Price war with partial spectrum sharing for competitive wireless service providers

Patrick Maillé, Bruno Tuffin

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Patrick Maillé
Institut Telecom; Telecom Bretagne
2, rue de la Châtaigneraie CS 17607
35576 Cesson Sévigné Cedex, France
Email: patrick.maille@telecom-bretagne.eu

Bruno Tuffin
INRIA Rennes - Bretagne Atlantique
Campus Universitaire de Beaulieu
35042 Rennes Cedex, France
Email: Bruno.Tuffin@inria.fr

***Abstract*—In 3G wireless technologies, competitive operators are assigned a fixed part of the spectrum from long-term auctions. This is known to lead to utilization inefficiencies because some providers can be congested while others are lightly used. Moreover it forbids the entrance of new candidate providers. There is now a stream of work dealing with spectrum sharing among providers to lead to a better utilization. In this paper, we study an intermediate model of price competition between two providers having a fixed (licensed) part of the spectrum, but where a remaining part (an unlicensed band) can be used in case of congestion, and is therefore shared. We discuss the existence and uniqueness of the Nash equilibrium in the pricing game when demand is distributed among providers according to Wardrop's principle so that users choose the least expensive *perceived* price (when congestion pricing is used), and investigate the influence of the shared band on social and user welfare.**

I. Introduction

Wireless technologies are becoming omnipresent, end users being connected from everywhere, and for a much broader use than just telephony. The network access can be provided by a set of technologies, ranging from 3G CDMA-based systems, WiFi, to WiMAX for instance. Another important issue is that competition is not only among technologies (which could be operated by a single provider), but among concurrent providers which operate on different or similar platforms. Users indeed have to choose between several service providers, their choice being based on a combination of price and quality of service (QoS). Modeling this competition requires the use of non-cooperative game theory [1], in order to understand the interactions between (pricing) strategy of each provider, depending also on the reactions of individual users. This competitive pricing game has recently been extensively studied in telecommunications, see among others [2], [3], [4], [5].

In wireless communications especially, resource, that is spectrum, is limited and congestion is likely to occur. It has been highlighted that the current spectrum sharing of 3G wireless networks, where providers own a long-term licensed fixed part of the spectrum, does not lead to an efficient utilization, since some providers can be congested while others are not fully used [6], [7]. It is now proposed to share at least a part of the spectrum (unlicensed bands) to cope with that situation and provide a better average QoS [8]. That kind of principle somewhat also justifies the development of cognitive networks where end-to-end performance is dynamically optimized by providers sensing and opportunistically accessing the under-utilized spectrum [9], [10].

This paper analyzes a specific pricing game between two providers having each a fixed and own spectrum, while an unlicensed part of the spectrum is shared and used when the fixed one is fully utilized. Spectrum is abstracted as a given capacity, and providers play on the price per sent packet they propose to users. We assume demand for service is not delay-sensitive and depends on the price to correctly send a packet, where packet losses occur as soon as demand exceeds capacity. Demand is split among the providers according to Wardrop's principle [11], meaning that users have negligible influence on total traffic, and choose the provider(s) with the smallest price for sending correctly a packet. Note that our model also represents the situation where each operator owns a 3G license and sends traffic on a (shared) WiFi network (transparently for users) as soon as the QoS using WiFi becomes as good as using 3G. The questions we aim at answering are: what is the user equilibrium (if any) for fixed prices? Then is there a Nash equilibrium in the pricing (non-cooperative) game, that is a price strategy for each provider such that none of them can improve his revenue by changing unilaterally his price? If it exists, is it unique? What is the influence of the proportion of spectrum left unlicensed on the outcomes of the game?

This work is to our knowledge the first attempt to deal with a model looking at a two-step game in wireless networks with (partially) shared spectrum/capacity. In the first step, demand is split among providers according to price and QoS and on top of that the pricing game is played among providers.

The remainder of the paper is organized as follows. In Section II we present the basic model and assumptions. Section III explains the users' behavior and their distribution among the different operators according to a combination of price and QoS. Using this user equilibrium, we then analyze the pricing game among providers thanks to non-cooperative game theory in Section IV. We illustrate how Nash equilibria can be derived and introduce social welfare (sum of utilities of *all* agents) and user welfare (dealing only with users). The influence of the spectrum proportion kept unlicensed is also discussed. Finally, we conclude and give some directions for future research in Section V.

II. Model

We consider two providers (operators) in competition, denoted by $i \in \{1,2\}$. Provider i owns a fixed (licensed) band of the spectrum. Using this band, he is able to serve a fixed number of packets C_i per time slot (assuming time is slotted). On the other hand, we assume that there is a remaining and unlicensed part of the spectrum, on which a capacity of C packets per time slot can be served, that the two providers can use as soon as demand exceeds capacity on their licensed band. Again, this can also represent the case where providers operate through a given technology where part of the spectrum is shared, or could represent the case where providers operate a licensed technology, say 3G, and may want to send some traffic transparently to users on another technology (provided their devices can support it) with shared spectrum, say WiFi, as soon as their licensed capacity is fully utilized.

Formally, let d_i be provider i's demand in a given time slot ($i \in \{1,2\}$). If $d_i \leq C_i$, then all packets are served. If $d_i > C_i$, demand in excess $d_i - C_i$ is sent to the shared band. As a consequence, total demand at this shared band is $[d_1-C_1]^+ + [d_2-C_2]^+$ where $[x]^+ = \max(0,x)$. We assume that packets in excess, when demand exceeds capacity, are lost and that the lost packets are uniformly chosen among the sent ones. Similarly, we assume that, at each provider, the packets sent on the shared band are uniformly chosen. As a consequence, the part of shared capacity devoted to provider i for $i \in \{1,2\}$ is

$$C_i' = \frac{[d_i - C_i]^+}{[d_1 - C_1]^+ + [d_2 - C_2]^+} C, \tag{1}$$

with the convention $0/0 = 0$, and a packet is correctly sent (that is, not lost) at provider i with probability

$$q_i = \min\left(1, \frac{C_i + C_i'}{d_i}\right) = \min\left(1, \frac{C_i + \frac{[d_i-C_i]^+}{[d_1-C_1]^+ + [d_2-C_2]^+} C}{d_i}\right). \tag{2}$$

Indeed, given the uniform choice, the probability of successful transmission is the ratio of correctly transmitted packets on submitted ones if demand exceeds capacity, and 1 if capacity is above demand. Figure 1 summarizes the way providers distribute demand.

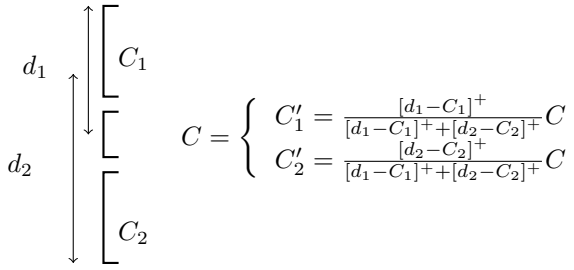

Fig. 1. Summary of used capacities

Similarly to [5], we assume that users are charged per *submitted* packet and not per correctly transmitted ones. That way, in periods of congestion charges for a given successful transmission are higher to incentivize users to restrict their traffic. It is therefore a kind of *congestion pricing*.

Denote by p_i the price that provider i fixes for each packet sent to his network. Given that each packet is successfully transmitted with probability q_i, the average number of attempts before success is $1/q_i$, the average value of a geometric distribution with parameter q_i, demand being considered the same in each time slot (this is not a limitation if time slots are very short with respect to demand fluctations in time). Thus a *perceived price* per correctly transmitted packet at provider i, i.e., the total average price to pay for sending a packet is

$$\bar{p}_i = p_i \frac{1}{q_i} = p_i \max\left(1, \frac{d_i}{C_i + \frac{[d_i-C_i]^+}{[d_1-C_1]^+ + [d_2-C_2]^+} C}\right). \tag{3}$$

Remark that it is not required to announce a price per sent packet (users are not likely to approve paying for rejected packets), but rather announce a congestion price $\bar{p}_i$ for successful transmission.

Our goal is now to investigate in next section how users will split themselves among the different providers at equilibrium, and what the total demand will be. Section IV deals with the price war among providers using the determined user equilibrium.

III. User equilibrium

Users are assumed to be infinitesimal, which means that their individual influence on the total traffic is negligible. Their behavior is to follow the so-called Wardrop's principle [11] taken from road transportation: demand is distributed in such a way that all users choose the available provider(s) with the lowest perceived price, and none if this perceived price is too expensive. Therefore all users perceive the same price

$$\bar{p} := \min(\bar{p}_1, \bar{p}_2). \tag{4}$$

Total demand, defined as the total number of packets for which the willingness to pay is larger than or equal to $\bar{p}$, is a function $D(\cdot)$ of the perceived price $\bar{p}$, and is assumed to be continuous and strictly decreasing.

We therefore end up with the following set of equations characterizing the Wardrop (user) equilibrium:

$$\bar{p}_1 = p_1 \max\left(1, \frac{d_1}{C_1 + \frac{[d_1-C_1]^+}{[d_1-C_1]^+ + [d_2-C_2]^+} C}\right) \tag{5}$$

$$\bar{p}_2 = p_2 \max\left(1, \frac{d_2}{C_2 + \frac{[d_2-C_2]^+}{[d_1-C_1]^+ + [d_2-C_2]^+} C}\right) \tag{6}$$

$$d_1 + d_2 = D(\min(\bar{p}_1, \bar{p}_2)) \tag{7}$$

$$\bar{p}_1 > \bar{p}_2 \Rightarrow d_1 = 0 \tag{8}$$

$$\bar{p}_2 > \bar{p}_1 \Rightarrow d_2 = 0. \tag{9}$$

Equations (5) and (6) express the perceived prices in terms of demand. Equations (8) and (9) express the fact that only cheapest providers get demand, otherwise some users would be better off switching. Equation (7) just relates total demand

to perceived price (which is then the same for both providers if they both have a positive demand).

We then have the following theorem.

Proposition 1: Whatever the price profile (p_1, p_2), there exists at least one Wardrop equilibrium. The corresponding perceived prices are unique.

Proof: The existence of a Wardop equilibrium is proved in a very general context in [12].

To show the uniqueness of the perceived price, assume that we have two Wardrop equilibria with demand (d_1, d_2) and (d_1', d_2') leading to respective perceived prices $(\bar{p}_1, \bar{p}_2)$ and $(\bar{p}_1', \bar{p}_2')$. Assume $(\bar{p}_1, \bar{p}_2) \neq (\bar{p}_1', \bar{p}_2')$ and let $\bar{p} = \min(\bar{p}_1, \bar{p}_2)$, $\bar{p}' = \min(\bar{p}_1', \bar{p}_2')$. Without loss of generality we can suppose that $\bar{p}_1 < \bar{p}_1'$, which implies several things:

$$\begin{aligned} d_1' &> C_1 + \frac{[d_1' - C_1]^+}{[d_1' - C_1]^+ + [d_2' - C_2]^+} C && (10) \\ C &< [d_1' - C_1]^+ + [d_2' - C_2]^+ && (11) \\ \bar{p}' &= \bar{p}_1'. && (12) \end{aligned}$$

Relation (10) is a direct consequence of (5) and $\bar{p}_1' > \bar{p}_1 \geq p_1$.

Moreover, $\bar{p}_1' > p_1$ means some packets are lost at provider 1 under the situation (d_1', d_2'). Due to our capacity allocation rule, packets can be lost only under (11).

Relation (12) comes from (10) and the fact that (d_1', d_2') is a Wardrop equilibrium, so only the cheapest (in terms of perceived costs) provider(s) have positive demand.

Relation (4) then implies that $\bar{p} = \min(\bar{p}_1, \bar{p}_2) \leq \bar{p}_1 < \bar{p}_1' = \bar{p}'$, therefore from (7) and the non-increasingness of the demand function, we have

$$d = d_1 + d_2 > d' = d_1' + d_2'. \quad (13)$$

Our goal is now to show that there is a contradiction: if total demand increases, then the perceived price increases too. We consider two cases, depending on the sign of $d_1 - d_1'$.

• If $d_1 > d_1'$, then we have $[d_1 - C_1]^+ + [d_2 - C_2]^+ \geq [d_1' - C_1]^+ + [d_2' - C_2]^+$: it is indeed obvious if $d_2 \geq d_2'$, whereas if $d_2 < d_2'$ we have

$$\begin{aligned} [d_2' - C_2]^+ - [d_2 - C_2]^+ &\leq d_2' - d_2 \\ &< d_1 - d_1' \\ &= [d_1 - C_1]^+ - [d_1' - C_1]^+, \end{aligned}$$

where the second inequality comes from (13), and the last equality stems from (10) and $d_1 > d_1'$.

Consequently we have[1], using also again $d_1 > d_1' > C_1$,

$$\begin{aligned} \frac{d_1}{C_1 + \frac{[d_1 - C_1]^+}{[d_1 - C_1]^+ + [d_2 - C_2]^+} C} &\geq \frac{d_1}{C_1 + \frac{[d_1 - C_1]^+}{[d_1' - C_1]^+ + [d_2' - C_2]^+} C} \\ &= \frac{d_1}{A d_1 + B}, \end{aligned}$$

where $A = \frac{1}{[d_1' - C_1]^+ + [d_2' - C_2]^+} C > 0$, and $B = C_1 \left(1 - \frac{1}{[d_1' - C_1]^+ + [d_2' - C_2]^+}\right) > 0$ from (11). Therefore $x \mapsto x/(Ax + B)$ is increasing in x, and thus

$$\frac{d_1}{C_1 + \frac{[d_1 - C_1]^+}{[d_1 - C_1]^+ + [d_2 - C_2]^+} C} \geq \frac{d_1'}{C_1 + \frac{[d_1' - C_1]^+}{[d_1' - C_1]^+ + [d_2' - C_2]^+} C},$$

meaning $\bar{p}_1 \geq \bar{p}_1'$ from (5), which is a contradiction.

• Now consider the case $d_1 \leq d_1'$. Then $d_2 > d_2'$ from (13), and

- if $[d_2' - C_2]^+ = 0$ then, from (6), $\bar{p}_2' = p_2 \leq \bar{p}_2$;
- if $[d_2' - C_2]^+ > 0$ then using the same reasoning as in the case $d_1 > d_1'$, just inverting the roles of the providers, we get that $\bar{p}_2' \leq \bar{p}_2$.

Since $d_2 > d_2'$, then $d_2 > 0$, and the Wardrop condition implies that $\bar{p} = \bar{p}_2 \leq \bar{p}_1$. On the other hand, $\bar{p}' \leq \bar{p}_2'$. Therefore, applying (12) and the fact that $\bar{p}_2' \leq \bar{p}_2$ we have $\bar{p}_1' \leq \bar{p}_1$, a contradiction. ∎

Remark 1: From the uniqueness of the perceived price $\bar{p}$, the uniqueness of demand distribution (d_1, d_2) can be discussed. First $d_1 + d_2 = D(\bar{p})$ is unique from (7). Next if both providers are strictly saturated (i.e., $\bar{p}_i > p_i$ for $i = 1, 2$), then the maximum is given by the right side in the max of (5) and (6), and (d_1, d_2) is the unique solution of

$$\bar{p} = p_i \frac{d_i}{C_i + \frac{d_i - C_i}{D(\bar{p}) - C_1 - C_2} C}, \qquad i = 1, 2.$$

Similarly, if only one provider is strictly saturated, say $i \in \{1, 2\}$, we have d_i solution of $\bar{p} = p_i \frac{d_i}{C_i + C}$, and demand at provider $j \neq i$ is $d_j = D(\bar{p}) - d_i$. Now the last case is when no provider is saturated and experiences losses. When $p_1 < p_2$ (without loss of generality), it means that 1 absorbs the whole demand $D(p_1)$ while 2 gets nothing. If $p_1 = p_2 = p$, we are in the only situation of non-uniqueness: users do not care choosing between the providers since they have the same price and none is congested, even if the unlicensed band is used by one of the provider. More formally, each point $(d_1 = x, d_2 = D(p) - x)$ is a Wardop equilibrium, for $\max(0, D(p) - C_2 - C) \leq x \leq \min(D(p), C_1 + C)$. In the numerical analyses of the next sections, for those rare cases we choose to spread demand among providers, proportionally to their capacities, i.e. we fix $d_i = D(p) \frac{C_i}{C_1 + C_2}$.

Figure 2 illustrates the Wardrop equilibrium characterization, for fixed values of the unit prices $p_1 = 1$, $p_2 = 2$ and a given demand function. The increasing curve gives the total demand level that would correspond to a given perceived price: if total demand D is below C_1, then all users choose provider 1 and perceive the price $\bar{p} = p_1$. Likewise, if $C_1 < D \leq C_1 + C$, then the needed extra capacity $D - C_1$ is available to provider 1 and the perceived price is still $\bar{p} = p_1$. When D exceeds $C_1 + C$, then provider 2 does not necessarily get some demand, since users may be better off choosing provider 1 even in case of losses. More precisely, if $C_1 + C < D < (C_1 + C)p_2/p_1$, then all users stay with provider 1 since the perceived price is then $\bar{p} = p_1 \frac{D}{C_1 + C} < p_2$. This situation is depicted in Figure 2 as case *a*. Provider 2 actually gets some demand when total demand exceeds $(C_1 + C)p_2/p_1$: if $(C_1 + C)p_2/p_1 < D \leq (C_1 + C)p_2/p_1 + C_2$

[1]Notice that $[d_1' - C_1]^+ + [d_2' - C_2]^+ > 0$ from (10), therefore the denominators are non-zero.

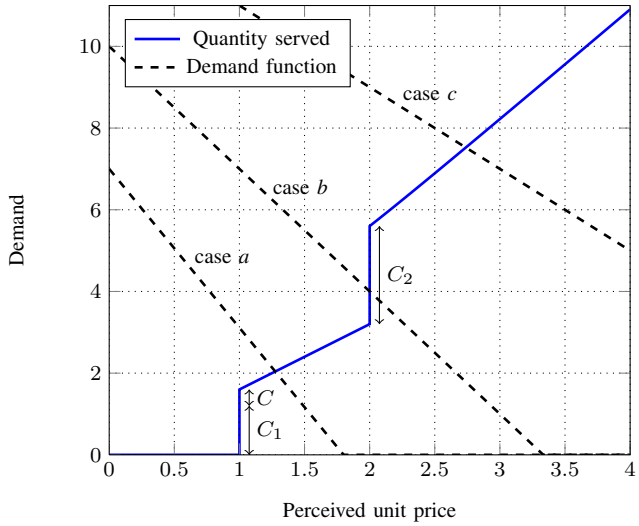

Fig. 2. Graphical representation of the Wardrop equilibrium demand and perceived price for three different demand functions.

then the perceived price is p_2 and provider 2 gets demand $d_2 = D - (C_1 + C)p_2/p_1$. This corresponds to case b in Figure 2. When $D > (C_1+C)p_2/p_1$ (this is case c in Figure 2, both providers become saturated, and the unlicensed band is shared among them according to (1). We therefore have $\bar{p}_1 = \bar{p}_2 = \bar{p}$, and (d_1, d_2) can be found by solving the system $(5) - (6)$. The perceived price at the Wardrop equilibrium is determined by the intersection of

- the increasing demand-perceived price relation due to congestion and the capacity allocation rule,
- and the decreasing demand-perceived price relation (7), corresponding to user willingness-to-pay for the service.

IV. Pricing game between providers

We now consider the competition game among providers. We assume providers know that for given prices p_1 and p_2, users will behave according to the Wardrop equilibrium described in the previous section. Providers therefore use that knowledge to determine their best pricing strategy. This makes the situation under study a two-stage game, where at a first stage providers set prices, and at the second stage users make their decision, determining the repartition (d_1, d_2) (see Remark 1 for the rare cases when demands are not unique). In this section, we focus on the upper-level game on prices, where the provider revenues are those obtained at Wardrop equilibrium.

We will illustrate the concepts introduced using some numerical results. Unless otherwise specified, the model parameters that we take are $C_1 = 1.2$, $C_2 = 2.4$, $C = 0.4$, and $D(p) = [10 - 3p]^+$. We will refer to that set of example values as S.

A. Provider utility and pricing game

The payoff we consider for provider i is simply his revenue

$$R_i(p_1, p_2) := p_i d_i \text{ for } i \in \{1, 2\}. \tag{14}$$

As pointed out in the previous section, the demand of provider i at the Wardrop equilibrium outcome depends not only on his price p_i, but also on the price of his opponent. The natural modeling framework is therefore that of non-cooperative game theory, and the equilibrium that of a Nash equilibrium [1]. A Nash equilibrium is a point of price strategies (p_1^*, p_2^*) such that no provider can increase his revenue by unilaterally changing his price, i.e., $\forall p_1, p_2 \geq 0$,

$$R_1(p_1^*, p_2^*) \geq R_1(p_1, p_2^*) \text{ and } R_2(p_1^*, p_2^*) \geq R_2(p_1^*, p_2).$$

The questions we wish to answer are then: is there a Nash equilibrium to this game? If so, is it unique?

Determining existence and uniqueness of a Nash equilibrium is difficult with this model, due to the non-derivability of the revenue functions and the multiple ratios involved. We therefore see how to analyze it numerically and investigate the influence of the unlicensed spectrum. Consider again the parameter set S. First, Figure 3 displays the revenue of provider 1 in terms of his price p_1 for different values of p_2, the other fixed values being those in S. It illustrates that

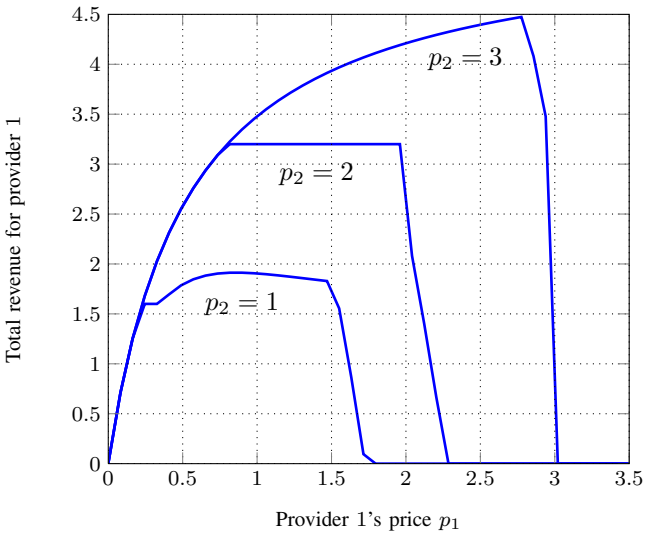

Fig. 3. Total utility of provider 1 as a function of p_1.

provider 1 revenue is first increasing and then decreasing. The three curves in Figure 3 are the same for low values of p_1: indeed, when p_1 is low enough then all users choose provider 1. More precisely, if $D(p_2) < (C_1 + C)p_2/p_1$, then the Wardrop intersection point $(\bar{p}, D)$, illustrated as case a in Figure 2, is for $\bar{p} < p_2$, implying $d_2 = 0$ from (9). On the contrary, when $p_1 > p_2 \frac{C_1+C}{D(p_2)}$, the curves in Figure 3 exhibit different behaviors:

- when $p_2 = 1$, the utility of provider 1 is maximal when p_1 is around 0.73. This corresponds to a situation where both providers are saturated at the Wardrop equilibrium (case c in Figure 2).
- When $p_2 = 2$, provider 2 is indifferent between all values in the range $[0.75 - 1.5]$. For those values of p_1, we are in the case b of Figure 2: the user perceived price is

p_2, provider 1 gets demand $d_2 = p_2/p_1(C_1 + C)$ while $d_2 = D(p_2) - p_2/p_1(C_1 + C)$. Then in this whole range for p_1, the revenue of provider 1 is $R_1 = p_2(C_1+C)$. For that case, there is not a unique best strategy for player 1 as a response to p_2, but a whole interval.

- When $p_2 = 3$, then the revenue of provider 1 is maximal for $p_1 = 2.83$, i.e. in the zone where $d_2 = 0$ (case a of Figure 2). If provider 1 increases his price, then the revenue loss due to the decrease of d_1 (because of total demand decrease or because some users switch to provider 2) exceeds the gain due to price increase.

Consider now the best reply of each provider $i \in \{1, 2\}$ as a function of $\text{BR}_i : \mathbb{R}^+ \mapsto \mathcal{P}(\mathbb{R}^+)$, such that

$$\begin{aligned} \text{BR}_1(p_2) &:= \arg\max_{p_1 \geq 0} R_1(p_1, p_2) \text{ and} \\ \text{BR}_2(p_1) &:= \arg\max_{p_2 \geq 0} R_2(p_1, p_2). \end{aligned}$$

In words, the best replies are the price values (not necessarily unique) maximizing the revenue of a provider, when the price of the opponent is fixed. Nash equilibria are therefore the set of points (p_1^*, p_2^*) for which $p_1^* \in \text{BR}_1(p_2^*)$ and $p_2^* \in \text{BR}_2(p_1^*)$, i.e., a fixed point of the best reply correspondance defined as $\overline{\text{BR}}(p_1, p_2) := \{(q_1, q_2) \in \mathbb{R}^+ \times \mathbb{R}^+ : q_1 \in \text{BR}_1(p_2), q_2 \in \text{BR}_2(p_1)\}$.

Figure 4 gives an example of those best replies for parameter values in S. First, we remark that best-replies are not

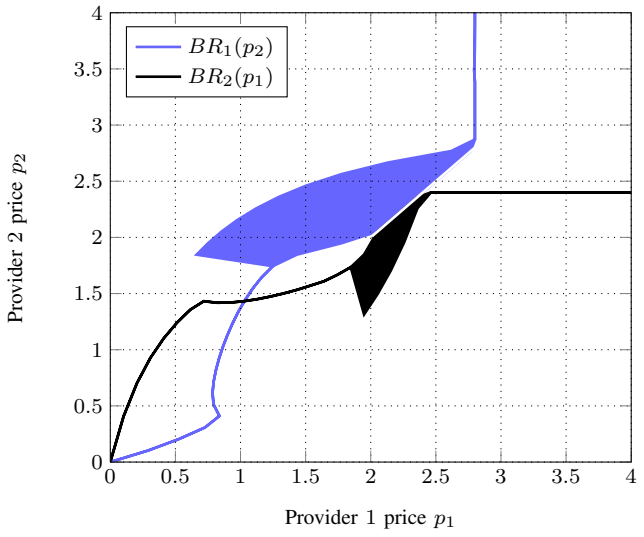

Fig. 4. Best reply curves of both providers, $C/C_{\text{total}} = 0.1$.

always unique; non-uniqueness actually correspond here to the situation of Figure 3 for $p_2 = 2$, where the concurrent provider is not saturated, so that revenue does not change over an interval. The best-reply correspondences seem to meet in three different zones: first at $(0, 0)$, then they cross at a point $p^* \approx (1.05, 1.45)$, and on the whole range $2 < p_1 = p_2 < 2.4$. Actually, for that latter zone, best-reply curves do not cross: the zone corresponds to the case $\max(C_1, C_2) < D(p) < C_1 + C_2 + C$, and therefore if providers have the same price p, each provider i would have an incentive to decrease one's price p_i by a small $\varepsilon > 0$, so as to fall into case b of Figure 2, and obtain demand $C_i + C$. Therefore, there is no equilibrium in that zone.

While $(0, 0)$ is a trivial Nash equilibrium, we can also notice that it is not stable: if one provider slightly deviates and sets a small positive price, then successive best replies of the providers lead to the second Nash equilibrium p^* that we pointed out, and that is stable.

Interestingly, the provider i with the largest own capacity C_i (here $i = 2$) sets a higher price, possibly because he is less in need for unlicensed band, and therefore is less affected by price competition. Notice also that the unique stable Nash equilibrium (p_1^*, p_2^*) corresponds to a case when both providers are saturated. Therefore the whole band (licensed + unlicensed) is used. Total demand even exceeds the overall transmission capacity $C_1 + C_2 + C$, and therefore there are some losses. We have seen how to compute a Nash equilibrium and interpret the pricing game; we now propose a measure of social welfare that takes those losses into account.

B. Social and user welfare considerations

We suggest to define social welfare as the overall "value" of the system, i.e. the sum of the utilities of all participants (including users and providers). Prices paid by users are received by providers, thus those monetary exchanges do not appear in our social welfare measure, and social welfare should reflect the sum of the willingness-to-pay values of all users that are served. We considered infinitesimal users, therefore such a value is calculated as an integral: if we define v as the inverse function of the demand, then $\int_0^x v$ is the overall user value if the x users with highest willingness-to-pay are served. When x users with highest willingness-to-pay are likely to be served but losses can occur with equal probability P_{loss} for those users, the corresponding sum of willingness-to-pay for served users is then

$$\text{SW} = (1 - P_{\text{loss}}) \int_0^x v. \tag{15}$$

Since v is decreasing and $1 - P_{\text{loss}} \leq \frac{C_1+C_2+C}{x}$ with equality for some repartitions of x over the providers, the optimal value of social welfare is $\text{SW}_{\max} = \int_0^{C_1+C_2+C} v$. For our case, at a Nash equilibrium the $D(\bar{p})$ users with highest willingness-to-pay send their data, and $P_{\text{loss}} = \left[1 - \frac{C_1+C_2+C}{D(\bar{p})}\right]^+$ for all of those users. We therefore have:

Proposition 2: The Social Welfare at Nash equilibrium is

$$\text{SW} = \min\left(1, \frac{C_1 + C_2 + C}{D(\bar{p})}\right) \int_0^{D(\bar{p})} v, \tag{16}$$

compared with the optimal one $\text{SW}_{\max} = \int_0^{C_1+C_2+C} v$.

Also, the overall User Welfare (user willingness-to-pay minus price paid) is $\text{UW} = \text{SW} - R_1 - R_2$.

C. Licensed vs unlicensed capacity

Let us now investigate the influence of the fraction μ of total available band that is unlicensed on the system outcomes. We assume that there is a total band C_{total}, that can be licensed to a provider or declared as unlicensed. We choose to fix the ratio $r = C_1/C_2$, so that

$$C = \mu C_{\text{total}}, \quad C_1 = \frac{r(1-\mu)}{1+r} C_{\text{total}}, \quad C_2 = \frac{1-\mu}{1+r} C_{\text{total}}.$$

In what follows, we consider as before $C_{\text{total}} = 4$ and $r = 0.5$. Figure 5 shows the stable Nash prices (p_1^*, p_2^*) and the corresponding user perceived price $\bar{p}$ when μ varies, whereas Figure 6 plots the provider revenues, user welfare UW, and overall social welfare SW versus μ. When the proportion

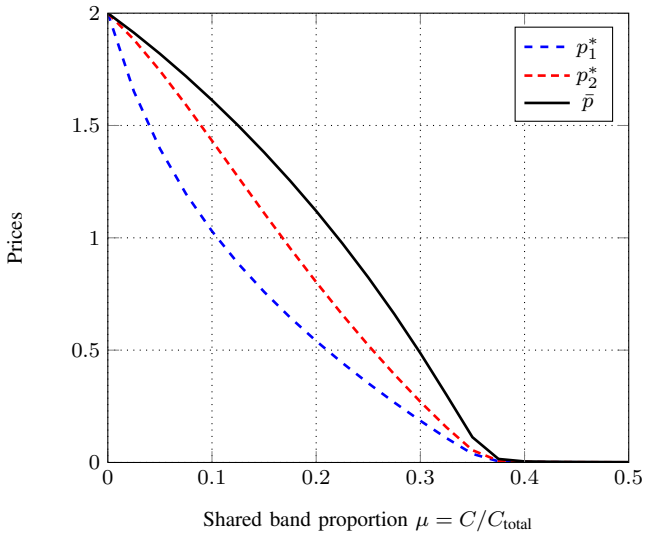

Fig. 5. Prices at Nash equilibrium when μ varies.

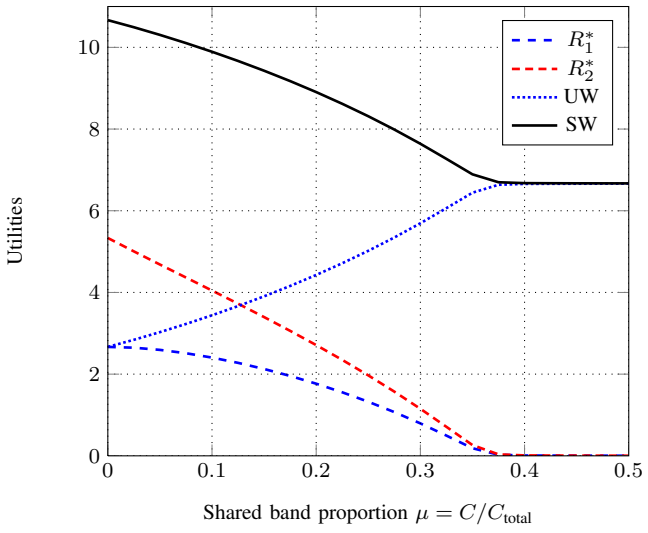

Fig. 6. Utilities at Nash equilibrium when μ varies.

of unlicensed band increases, the revenue of both providers decreases since they have to drastically lower their price in order to stay competitive (see Figure 5). This benefits to users, who perceive a lower price $\bar{p}$ and experience a larger user welfare. However, the overall social welfare decreases, which means that resources are less efficiently used. Using that kind of study, a central authority (a government for instance) could fix a given amount of shared spectrum so as to favor user welfare over provider revenue. However, setting μ too large leads prices and provider revenues tend to 0, thus a trade-off has to be found to still ensure provider rentability.

V. Conclusions

This paper has investigated the price war among wireless providers when part of the spectrum is unlicensed and can be freely used. We have shown the existence of a user equilibrium and described how to determine a Nash equilibrium for the price war. We have also shown how the choice of the unlicensed spectrum weight affects the trade-off between user welfare and overall social welfare.

Next steps of this work are numerous. A relevant aspect is to see what happens if the unlicensed spectrum is charged to providers: could it help to improve the general behavior of the system? Similarly, we would like to investigate the best proportion of unlicensed band if there is a different weight for user and provider utilities in a global objective measure for a regulating authority.

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