancellation of the shaking moment is a more complicated task and can be obtained using three main different methods: (i) shaking moment balancing using counter-rotations [12-17] (Fig. 1a) (ii) shaking moment balancing by adding four bar linkages [18-22] (Fig.1b) and (iii) shaking moment balancing by optimal trajectory planning [17, 23, 24].

Previous works have been devoted to the study of parallel manipulators with revolute joints and until now, to our knowledge, no study has been carried out on complete shaking force and shaking moment balancing of parallel manipulators with prismatic pairs.



(a) balancing by adding counter-rotations [17] (b) balancing by adding four-bar linkages [18]

Figure 1. Complete shaking force and shaking moment balanced 3-RRR planar parallel manipulators.

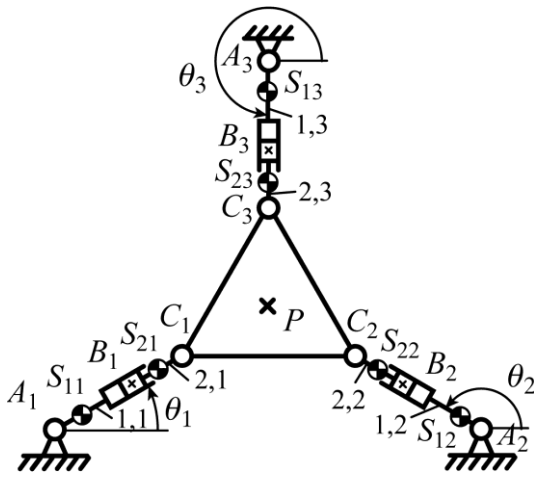
In this paper, for the first time, we propose solutions for complete shaking force and shaking moment balancing of planar parallel manipulators with prismatic pairs. We illustrate these solutions via the 3-RPR parallel manipulator. All obtained results are validated using ADAMS software simulations.

2. COMPLETE SHAKING MOMENT AND SHAKING FORCE BALANCING BY ADDING AN IDLER LOOP BETWEEN THE BASE AND THE PLATFORM

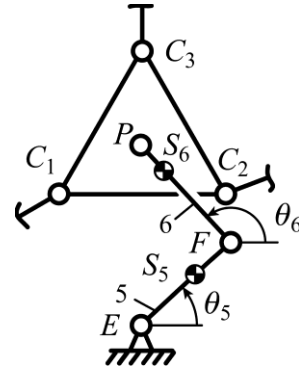
Inertia force balancing by adding an idler loop is known to be used for 1-degree-of-freedom (DOF) mechanisms [25-29]. With regard to planar manipulators, such an approach has only been used in the balancing of gravitational and inertia forces [9, 10, 30, 31].

In this section, the complete shaking force and shaking moment balancing of planar manipulators by adding an idler loop is discussed. The added balancing loop is mounted between the base and the platform of the mechanism. We illustrate the suggested balancing technique on a 3-*RPR* mechanism (Fig. 2). Please note that we do not mention the type of actuation of the mechanism as it has no influence on the balancing.

Firstly, let us analyze the cancellation of the dynamic reactions of the 3-*RPR* planar parallel mechanism (Fig. 2a). Such a mechanism has 3 DOF (two translations in the Oxy plane and one rotation of the moving platform around an axis perpendicular to Oxy) and is composed of three identical legs, each being composed of a revolute joint attached to the base at point A_i (in the remainder of this report, $i = 1, 2, 3$), one moving prismatic guide, located at point B_i , and another revolute joint attached to the platform at C_i . The base and platform triangles, denoted $A_1A_2A_3$ and $C_1C_2C_3$, are equilateral. On this manipulator, typically, the actuated joints are the first revolute joint at A_i or the linear guide at B_i .



(a) Schematics of the 3-*RPR* mechanism.



(b) Schematics of the added idler loop (*RRR* chain).

Figure 2. Schematic of the 3-*RPR* robot under study.

Considering that the x axis is directed along the line A_1A_2 , the y axis being perpendicular to the x axis and the origin of the base frame located at point O , the centre of the circumcircle of triangle $A_1A_2A_3$, one can define the coordinate x , y and ϕ of the platform, as being respectively the coordinates of point P along the x and y axes and the angle between the line C_1C_2 and A_1A_2 .

The length B_iC_i is denoted L_i . Let us also denote as S_{ji} the centre of mass of link ij ($j = 1, 2$), which has a mass m_j and an axial moment of inertia I_j . The centre of the mass of the platform is located at point P . The mass of the platform is m_p and its axial moment of inertia I_p .

In order to cancel the shaking forces and shaking moment of the manipulator, an idler loop is added between the base and the platform (Fig. 2b). The lengths EF and FP are denoted L_5 and L_6 respectively. The centre of mass of elements 5 and 6 of the idler loop are denoted S_3 and S_4 with masses m_5 and m_6 and axial moments of inertia I_5 and I_6 , respectively. The positions of the centre of masses are $\mathbf{d}_{A_iS_{1i}} = r_1 L_1 \mathbf{u}_i$, $\mathbf{d}_{C_iS_{2i}} = (r_2-1) L_1 \mathbf{u}_i$, $\mathbf{d}_{E S_5} = r_5 \mathbf{d}_{EF}$, and $\mathbf{d}_{F S_6} = r_6 \mathbf{d}_{FP}$, r_1 , r_2 , r_5 , and r_6 being dimensionless coefficients, and \mathbf{u}_i a unit vector directed along B_iC_i .

Thus, considering the shaking force \mathbf{F} of leg 1, its expression is:

$$\mathbf{F} = \sum_{i=1}^3 \sum_{j=1}^2 m_j \ddot{\mathbf{d}}_{S_{ji}} + m_p \ddot{\mathbf{d}}_P + m_5 \ddot{\mathbf{d}}_{S_5} + m_6 \ddot{\mathbf{d}}_{S_6} \quad (1)$$

where $\ddot{\mathbf{d}}_{S_{ji}}$, $\ddot{\mathbf{d}}_P$, $\ddot{\mathbf{d}}_{S_5}$ and $\ddot{\mathbf{d}}_{S_6}$ are the acceleration of the centre of mass S_{ij} , of P , of S_5 and of S_6 respectively.

After a simple development of expression (1), it can be seen that the shaking force \mathbf{F} can be expressed as:

$$\mathbf{F} = (m_1 r_1 - m_2 (1 - r_2)) \sum_{i=1}^3 \mathbf{a}_i + (3m_2 + m_p + m_6 r_6) \mathbf{a}_4 + (3m_2 + m_p + m_5 r_5 + m_6) \ddot{\mathbf{d}}_F \quad (2)$$

with

$$\mathbf{a}_i = L_1 \left(\ddot{\theta}_i \begin{bmatrix} -\sin \theta_i \\ \cos \theta_i \end{bmatrix} - \dot{\theta}_i^2 \begin{bmatrix} \cos \theta_i \\ \sin \theta_i \end{bmatrix} \right), \quad (3a)$$

$$\mathbf{a}_4 = L_6 \left(\ddot{\theta}_6 \begin{bmatrix} -\sin \theta_6 \\ \cos \theta_6 \end{bmatrix} - \dot{\theta}_6^2 \begin{bmatrix} \cos \theta_6 \\ \sin \theta_6 \end{bmatrix} \right), \quad (3b)$$

and $\ddot{\mathbf{d}}_F$ is the acceleration of point F .

At this step, only five counterweights are needed in the cancellation of the shaking force, but it could be seen after more derivations that three others are necessary for the cancellation of the shaking moment. Therefore, we propose directly adding three supplementary counterweights (Fig. 3). The positions of the eight masses are $\mathbf{d}_{AiMcp1i} = r_{cp1} L_1 \mathbf{u}_i$, $\mathbf{d}_{BiMcp2i} = r_{cp2} L_1 \mathbf{u}_i$, $\mathbf{d}_{EMcp5} = r_{cp5} \mathbf{d}_{EF}$, and $\mathbf{d}_{FMcp6} = r_{cp} \mathbf{d}_{FP}$, r_{cp1} , r_{cp2} , r_{cp5} , and r_{cp6} being dimensionless coefficients. Their masses are respectively denoted m_{cp1} , m_{cp2} , m_{cp5} and m_{cp6} . With the addition of the counterweights, the shaking force becomes:

$$\mathbf{F}^{bal} = \mathbf{F} + \left(m_{cp1} r_{cp1} - m_{cp2} (1 - r_{cp2}) \right) \sum_{i=1}^3 \mathbf{a}_i + m_{cp6} r_{cp6} \mathbf{a}_4 + (m_{cp6} + m_{cp5} r_{cp5}) \ddot{\mathbf{d}}_F \quad (4)$$

Thus, the shaking force is cancelled if:

$$m_{cp1} = -\frac{m_1 r_1}{r_{cp1}}, \quad (5a)$$

$$m_{cp2} = -\frac{m_2 (1 - r_2)}{1 - r_{cp2}}, \quad (5b)$$

$$m_{cp6} = -\frac{3(m_2 + m_{cp2}) + m_p + r_6 m_6}{r_{cp6}}, \quad (5c)$$

and

$$m_{cp5} = -\frac{3(m_2 + m_{cp2}) + m_p + m_6 + m_{cp6} + r_5 m_5}{r_{cp5}}. \quad (5d)$$

The expression of the shaking moment M_O of the modified structure (expressed at point O) can be written as:

$$M_O = \frac{dH_O}{dt}, \quad (6)$$

where H_O is the angular momentum of the leg (expressed at point O). Thus, in order to cancel the shaking moment, the angular momentum is held constant over time.

The expression of the angular momentum H_O is:

$$H_O = \sum_{i=1}^3 \sum_{j=1}^2 (m_j (x_{OSji} \dot{y}_{OSji} - y_{OSji} \dot{x}_{OSji}) + I_j \dot{\theta}_i) + \sum_{i=1}^3 \sum_{j=1}^2 (m_{cpj} (x_{OMji} \dot{y}_{OMji} - y_{OMji} \dot{x}_{OMji})) + I_p \dot{\phi} \\ + \sum_{j=5}^6 (m_j (x_{OSj} \dot{y}_{OSj} - y_{OSj} \dot{x}_{OSj}) + m_{cpj} (x_{OMj} \dot{y}_{OMj} - y_{OMj} \dot{x}_{OMj}) + I_j \dot{\theta}_j) \quad (7)$$

where x_{OQ} , y_{OQ} , \dot{x}_{OQ} and \dot{y}_{OQ} are the position and velocities of any point Q along x and y axes, respectively, Q being either point S_{ji} , M_{ji} , ($j = 1, 2$), S_j or M_j ($j = 5, 6$).

Developing and introducing (5) into (7) yields

$$H_O = \sum_{i=1}^3 (I_1 + I_2 + (m_1 r_1^2 + m_{cp1} r_{cp1}^2 + m_2 (1 - r_2)^2) L_1^2) \dot{\theta}_i + \sum_{i=1}^3 (m_{cp2} (1 - r_{cp2})^2 L_1^2) \dot{\theta}_i \\ + (I_p + 3(m_2 + m_{cp2}) R_p^2) \dot{\phi} + (I_6 + (m_6 r_6^2 + m_{cp6} r_{cp6}^2 + m_p + 3(m_2 + m_{cp2}))) L_6^2 \dot{\theta}_6 \\ + (I_5 + (m_5 r_5^2 + m_{cp5} r_{cp5}^2 + m_6 + m_{cp6} + m_p) L_5^2) \dot{\theta}_5 + 3(m_2 + m_{cp2}) L_5^2 \dot{\theta}_5 \quad (8)$$

After such modifications of the RRR chain, the angular momentum of the legs of the mechanism and of the RRR chain can be balanced using six counter-rotations (Fig. 3), which have an axial moment of inertia equal to:

$$I_{cr1} = I_1 + I_2 + (m_1 r_1^2 + m_{cp1} r_{cp1}^2 + m_2 (1 - r_2)^2 + m_{cp2} (1 - r_{cp2})^2) L_1^2 \quad (9a)$$

$$I_{cr2} = I_p + 3(m_2 + m_{cp2}) R_p^2 \quad (9b)$$

$$I_{cr3} = (m_6 r_6^2 + m_{cp6} r_{cp6}^2 + m_p + 3(m_2 + m_{cp2})) L_6^2 + 2I_{cr2} + I_6 \quad (9c)$$

$$I_{cr4} = (m_5 r_5^2 + m_{cp5} r_{cp5}^2 + m_6 + m_{cp6} + m_p + 3(m_2 + m_{cp2})) L_5^2 + 2I_{cr3} + I_5 \quad (9d)$$

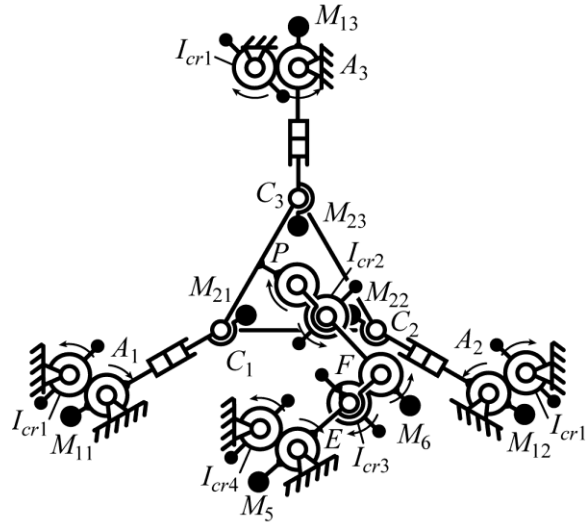


Figure 3. Schematics of the 3-RPR mechanism with the added RRR chain used for the cancellation of the shaking force and shaking moment.

Numerical application

Let us illustrate the suggested balancing approach using numerical simulations carried out with ADAMS software. For this purpose, non balanced and balanced 3-RPR parallel manipulators will be compared.

The chosen trajectory for simulations is a straight line of the controlled point of the platform, achieved in $t_f = 0.25$ s, between $P_0 = (x_0, y_0) = (-0.05 \text{ m}, 0)$ and $P_f = (x_f, y_f) = (-0.2 \text{ m}, 0)$ with a

rotation of the platform from $\phi_0 = 0^\circ$ to $\phi_f = 30^\circ$. For the displacement of the mechanism, fifth order polynomial laws are used and therefore the trajectory is defined by the following expressions:

$$x(t) = x_0 + (x_f - x_0) \left(10 \left(\frac{t}{t_f} \right)^3 - 15 \left(\frac{t}{t_f} \right)^4 + 6 \left(\frac{t}{t_f} \right)^5 \right), \quad (10a)$$

$$y(t) = 0, \quad (10b)$$

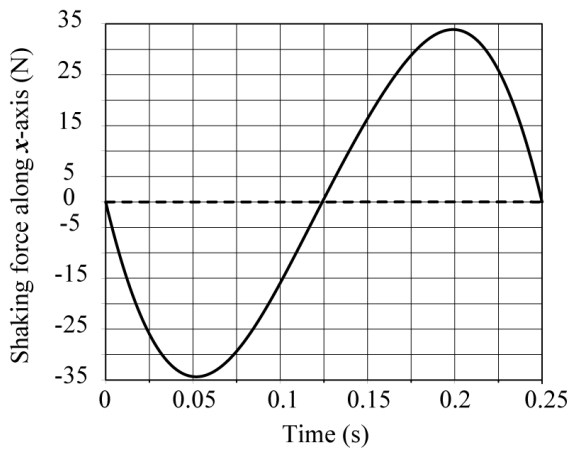
$$\phi(t) = \phi_0 + (\phi_f - \phi_0) \left(10 \left(\frac{t}{t_f} \right)^3 - 15 \left(\frac{t}{t_f} \right)^4 + 6 \left(\frac{t}{t_f} \right)^5 \right). \quad (10c)$$

The parameters used for the simulations are the following:

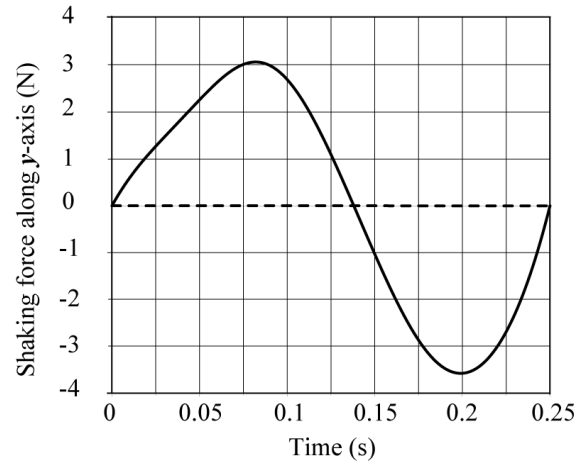
- radii of the circumcircles of the base triangle $A_1A_2A_3$ and the platform triangle $C_1C_2C_3$, $R_b = 0.35$ m, $R_p = 0.1$ m;
- $L_1 = 0.05$ m; $L_5 = 0.15$ m; $L_6 = 0.1581$ m;
- $r_2 = r_5 = r_6 = 0.5$; $r_1 = 2$;
- $m_1 = 0.75$ kg; $m_2 = 0.37$ kg; $m_5 = 0.42$ kg; $m_6 = 0.47$ kg; $m_p = 1$ kg;
- $I_1 = 0.00344$ kg.m²; $I_2 = 0.00025$ kg.m²; $I_5 = 0.00122$ kg.m²; $I_6 = 0.00146$ kg.m²; $I_p = 0.00436$ kg.m²;
- point E is located at point O .

For such parameters and such a trajectory, the shaking force and shaking moment are computed using ADAMS software and are presented in Fig. 4 (solid line). Then, we add the counterweights and the idler loop EFP to the mechanism. The position coefficients of the counterweights are all equal to $r_{cpj} = 0.5$ ($j = 1, 2, 5, 6$). Therefore, the added masses are equal to $m_{cp1} = 0.75$ kg, $m_{cp2} = 0.37$ kg, $m_{cp5} = 6.92$ kg, $m_{cp6} = 21.66$ kg. The new values of the shaking force and moment are presented in Fig. 4 (dashed line). It is possible to see that with the added counterweights the

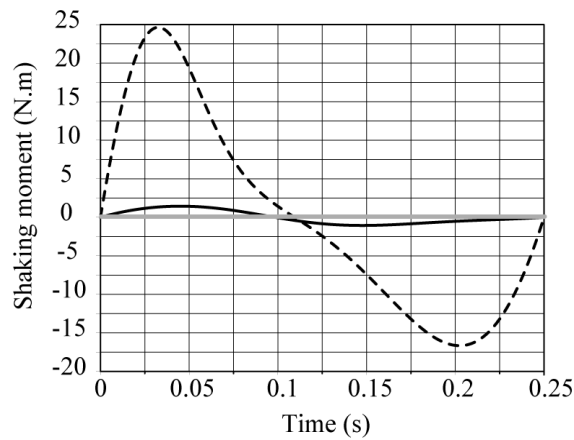
shaking efforts are cancelled, while the maximal value of shaking moment is increased by a factor 17. Finally, we add the counter-rotations. Their values are equal to $I_{cr1} = 0.01917 \text{ kg.m}^2$, $I_{cr2} = 0.02665 \text{ kg.m}^2$, $I_{cr3} = 0.18169 \text{ kg.m}^2$, $I_{cr4} = 0.72781 \text{ kg.m}^2$. With such counter-rotations, the shaking moment is balanced (in gray line in Fig. 4c).



(a) shaking force along x -axis



(b) shaking force along y -axis



(c) shaking moment along z -axis

Figure 4. Shaking force and shaking moment before (solid line) and after (dashed line) the addition of the counterweights, and after the addition of the counter-rotations (gray line).

3. COMPLETE SHAKING FORCE AND SHAKING MOMENT BALANCING VIA SCOTT-RUSSELL MECHANISM

In this section another approach for complete shaking force and shaking moment balancing is developed, which consists of adding Scott-Russell mechanisms to the initial architecture of a manipulator. This approach enables a reduction in the number of counter-rotations.

3.1. Properties of the Scott-Russell mechanism.

Firstly, let us observe a simple slider-crank mechanism (Fig. 5). Let us denote the lengths AB and BC as respectively L_1 and L_2 , and the centre of masses of link i ($i = 1, 2, 3$) as S_i , which has a mass m_i and an axial moment of inertia I_i . The positions of the centres of mass are $\mathbf{d}_{AS1} = r_1 \mathbf{d}_{AB}$, $\mathbf{d}_{AS2} = \mathbf{d}_{AB} + r_2 \mathbf{d}_{BC}$, $\mathbf{d}_{AS3} = \mathbf{d}_{AC} + \mathbf{d}_{CS3}$, r_1 and r_2 being dimensionless coefficients, $\mathbf{d}_{CS3} = L_3 r_3 \mathbf{x}$, (L_3 is a constant).

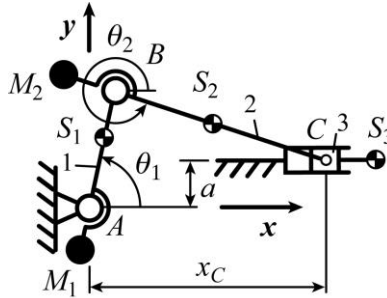


Figure 5. A general slider-crank mechanism.

It is known that the complete shaking force and shaking moment balancing of a general slider-crank mechanism can be obtained by adding two counterweights mounted on the links and two pairs counter-rotation inertia-counterweights. However, it is possible to balance this mechanism

without counter-rotation inertia-counterweights if it has specific geometrical parameters, as in Scott-Russell mechanisms ($a=0, L_1=L_2$).

Let us consider the balancing of this mechanism. The expression of the shaking force \mathbf{F} of a slider-crank mechanism can be written as:

$$\mathbf{F} = \sum_{i=1}^3 m_i \ddot{\mathbf{d}}_{S_i} \quad (11)$$

where $\ddot{\mathbf{d}}_{S_i}$ is the acceleration of the centre of mass S_i .

Developing (11), the expression of \mathbf{F} becomes:

$$\mathbf{F} = (m_1 r_1 + m_2) \ddot{\mathbf{d}}_B + (m_2 r_2 + m_3) \mathbf{a} \quad (12)$$

with

$$\mathbf{a} = L_2 \left(\ddot{\theta}_2 \begin{bmatrix} -\sin \theta_2 \\ \cos \theta_2 \end{bmatrix} - \dot{\theta}_2^2 \begin{bmatrix} \cos \theta_2 \\ \sin \theta_2 \end{bmatrix} \right). \quad (13)$$

$\ddot{\mathbf{d}}_B$ is the acceleration of point B .

The constant terms of eq. (12) can be cancelled by the addition of two counterweights M_j , ($j = 1, 2$) (Fig. 5), of which the masses are m_{cpj} . Their positions are: $\mathbf{d}_{AMcp1} = r_{cp1} \mathbf{d}_{AB}$, $\mathbf{d}_{BMcp2} = r_{cp2} \mathbf{d}_{BC}$, r_{cp1} and r_{cp2} being dimensionless coefficients. With the addition of the counterweights, the shaking force becomes:

$$\mathbf{F}^{bal} = \mathbf{F} + (m_{cp1} r_{cp1} + m_{cp2}) \ddot{\mathbf{d}}_B + m_{cp2} r_{cp2} \mathbf{a} \quad (14)$$

Thus, the shaking force vanishes if:

$$m_{cp2} = -\frac{m_2 r_2 + m_3}{r_{cp2}} \quad \text{and} \quad m_{cp1} = -\frac{m_1 r_1 + m_2 + m_{cp2} + m_3}{r_{cp1}}. \quad (15)$$

The expression of the angular momentum H_A (expressed at point A) is:

$$H_A = \sum_{j=1}^3 (m_j (x_{ASj} \dot{y}_{ASj} - \dot{x}_{ASj} y_{ASj})) + \sum_{j=1}^2 (I_j \dot{\theta}_j + m_{cpj} (x_{MSj} \dot{y}_{MSj} - \dot{x}_{MSj} y_{MSj})) \quad (16)$$

where x_{AQ} , y_{AQ} , \dot{x}_{AQ} and \dot{y}_{AQ} are the position and velocities of any point Q along x and y axes, respectively, Q being either point S_j or M_j ($j = 1, 2, 3$).

Developing and introducing (15) into (16),

$$H_A = (I_1 + (m_1 r_1^2 + m_{cp1} r_{cp1}^2 + m_2 + m_{cp2} + m_3) L_1^2) \dot{\theta}_1 + (I_2 + (m_2 r_2^2 + m_{cp2} r_{cp2}^2 + m_3) L_2^2) \dot{\theta}_2 \quad (17)$$

with

$$\dot{\theta}_2 = - \frac{\dot{y}_{AB} (x_{AC} - x_{AB}) + (a - y_{AB}) (\dot{x}_{AC} - \dot{x}_{AB})}{L_2^2}. \quad (18)$$

where x_{AC} , x_{AB} , y_{AB} are the coordinates of points C and B , respectively, and \dot{x}_{AC} , \dot{x}_{AB} , \dot{y}_{AB} their velocities.

In order to cancel the shaking moment M_A , the angular momentum has to be constant or null.

Developing (18), one notices that this can be obtained if:

$$a = 0 \text{ and } L_1 = L_2. \quad (19)$$

In such a case, $\dot{\theta}_1 = -\dot{\theta}_2$. Therefore, the shaking moment is cancelled if:

$$I_1 + (m_1 r_1^2 + m_{cp1} r_{cp1}^2 + m_2 + m_{cp2}) L_1^2 - I_2 - (m_2 r_2^2 + m_{cp2} r_{cp2}^2) L_1^2 = 0 \quad (20)$$

3.2. Balancing of a manipulator's leg using a Scott-Russell mechanism.

Now let us consider a manipulator's leg with an added Scott-Russell mechanism (Fig. 6). Let us denote as S_4 the centre of mass of link 4, which has a mass m_4 and an axial moment of inertia I_4 . The position of S_4 is: $\mathbf{d}_{CS4} = L_3 r_4 \mathbf{u}$, r_4 being a dimensionless coefficient and \mathbf{u} a unit vector along \mathbf{d}_{CS3} .

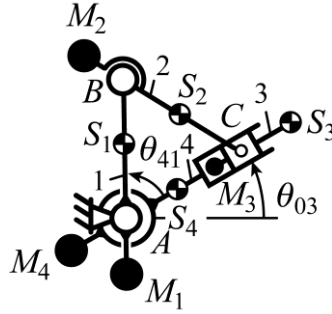


Figure 6. A manipulator leg with added Scott-Russell mechanism.

Now the shaking force becomes:

$$\mathbf{F} = (m_1 r_1 + m_{cp1} r_{cp1} + m_2 + m_{cp2} + m_3) \ddot{\mathbf{d}}_B + (m_3 r_3 + m_4 r_4) \mathbf{a}_1 + (m_2 r_2 + m_{cp2} r_{cp2} + m_3) \mathbf{a}_2 \quad (21)$$

with

$$\mathbf{a}_1 = L_3 \left(\ddot{\theta}_{03} \begin{bmatrix} -\sin \theta_{03} \\ \cos \theta_{03} \end{bmatrix} - \dot{\theta}_{03}^2 \begin{bmatrix} \cos \theta_{03} \\ \sin \theta_{03} \end{bmatrix} \right) \quad (22)$$

and

$$\mathbf{a}_2 = L_1 \left((\ddot{\theta}_{03} - \ddot{\theta}_{41}) \begin{bmatrix} -\sin(\theta_{03} - \theta_{41}) \\ \cos(\theta_{03} - \theta_{41}) \end{bmatrix} - (\dot{\theta}_{03} - \dot{\theta}_{41})^2 \begin{bmatrix} \cos(\theta_{03} - \theta_{41}) \\ \sin(\theta_{03} - \theta_{41}) \end{bmatrix} \right). \quad (23)$$

At this step, only one counterweight is needed for the cancellation of the shaking force, but it could be seen after more derivations that another is necessary for the cancellation of the shaking moment. Therefore, we propose adding this supplementary counterweight directly. The positions

of the two masses are: $\mathbf{d}_{AMcp3} = r_{cp3} L_3 \mathbf{u}$, $\mathbf{d}_{CMcp4} = r_{cp4} L_3 \mathbf{u}$, r_{cp3} and r_{cp4} being dimensionless coefficients. Their masses are respectively denoted m_{cp3} and m_{cp4} . With the addition of the counterweights, the shaking force becomes:

$$\mathbf{F}^{bal} = \mathbf{F} + m_{cp3} \ddot{\mathbf{d}}_B + m_{cp3} \mathbf{a}_1 + (m_{cp3} r_{cp3} + m_{cp4} r_{cp4}) \mathbf{a}_2. \quad (24)$$

Thus, the shaking force is cancelled if:

$$m_{cp4} = -\frac{m_4 r_4}{r_{cp4}}, \quad (25a)$$

$$m_{cp3} = -\frac{m_3 r_3}{r_{cp3}}, \quad (25b)$$

$$m_{cp2} = -\frac{m_2 r_2 + m_3 + m_{cp3}}{r_{cp2}} \quad (25c)$$

$$\text{and } m_{cp1} = -\frac{m_1 r_1 + m_2 + m_{cp2} + m_3 + m_{cp3}}{r_{cp1}}. \quad (25d)$$

Developing and simplifying, the expression of the angular momentum is:

$$H_A = I_{eq1} \dot{\theta}_{03} + I_{eq2} \dot{\theta}_{41} \quad (26)$$

with

$$I_{eq1} = (m_1 r_1^2 + m_{cp1} r_{cp1}^2 + m_2 (1 - r_2)^2 + m_{cp2} (1 - r_{cp2})^2) L_1^2 + I_1 + I_2 + I_3 + I_4 \\ + (m_3 r_3^2 + m_{cp3} r_{cp3}^2 + m_4 r_4^2 + m_{cp4} r_{cp4}^2) L_3^2 \quad (27)$$

$$I_{eq2} = I_1 + (m_1 r_1^2 + m_{cp1} r_{cp1}^2 + m_2 + m_{cp2}) L_1^2 - I_2 - (m_2 r_2^2 + m_{cp2} r_{cp2}^2) L_1^2. \quad (28)$$

From (20), $I_{eq2} = 0$. Therefore, the shaking moment of the slider-crank can be cancelled using a simple counter-rotation I_{cr} with an axial moment of inertia equal to I_{eq1} .

3.3. Shaking moment and shaking force balancing of the 3-RPR manipulator

Now, let us apply such an approach to the 3-RPR mechanism. First of all, let us substitute the platform mass by three points masses located at C_1 , C_2 and C_3 , with the values of mass equal to m_{p1} , m_{p2} and m_{p3} respectively [13, 31, 32]. Such a condition can be obtained if:

$$m_{pi} = m_p / 3 \text{ and } I_p = 3 m_{pi} R_p^2. \quad (29)$$

where R_p is the radius of the circumcircle of $C_1C_2C_3$. Such a decomposition of the platform enables us to consider the shaking force and shaking moment balancing of each leg of the mechanism. Then, modifying each leg in order to obtain a mechanism similar to a slider-crank linkage, (i.e., by adding an idler loop to each leg), the shaking force and shaking moment are cancelled if:

$$m_{cp4} = -\frac{m_4 r_4}{r_{cp4}}, \quad (30a)$$

$$m_{cp3} = -\frac{m_3 r_3 + m_{pi}}{r_{cp3}}, \quad (30b)$$

$$m_{cp2} = -\frac{m_2 r_2 + m_3 + m_{cp3} + m_{pi}}{r_{cp2}}, \quad (30c)$$

$$m_{cp1} = -\frac{m_1 r_1 + m_2 + m_{cp2} + m_3 + m_{cp3} + m_{pi}}{r_{cp1}}, \quad (30d)$$

$$0 = I_1 + (m_1 r_1^2 + m_{cp1} r_{cp1}^2 + m_2 + m_{cp2})L_1^2 - I_2 - (m_2 r_2^2 + m_{cp2} r_{cp2}^2)L_1^2, \quad (30e)$$

and

$$I_{cr} = (m_1 r_1^2 + m_{cp1} r_{cp1}^2 + m_2 (1-r_2)^2 + m_{cp2} (1-r_{cp2})^2) L_1^2 + I_1 + I_2 + I_3 + I_4 + (m_3 r_3^2 + m_{cp3} r_{cp3}^2 + m_4 r_4^2 + m_{cp4} r_{cp4}^2 + m_{pi}) L_3^2 \quad (30f)$$

taking into account that I_{cr} is the axial moment of inertia of the counter-rotations (Fig. 7).

Thus, with this approach it is possible to create a fully-balanced shaking force and shaking moment 3-RPR mechanism with only three counter-rotations (Fig. 7), i.e., this method enables a reduction in the number of counter-rotations by a factor of two.

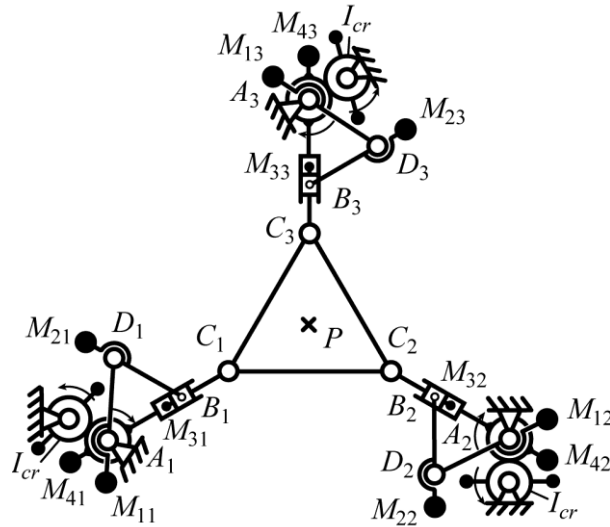


Figure 7. Schematics of a shaking force and shaking moment balanced 3-RPR mechanism.

3.4. Numerical application

The parameters used for the simulations are the followings:

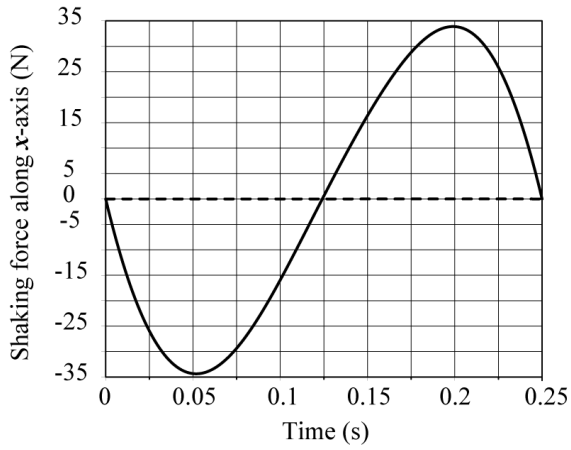
- radii of the circumcircles of the base triangle $A_1A_2A_3$ and the platform triangle $C_1C_2C_3$ R_b
 $= 0.35$ m, $R_p = 0.1$ m;
- $L_1 = L_2 = 0.25$ m; $L_3 = 0.025$ m

- $r_1 = r_2 = 0.5; r_3 = 0; r_4 = 4;$
- $m_1 = 1.09 \text{ kg}; m_2 = 1.1 \text{ kg}; m_3 = 0.37 \text{ kg}; m_4 = 0.75 \text{ kg}; m_p = 1 \text{ kg};$
- $I_1 = 0.00738 \text{ kg.m}^2; I_2 = 0.58389 \text{ kg.m}^2; I_3 = 0.00344 \text{ kg.m}^2; I_6 = 0.00025 \text{ kg.m}^2; I_p = 0.01 \text{ kg.m}^2.$

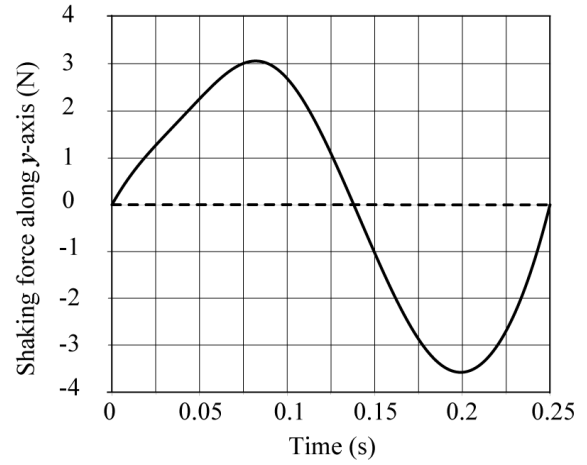
For these new parameters and for the trajectory used previously, taking into account that the position coefficients of the counterweights are equal to $r_{cpj} = -0.5$ ($j = 1, 3, 4$), $r_{cp2} = -1$, the new values of the counterweights are: $m_{cp1} = 3.17 \text{ kg}$, $m_{cp2} = 11.71 \text{ kg}$, $m_{cp3} = 0.33 \text{ kg}$, $m_{cp4} = 0.75 \text{ kg}$. The shaking force and shaking moment are then computed (dashed line in Fig. 8). It is possible to see that, with the counterweights, the shaking efforts are cancelled, while the maximal value of the shaking moment is increased by a factor 28. Finally, we add the counter-rotations. Their values are equal to $I_{cr} = 1.56907 \text{ kg.m}^2$. With such counter-rotations, the shaking moment is balanced (gray line in Fig. 8c).

Finally, it should be noted that the combination of the proposed two techniques of balancing enables the creation of fully balanced parallel manipulators with modified legs. As examples, different structures of balanced manipulators are presented in Fig. 9 (3-RPR, 3-PRR and 3-PRP) in which one leg with a prismatic pair is replaced by a leg with only revolute joints. Such a modification allows displacing the centre of mass of the manipulator to C_3 and then to balance the manipulator via the modified leg $C_3 B_3 A_3$.

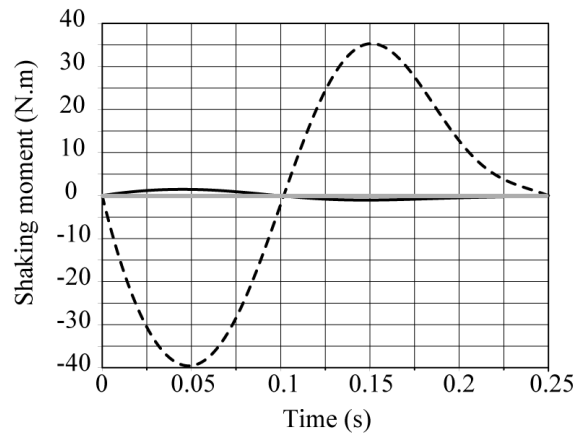
In the same way, it is possible to balance a parallel manipulator with prismatic pairs by adding fewer Scott-Russell mechanisms. The balancing schemes for several parallel manipulators are presented in Fig. 10.



(a) shaking force along x -axis

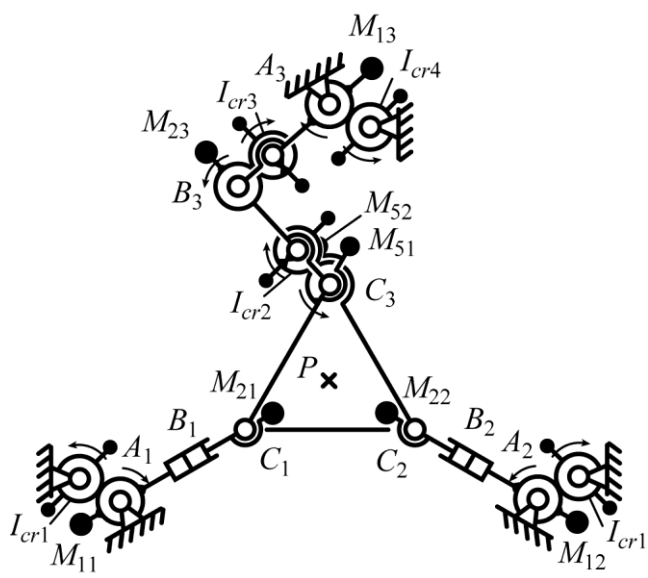


(b) shaking force along y -axis

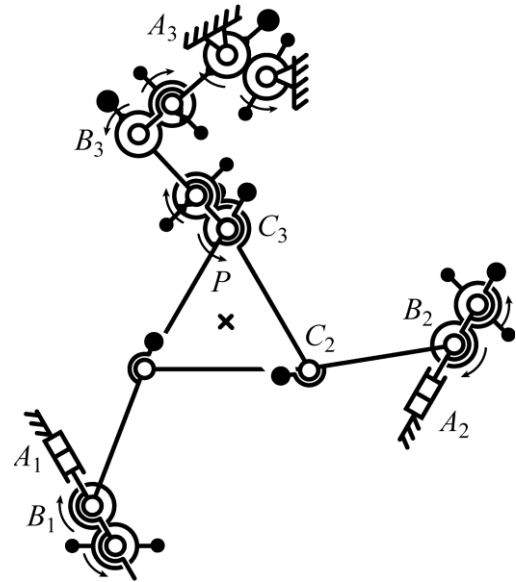


(c) shaking moment along z -axis

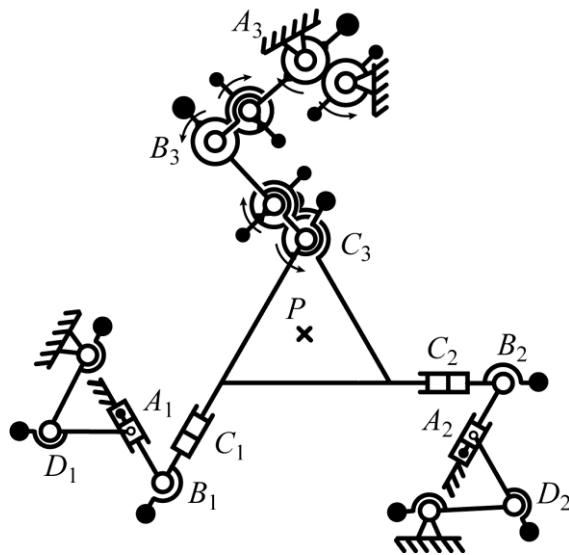
Figure 8. Shaking force and shaking moment, before (solid line) and after (dashed line) the addition of the counterweights, and after the addition of the counter-rotations (gray line).



a) Modified 3-RPR parallel manipulator

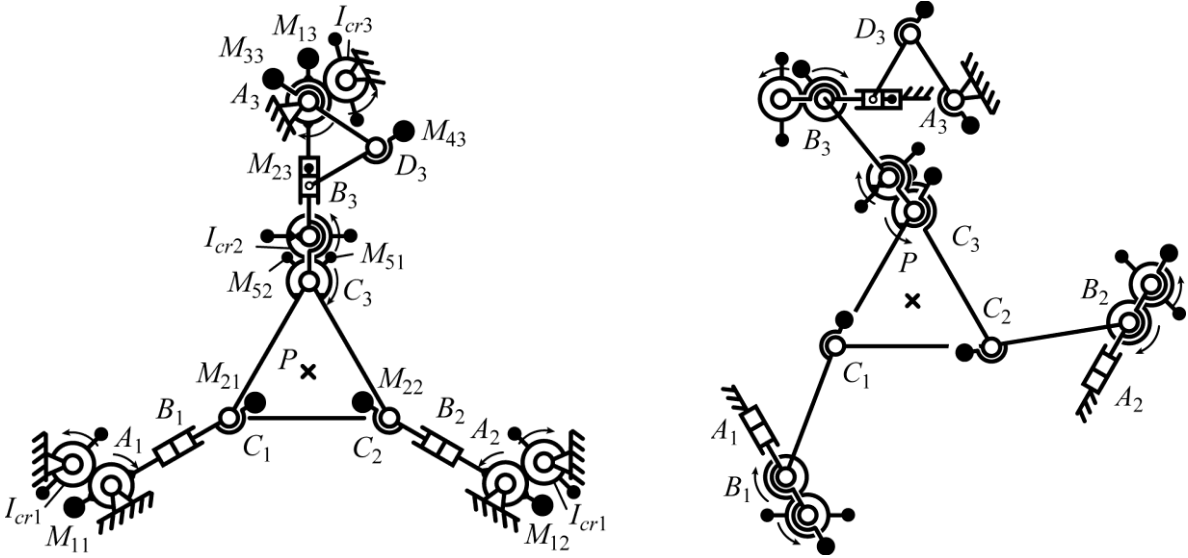


b) Modified 3-PRR parallel manipulator



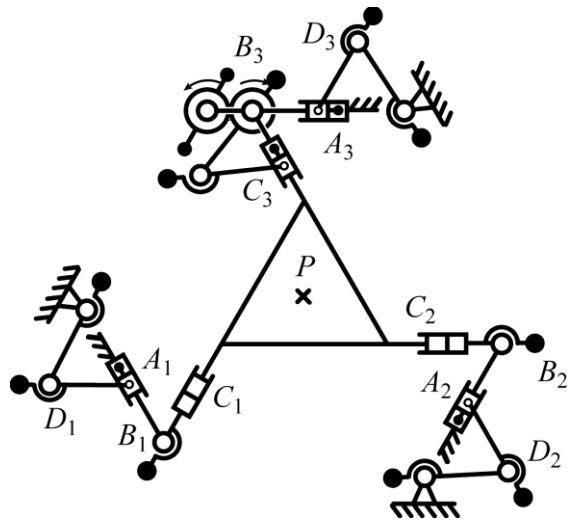
c) Modified 3-PRP parallel manipulator

Figure 9. Complete shaking force and shaking moment balancing of planar manipulators with prismatic pairs via structural modification of one leg.



a) Balancing of 3-RPR parallel manipulator

b) Balancing of 3-PRR parallel manipulator



c) Balancing of 3-PRP parallel manipulator

Figure 10. Complete shaking force and shaking moment balancing of planar manipulators with prismatic with reduced number of Scott-Russell mechanisms.

4. CONCLUSIONS

This paper presents the complete shaking force and shaking moment balancing of planar parallel manipulators with prismatic pairs. Two approaches are discussed: balancing via adding an idler loop mounted between the platform and the base of the manipulator and balancing via the Scott-Russell mechanism, which enables a reduction in the number of counter-rotations by a factor of two. All studied balancing techniques are validated by simulations carried out using ADAMS software. The obtained results show that parallel manipulators balanced using the suggested methods transmit no inertia loads to their bases, i.e., the sum of all ground forces and their moments are zero.

Finally, we would like to mention that using Scott-Russell mechanisms remains using some modified 3-RRR manipulator. Thus, if we do not consider the type of actuation, the prismatic guides could be suppressed and our work could be of no interest. But the goal of our study is to propose the complete shaking force and shaking moment balancing of manipulators for applications where an actuation via a prismatic motor is needed, such as in high load carrying (using hydraulic devices). Therefore, we think the proposed solutions are of great interest to the scientific community.

REFERENCES

- 1 **Lowen, G.G., Tepper, F.R. and Berkof, R.S.** Balancing of linkages - an update. *Mechanism and Machine Theory*, 1983, **18**(3), 213–230.
- 2 **Arakelian, V. and Smith, M.** Shaking force and shaking moment balancing of mechanisms: an historical review with new examples. *Journal of Mechanical Design*, 2005, **127**(2), 334–339 (see also ERRATUM. 2005, **127**(5), 1034–1035).

- 3 **Arakelian, V., Dahan, M. and Smith, M.R.** A historical review of the evolution of the theory on balancing of mechanisms. *International Symposium on History of Machines and Mechanisms - Proceedings HMM2000*, Kluwer Academic Publishers, Dordrecht/Boston/London, 2000, pp. 291–300.
- 4 **Agrawal, S.K. and Fattah, A.** Reactionless space and ground robots: novel designs and concept studies. *Mechanism and Machine Theory*, 2004, **39**(1), 25–40.
- 5 **Wang, J. and Gosselin, C.M.** Static balancing of spatial three-degree-of-freedom parallel mechanisms. *Mechanism and Machine Theory*, 1999, **34**(3), 437–452.
- 6 **Newman, W.S. and Hogan, N.** The optimal control of balanced manipulators. *Proc. winter annual meeting of the ASME*, 1986, Anaheim, California.
- 7 **Laliberté, T., Gosselin, C.M. and Jean, M.** Static balancing of 3-DOF planar parallel mechanisms. *IEEE/ASME Transactions on Mechatronics*, 1999, **4**(4), 363–377.
- 8 **Fujikoshi, K.** Balancing apparatus for jointed robot. *Japanese Patent JP51-122254*, 26 October 1976.
- 9 **Wang, J. and Gosselin, C.M.** Static balancing of spatial four-degree-of-freedom parallel mechanisms. *Mechanism and Machine Theory*, 2000, **35**(4), 563–592.
- 10 **Russo, A., Sinatra, R. and Xi, F.** Static balancing of parallel robots. *Mechanism and Machine Theory*, 2005, **40**(2), 191–202.
- 11 **Ouyang, P. and Zhang, W.J.** Force Balancing of Robotic Mechanisms based on Adjustment of Kinematic Parameters. *ASME Journal of Mechanical Design*, 2005, **127**(3), 433–440.
- 12 **Berkof, R.S.** Complete force and moment balancing of inline four-bar linkages. *Mechanism and Machine Theory*, 1973, **8**(3), 397–410.
- 13 **Arakelian, V. and Smith, M.R.** Complete shaking force and shaking moment balancing of linkages. *Mechanism and Machine Theory*, 1999, **34**(8), 1141–1153.

- 14 **Dresig, H., Naake, S. and Rockausen, L.** Vollständiger und harmonischer Ausgleich ebener Mechanismen. *VDI Verlag*, Düsseldorf, 1994.
- 15 **Herder, J.L. and Gosselin, C.M.** A counter-rotary counterweight for light-weight dynamic balancing. *Proc. ASME 2004 DETC/CIEC Conference*, 28 September – 2 October 2004, Salt Lake City, Utah, USA, pp. 659–667.
- 16 **Kochev, I.S.** General theory of complete shaking moment balancing of planar linkages: a critical review. *Mechanism and Machine Theory*, 2000, **35**(11), 1501–1514.
- 17 **Fattah, A. and Agrawal, S.K.** On the design of reactionless 3-DOF planar parallel mechanisms. *Mechanism and Machine Theory*, 2006, **41**(1), 70–82.
- 18 **Ricard, R. and Gosselin, C.M.** On the development of reactionless parallel manipulators. *Proc. ASME 2000 DETC*, 10–13 September 2000, Baltimore, Maryland, USA.
- 19 **Wu, Y. and Gosselin, C.M.** On the synthesis of a reactionless 6-DOF parallel mechanism using planar four-bar linkages. *Proc. Workshop on Fundamental Issues and Future Research Directions for Parallel Mechanisms and Manipulators*, 3–4 October 2002, Quebec City, Quebec, Canada.
- 20 **Gosselin, C.M., Côté, G. and Wu, Y.** Synthesis and design of reactionless three-degree-of-freedom parallel mechanisms. *IEEE Transaction on Robotics and Automation*, 2004, **20**(2), 191–199.
- 21 **Foucault, S. and Gosselin, C.M.** Synthesis, design, and prototyping of a planar three degrees-of-freedom reactionless parallel mechanism. *Journal of Mechanical Design*, 2004, **126**(6), 992–999.
- 22 **Wu, Y. and Gosselin, C.M.** Design of reactionless 3-DOF and 6-DOF parallel manipulators using parallelepiped mechanisms. *IEEE Transaction on Robotics and Automation*, 2005, **21**(5), 821–833.

- 23 Papadopoulos, E. and Abu-Abed, A.** Design and motion planning for a zero-reaction manipulator. *Proc. IEEE International Conference on Robotics and Automation (ICRA)* , 1994, San Diego, CA, USA, pp. 1554–1559.
- 24 Arakelian, V. and Briot, S.** Dynamic balancing of the SCARA robot. *The 17th CISM-IFTToMM Symposium on Robot Design, Dynamics, and Control (RoManSy 2008)*, 5–9 July 2008, Tokyo, Japan.
- 25 Bagci, C.** Complete shaking force and shaking moment balancing of link mechanisms using balancing idler loops. *ASME Journal of Mechanical Design*, 1982, **104**, 482-493.
- 26 Frolov, K.V.** Theory of mechanisms and machines. Moscow, ed. Vishaya shkola, 1987.
- 27 Doronin, V.I. and Pospelov, A.I.** Balanced slider-crank mechanism”, *Patent SU1627769*, 15 February 1991.
- 28 Hilpert, H.** Weight balancing of precision mechanical instruments. *Mechanisms*, 1968, **3**(4), 289-302.
- 29 Arakelian, V.** Equilibrage dynamique complet des mécanismes. *Mechanism and Machine Theory*, 1998, **33**(4), 425–436.
- 30 Leblond, M. and Gosselin, C.M.** Static balancing of spatial and planar parallel manipulators with prismatic actuators. *Proc. ASME 1998 DETC Conference*, 1998, Atlanta, Georgia, USA , pp. 1-12.
- 31 Baradat, C., Arakelian, V., Briot, S. and Guegan, S.** Design and prototyping of a new balancing mechanism for spatial parallel manipulators. *ASME Journal of Mechanical Design*, 2008, (to be published).
- 32 Seyferth, W.** Massenersatz durch punktmassen in räumlichen getrieben. *Mechanism and Machine Theory*, 1974, **9**(1), 49-59.

33 Wu, Y. and Gosselin, C.M. On the dynamic balancing of multi-DOF parallel mechanisms with multiple legs. *Transaction of the ASME. Journal of Mechanical Design*, 2007, **129**(2), 234-238.

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