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► **To cite this version:**

Patrick Maillé, Maurizio Naldi, Bruno Tuffin. Price war with migrating customers. MASCOTS 2009 :17th IEEE International Symposium on Modelling, Analysis and Simulation of Computer and Telecommunication Systems, Sep 2009, London, United Kingdom. 10.1109/MASCOT.2009.5362674 . hal-00447066

**HAL Id: hal-00447066**

**<https://hal.science/hal-00447066v1>**

Submitted on 14 Jan 2010

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# Price war with migrating customers

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**Abstract**—In the telecommunication world, competition among providers to attract and keep customers is fierce. On the other hand customers churn between providers due to better prices, better reputation or better services. We propose in this paper to study the price war between two providers in the case where users' decisions are modeled by a Markov chain, with price-dependent transition rates. Each provider is assumed to look for a maximized revenue, which depends on the strategy of the competitor. Therefore, using the framework of non-cooperative game theory, we show how the price war can be analyzed and show the influence of various parameters.

## I. INTRODUCTION

The migration of customers from a service provider to another, a.k.a. as *churn*, has become a relevant phenomenon since the liberalization of telecommunications service and the ensuing proliferation of network operators. Churn is especially large in mobile networks, where yearly migration rates as high as 25% are not uncommon [1]. The migration of each customer, to the benefit of another service provider, implies both the loss of the stream of future revenues associated to that customer and of the acquisition cost. Service providers are therefore very keen on retaining their customers as well as on attracting new ones. In doing so they can rely both on preventive and on reactive strategies. An example of the latter is given by unfair practices such as the malicious introduction of delays in the migration process [2] [3]. Preventive strategies rely instead on the identification of the factors having a major influence on the churning decision (the churn determinants) [4] [5] and on successive actions on those factors. Among the identified churn determinants, price plays the most relevant role. Price always appears as a major factor: in the context of mobile number portability (a mechanism allowing to switch provider with minimal discontinuity, since the telephone number is retained), price is stated as a key element in spurring churn [6]. Another example is provided in [7], where retention and attrition phenomena are studied in an experimental setting by proposing different pricing plans to test customers. We can then expect that providers may compete for retaining customers by acting primarily on price. In this paper we propose a model for the competition among service providers based on price, the competition being here limited to two service providers. To our knowledge, it is the first time that such a competition model is introduced in telecommunications to model the price war. While most

research efforts on telecommunication pricing are concerned with congestion externality for usage-based pricing [8], here we focus more on subscription-fee based pricing, where users are charged for the amount of time they stay with a provider, regardless of their usage (e.g., the Internet subscription fee incorporated in most of the current pricing packages). Our model makes use of a Markov chain to mimic the churn behaviour of a customer in terms of prices and other parameters. Basically, the user can be with any of the providers or none of them if not satisfied with their combination of price and services. The per-user revenue of each provider can then be easily computed from steady-state probabilities, considering a single user without loss of generality. Indeed, assuming that customers behave independently and according to the same Markov chain, the expected revenue of providers is exactly the expected revenue per customer times the total population. Those state probabilities depending on both prices, so are the revenues of providers. As a consequence, the natural framework for analyzing the competition between providers seeking to maximize their revenue is non-cooperative game theory. We show how to solve this game, and illustrate the influence of parameters such as impact of other churn determinants, and (social or financial) cost for not getting any service due to excessive prices.

The paper is organized as follows. Section II presents the Markov chain model representing the user behaviour and computes the associated steady-state probabilities. Section III explains in full generality why non-cooperative come into play and how it can be solved, either analytically or numerically. Section IV then shows how, in a simplified setting, the game can be solved analytically. Section V on the other hand makes use of a description of rates issued from logit models. In this case, a numerical analysis is performed. In addition, we investigate the sensitivity of the resulting equilibria to the degree of asymmetry between the two providers in attracting customers and to the relevance of the price factor for the customers. Finally, Section VI describes our conclusions.

## II. MARKOV CHAIN MODEL OF USERS' BEHAVIOR

We assume that the behavior of a customer is represented by the continuous time Markov chain<sup>1</sup> that is depicted in

<sup>1</sup>We therefore implicitly assume that all events leading to a state change occur after an exponentially distributed time.

$$Q := \begin{pmatrix} -2\alpha & \alpha & \alpha \\ \lambda_{10}(p_1, p_2) & -(\lambda_{10}(p_1, p_2) + \lambda_{12}(p_1, p_2)) & \lambda_{12}(p_1, p_2) \\ \lambda_{20}(p_1, p_2) & \lambda_{21}(p_1, p_2) & -(\lambda_{20}(p_1, p_2) + \lambda_{21}(p_1, p_2)) \end{pmatrix}.$$

Figure 2: Infinitesimal generator of the Markov chain

Figure 1<sup>2</sup>. Here state 1 means that the customer is with provider 1, state 2 that he is with provider 2 and state 0 that he does not use any service. The parameter  $\alpha$  is a constant rate independent of prices. Remark on the other hand that other rates will be considered in the numerical section, but the resulting steady-state probabilities will be computed in the same way. The resulting infinitesimal generator  $Q$  is given in

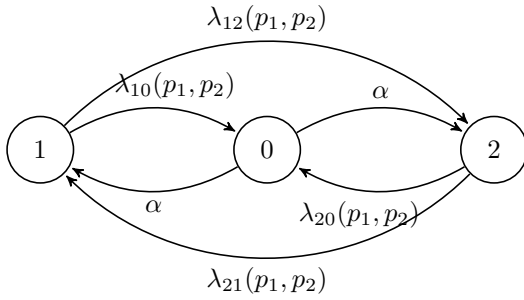


Figure 1: Continuous time Markov chain model of the customer's switching behaviour

Figure 2. From standard Markov chain analysis, the steady-state probabilities for each of the three states grouped in the row vector  $\pi = (\pi_i)_{i=0,\dots,2}$  exist and are given by the solution of equations

$$\pi Q = 0, \quad \sum_{i=0}^2 \pi_i = 1.$$

If

$$c = \alpha (2\lambda_{12}(p_1, p_2) + 2\lambda_{21}(p_1, p_2) + \lambda_{10}(p_1, p_2) + \lambda_{20}(p_1, p_2)) + \lambda_{10}(p_1, p_2)\lambda_{21}(p_1, p_2) + \lambda_{20}(p_1, p_2)\lambda_{12}(p_1, p_2) + \lambda_{10}(p_1, p_2)\lambda_{20}(p_1, p_2),$$

we have

$$\begin{aligned} \pi_0 &= \frac{\lambda_{10}(p_1, p_2)\lambda_{21}(p_1, p_2) + \lambda_{10}(p_1, p_2)\lambda_{20}(p_1, p_2)}{c} \\ &\quad + \frac{\lambda_{12}(p_1, p_2)\lambda_{20}(p_1, p_2)}{c} \\ \pi_1 &= \frac{(\lambda_{20}(p_1, p_2) + 2\lambda_{12}(p_1, p_2))\alpha}{c} \\ \pi_2 &= \frac{\alpha(\lambda_{10}(p_1, p_2) + 2\lambda_{21}(p_1, p_2))}{c}. \end{aligned}$$

<sup>2</sup>Remark that other distributions for sojourn times can be used as well. In that case the model requires to be handled by simulation, while here steady-state probabilities can be computed analytically and as a consequence the game is solved using simple numerical analysis tools.

### III. NON-COOPERATIVE GAME FROM THE PROVIDERS' SIDE

The previous section describes the behaviour of a customer as a function of prices set by providers. The question is now to define the best pricing strategy for each provider knowing that behaviour. We therefore have a so-called Stackelberg game [9], with leaders (the providers) choosing their prices knowing the consequences they would have on users' behaviour, and the followers (the users) whose reaction is a direct consequence of providers' prices. This means that providers play first, but using backward induction, they anticipate the resulting strategy of end users who actually make the last move.

It is important to stress that our model, considering a single customer in front of two providers, is sufficient if assuming that each user has a behaviour independent of others'. The case of  $N$  users can then indeed be easily derived by multiplying the expected revenue by  $N$  (thanks to the independence).

In the first step of the Stackelberg game, each provider tries to maximize its revenue. There is a trade-off to be analyzed between the fact that increasing the price will increase the revenue per customer, but on the other hand potentially reduce the number of customers (i.e., the probability of having the user as customer in our case). The revenue per customer  $R_i$  for provider  $i \in \{1, 2\}$  is therefore expressed formally as the price charged multiplied by the probability that this customers is indeed with provider  $i$ , i.e.,  $R_i = p_i \pi_i \forall i \in \{1, 2\}$ , or more exactly using the expressions for the steady-state probabilities of the Markov chain:

$$\begin{aligned} R_1(p_1, p_2) &= p_1 \frac{\alpha(\lambda_{20}(p_1, p_2) + 2\lambda_{12}(p_1, p_2))}{c} \\ R_2(p_1, p_2) &= p_2 \frac{\alpha(\lambda_{10}(p_1, p_2) + 2\lambda_{21}(p_1, p_2))}{c}. \end{aligned}$$

From those expressions, it is clear that the revenue of a provider depends on the price strategy of the concurrent. Indeed, steady-state probabilities are functions of rates which themselves depend on both prices. As a consequence, this fits the framework of *non-cooperative game theory* [9]. Each provider strives to find its best strategy, i.e., its price maximizing its revenue, which can be modified by the strategy of the competitor. The solution concept is that of a *Nash equilibrium*: a Nash equilibrium is a price profile  $(p_1^*, p_2^*)$  such that no provider can unilaterally increase its revenue, i.e.,

$$\begin{aligned} R_1(p_1^*, p_2^*) &= \max_{p_1 \geq 0} R_1(p_1, p_2^*) \\ R_2(p_1^*, p_2^*) &= \max_{p_2 \geq 0} R_2(p_1^*, p_2). \end{aligned}$$

In general the existence of a Nash equilibrium cannot be ensured without assumptions, nor its uniqueness when existence is shown. In the case where rate functions are simple enough in terms of prices, we may find the form of the Nash equilibria analytically (see next section). Otherwise, the computations can be performed numerically using the following algorithm. We define the best response of each provider as a function of the strategy of its opponent by

$$\begin{aligned} \text{BR}_1(p_2) &:= \arg \max_{p_1 \geq 0} R_1(p_1, p_2) \text{ and} \\ \text{BR}_2(p_1) &:= \arg \max_{p_2 \geq 0} R_2(p_1, p_2). \end{aligned}$$

In this setting, a Nash equilibrium is just a point  $(p_1^*, p_2^*)$  such that  $\text{BR}_1(p_2^*) = p_1^*$  and  $\text{BR}_2(p_1^*) = p_2^*$  (if best responses are not unique, it means that  $p_1^*$  in the set of best responses when provider's price is  $p_2^*$ , and reciprocally). Algorithm 1 describes how to algorithmically and graphically determine Nash equilibria (if any)

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**Alg. 1** Graphically finding the Nash equilibria of the game

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Input: transition rates of the Markov chain in Figure 1, as functions of prices  $p_1$  and  $p_2$ .

- 1) For all possible values of  $p_2 \geq 0$ , find the set  $\text{BR}_1(p_2)$  of  $p_1$  values maximizing  $R_1(p_1, p_2)$ .
  - 2) For all possible values of  $p_1 \geq 0$ , find the set  $\text{BR}_2(p_1)$  of  $p_2$  values maximizing  $R_2(p_1, p_2)$ .
  - 3) On a same graphic, plot the best response functions  $p_1 = \text{BR}_1(p_2)$  and  $p_2 = \text{BR}_2(p_1)$ , as illustrated Figure 3.
  - 4) The set of Nash equilibria is the (possibly empty) set of intersection points of those functions.
- 

Two practical remarks can be made:

- Instead of assuming the set  $[0, \infty)$  for each price, we will limit ourselves to  $[0, p_{max}]$  since customers are unlikely to come to the provider if price is too high.
- When analytical derivation cannot be performed and solved to determine the best response functions, only a finite number of values can be tried in practice in each case, and the best responses determined at these points, and the solution investigated on the corresponding lattice.

#### IV. ANALYSIS FOR SPECIFIC BUT REALISTIC RATES

This section is dedicated to the case where rates of the Markov chain are simple, but realistic enough, to derive analytical expressions for the Nash equilibria. We more specifically assume that

$$\begin{aligned} \lambda_{10}(p_1, p_2) &= p_1 \\ \lambda_{20}(p_1, p_2) &= p_2 \\ \lambda_{12}(p_1, p_2) &= \zeta p_1 / p_2 \\ \lambda_{21}(p_1, p_2) &= p_2 / p_1, \end{aligned}$$

with  $\zeta$  a strictly positive real number. Expressions for  $\lambda_{10}$  and  $\lambda_{20}$  mean that a customer is more likely to leave a provider

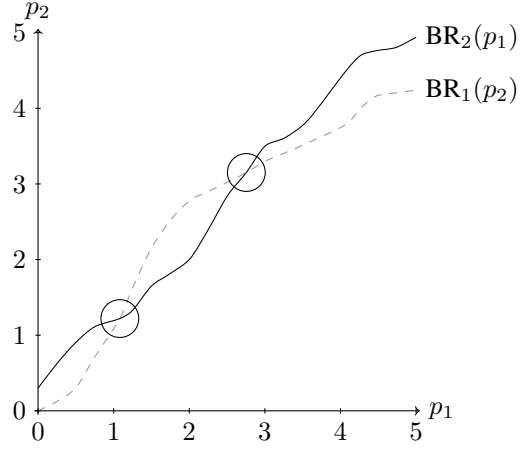


Figure 3: Graphical determination of Nash equilibria. Two solutions here (circled points).

for no service if its price is high, and depend linearly (and only) on the price at the incumbent provider. Values for  $\lambda_{12}$  and  $\lambda_{21}$  mean that swapping between providers depend on the ratios of prices. The insertion of parameter  $a$  is to introduce some asymmetry, because provider 1 may have a better (worse) reputation and it is therefore less (more) likely to be left for provider 2 when  $\zeta < 1$  ( $\zeta > 1$ ).

In that case, we end up, after simple computations, with

$$\begin{aligned} R_1 &= \frac{\alpha p_2^2 p_1 (p_1 + 2)}{2\alpha p_2^2 + (\alpha + 1)p_2 p_1^2 + (1 + \alpha)p_1 p_2^2 + p_1^2 p_2^2 + 2\zeta \alpha p_1^2} \\ R_2 &= \frac{\alpha p_1^2 p_2 (p_2 + 2\zeta)}{2\alpha p_2^2 + (\alpha + 1)p_2 p_1^2 + (1 + \alpha)p_1 p_2^2 + p_1^2 p_2^2 + 2\zeta \alpha p_1^2}. \end{aligned}$$

In order to determine the existence of a Nash equilibrium, we compute the derivatives. Using  $D = 2\alpha p_2^2 + (\alpha + 1)p_2 p_1^2 + (1 + \alpha)p_1 p_2^2 + p_1^2 p_2^2 + 2\zeta \alpha p_1^2$ , we have

$$\begin{aligned} \frac{\partial R_1}{\partial p_1} &= -\alpha p_2^2 \left[ \frac{p_1^2 ((1 - \alpha)p_2^2 + 2(\alpha + \zeta)p_2 + 4\zeta\alpha)}{D^2} + \frac{p_1(-4\alpha p_2^2) + (-4\alpha p_2^2)}{D^2} \right] \\ \frac{\partial R_2}{\partial p_2} &= \alpha p_1^2 \left[ \frac{p_2^2 (-2\zeta(1 + \alpha)p_1 + (\alpha - \zeta)p_1^2 - 4\zeta\alpha)}{D^2} + \frac{p_2(4\zeta\alpha p_1^2) + (4\zeta^2\alpha p_1^2)}{D^2} \right]. \end{aligned}$$

Note that  $(p_1 = 0, p_2 = 0)$  is always a solution of the system  $\frac{\partial R_1}{\partial p_1} = 0$  and  $\frac{\partial R_2}{\partial p_2} = 0$ , and a Nash equilibrium. On the other hand, equating the numerators to zero, any solution with  $(p_1, p_2) \neq (0, 0)$ , i.e., any non-degenerate strictly positive and finite Nash equilibrium, then solves the system of second degree equations

$$p_1^2 ((1 - \alpha)p_2^2 + 2(\alpha + \zeta)p_2 + 4\zeta\alpha) + p_1(-4\alpha p_2^2) + (-4\alpha p_2^2) = 0 \quad (1)$$

$$p_2^2 (-2\zeta(1 + \alpha)p_1 + (\alpha - \zeta)p_1^2 - 4\zeta\alpha) + p_2(4\zeta\alpha p_1^2) + (4\zeta^2\alpha p_1^2) = 0. \quad (2)$$

The unique strictly positive solution of this system is given by

$$p_1 = -2 \frac{\alpha(-\zeta^2 + \zeta + \alpha^2\zeta - \alpha^2)}{\zeta\alpha^3 - 2\alpha^2\zeta - \alpha^2 + 3\zeta\alpha - \alpha^2} \quad (3)$$

$$p_2 = 2 \frac{\zeta\alpha(-\zeta^2 + \zeta + \alpha^2\zeta - \alpha^2)}{3\zeta^2\alpha - \zeta^2 - \zeta^2\alpha^2 - 2\alpha^2\zeta + \alpha^3}, \quad (4)$$

provided those values are positive. At most one Nash equilibrium with strictly positive prices is possible in that case.

## V. NUMERICAL RESULTS

### A. Churn rates and prices in the literature

In Section II the transition rates that mark the passage from a provider to the other are shown to depend on the prices offered by the providers and Section IV illustrates that, in very simplified cases, the Nash equilibrium can be obtained analytically, though not easily. In order to adopt a model as close as possible to reality, we briefly review the related literature on the mathematical relationship between churn rates and prices in this sub-section, that which will be adopted during our numerical analysis.

Significant efforts have been spent to identify the most relevant factors in determining churn (often named churn determinants). In order to model the relationship between prices and churn rates in a quantitative fashion both parametric and non-parametric approaches have been proposed in the literature. Among the non-parametric approaches we can cite [10], where neural networks and decision trees are employed, and [11] where a novel evolutionary learning algorithm is proposed. Since we need a closed form relationship here we are more interested in parametric approaches. The most widespread model adopted in the literature to represent that relationship is the logit model, which employs a logistic probability distribution function [12] [13] [14]. The argument of the logistic function is a linear function of a number of churn determinants. The most general expression of the probability that a user churns in the next period (e.g., a year as in [12]) is then

$$\mathbb{P}_{\text{churn}} = \frac{1}{1 + e^{-I}}, \quad (5)$$

where  $I$  is the logit factor, in turn given by

$$I = \sum_{i=1}^n \beta_i X_i, \quad (6)$$

where  $X_i$ ,  $i = 1, \dots, n$  are the explanatory variables (churn determinants) and  $\beta_i$ ,  $i = 1, \dots, n$ , are the coefficients representing the relative importance of those determinants. In this paper we have focussed on the price factor, so that we can group the impact of the other churn determinants in the overall term  $\gamma$ , arriving at the simpler expression

$$\mathbb{P}_{\text{churn}} = \frac{1}{1 + \gamma e^{-\beta p_1/p_2}} \quad (7)$$

for the probability that in the specified period the user switches from Provider 1 to Provider 2. We may employ that expression for a time period of any duration, so we can adopt it in the

Markov chain model described in Section II. We note that, according to expression (7), there is a non zero probability, namely  $\mathbb{P}_{\text{churn}} = \frac{1}{1+\gamma}$ , that the user switches provider due to the ensemble of other dissatisfaction factors, even when the service offered by the losing provider is free.

### B. Transition rates

The transition rates that we assume now are chosen to reflect the conclusions of the previous subsection. However, the data and conclusions drawn from the literature do not provide us with a complete description of all the transition rates we need, in particular the state where users do not subscribe to the service is not encompassed in previous results. Moreover, the literature considers discretized time, whereas we focus here on a continuous-time model.

That latter difficulty is addressed here by assuming that time periods considered in Subsection V-A, are short with respect to the mean sojourn time in a given state. This implies that the discrete time transition probabilities are approximately the continuous-time transition rates multiplied by the period duration. Consequently, we would like to consider transition rates from state  $i \in \{1, 2\}$  to state  $j \in \{1, 2\} \setminus \{i\}$  of the form  $\frac{\kappa}{1 + \gamma_i e^{-\beta p_i/p_j}}$ , where  $\kappa > 0$  represents the inverse of the period duration. Since  $\beta$  represents the user sensitivity to prices, we consider it is the same for the different states of the model. However, such an expression would imply that all transition rates be in an interval  $[\kappa/(1 + \gamma_i), \kappa]$  regardless of the price values. This is not realistic, since it would imply that a provider could ensure an arbitrarily large revenue by setting a very large price. We therefore need that the transition rates to a provider  $i$  tend to 0 and/or that the rates from provider  $i$  tend to  $\infty$ , when  $p_i \rightarrow \infty$ . To that end, we slightly modify the previous expression, and take transition rates of the form

$$\lambda_{ij}(p_i, p_j) = \frac{\kappa}{\gamma_i e^{-\beta p_i/p_j}} = \frac{\kappa}{\gamma_i} e^{\beta p_i/p_j}, \quad (8)$$

We introduce asymmetry among providers through the parameter  $\gamma_i$ : as explained before, this parameter encompasses the reasons other than price (e.g., Quality of Service, reputation, ...), why a user should leave state  $i$ .

We propose to address the former difficulty (no hints regarding the transitions to/from our state 0) by assuming that being in state 0 corresponds to perceiving a *cost*  $p_0$ , that reflects the inconvenience for not benefitting from the service. We therefore treat state 0 as the two other states, but considering  $p_0$  as a fixed value instead of a strategic variable. As a result, the transition rate we assume from any state  $i \in \{0, 1, 2\}$  to state  $j \in \{0, 1, 2\} \setminus \{i\}$  is given by (8).

The model parameters that we consider are then:

- the user sensitivity to price  $\beta$ ,
- the likeliness  $\gamma_i$  to stay in current state  $i$ ,  $i = 0, 1, 2$ ,
- the inverse of period duration  $\kappa$  (this parameter should not play a role in our model, since by a time unit change we can assume  $\kappa = 1$ ),
- the user perceived cost  $p_0$  for not benefitting from the service.

### C. Numerical analysis of the game

In this subsection, we suggest to study the game while considering the previous expressions of the transition rates. The dependence of those rates on provider prices are too complicated to solve the problem analytically, therefore we perform here a numerical study. Unless otherwise stated, the parameter values considered in this section are the following:  $p_0 = 1, \kappa = 1, \beta = 0.5, \gamma_1 = 1, \gamma_2 = 2, \gamma_0 = 1$ . We will refer to this set of parameter values as  $S$ .

Figure 4 plots the steady-state revenue  $R_1$  of provider 1 when its price  $p_1$  varies, for different values of the opponent price  $p_2$ . We remark that the revenue of provider 1 is first

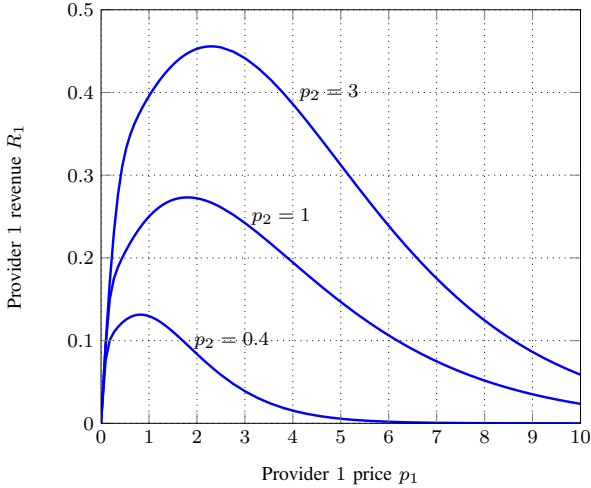


Figure 4: Provider 1 revenue versus  $p_1$  for fixed  $p_2$ .

increasing, then decreasing in  $p_1$ . However, this is not always the case: for some parameter values, the revenue of a provider as a function of its price may have two local maxima, and depending on the opponent's price, the first or the second local optimum is a global one. This is exemplified in Figure 5 for  $\gamma_2 = 7$  and the other parameter values in  $S$ . Nevertheless, it appears that the revenue of provider 1 tends to 0 as its price tends to infinity, which implies that there exists a finite price  $p_1$  maximizing  $R_1$ . That revenue-maximizing price constitutes the best reply of provider 1 to the price set by provider 2.

As explained in Section III, plotting the best-reply curves of both providers on the same graph highlights the Nash equilibria of the game. Those curves are shown in Figure 6 for the parameter values in  $S$ . Figure 6 shows that there are two Nash equilibria of the game, namely  $(0, 0)$  and  $p^* \approx (2.29, 2.84)$ . However, we notice that  $(0, 0)$  is not a satisfying situation, since it brings no revenue to the providers, and moreover it is not a stable Nash equilibrium: if any of the two providers slightly deviates from that situation by setting a strictly positive price, then successive best replies lead to the other (stable) Nash equilibrium  $p^*$ . We will consequently focus on that equilibrium in the following, when it exists. Indeed, notice that as the example of Figure 5 illustrates, the best reply correspondence  $p_2 \mapsto BR_1(p_2)$  may not be continuous

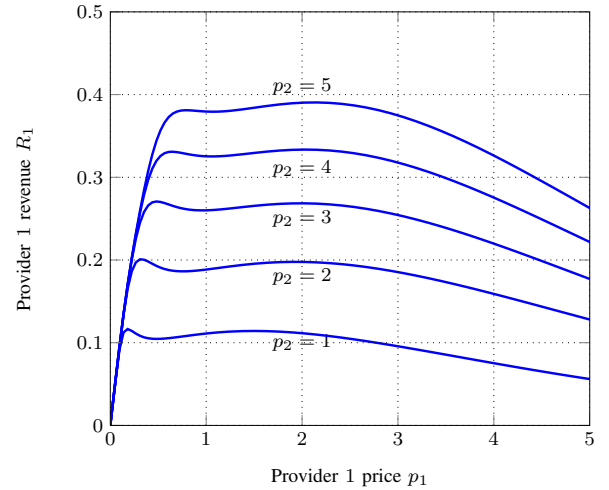


Figure 5: Provider 1 revenue versus  $p_1$  for fixed  $p_2$  and  $\gamma_2 = 7$  (other parameters taken from  $S$ ).

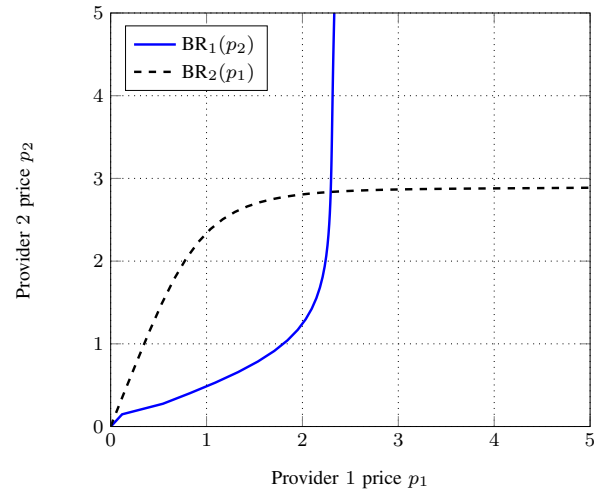


Figure 6: Best-reply curves of both providers.

due to the fact that the global maximum can switch from one local maximum to the other. Figure 7 plots the best-reply correspondences with the same parameter values as for Figure 5. Interestingly, for that set of value there are two stable Nash equilibrium: one around  $(p_1, p_2) = (0.14, 0.7)$  and the other one near  $(p_1, p_2) = (2, 3.9)$ . Nevertheless, those cases were rarely met in our numerical computations, and were not met with the “reasonable” values that we used.

### D. Influence of the parameter $p_0$

In Subsection V-B we have interpreted  $p_0$  as the user perceived cost for not benefitting from the service. When the service in question concerns a rapidly time-evolving sector such as telecommunications, it is very likely that this perceived cost  $p_0$  changes. For example, Internet access or cellular telephony are now almost priceless services for many users, because those tools are extensively used and becoming

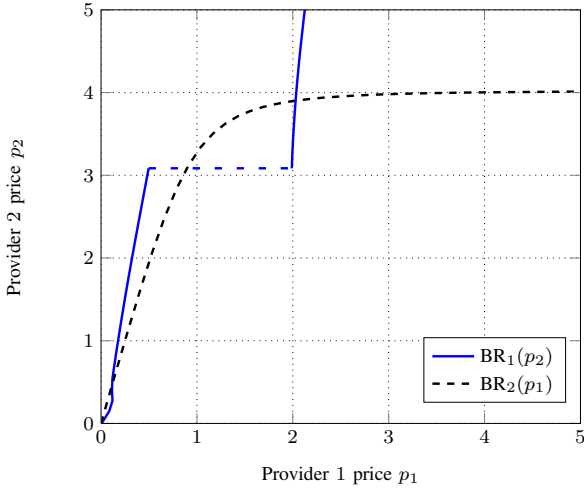


Figure 7: Best-reply curves of both providers when  $\gamma_2 = 7$  (other parameter values taken from  $S$ ).

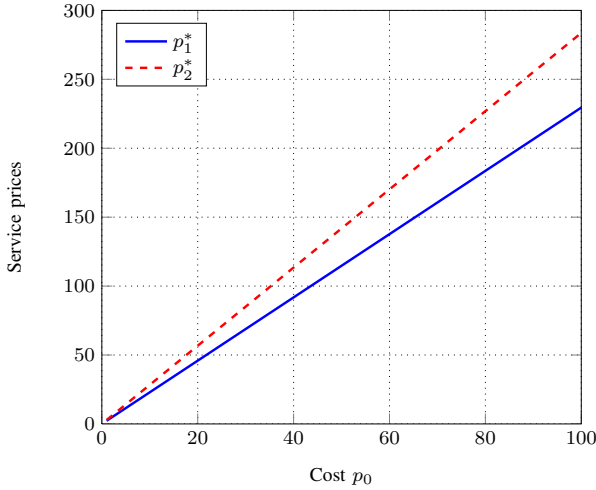


Figure 8: Nash equilibrium prices when the no-service cost  $p_0$  varies.

mandatory for regular business. This was not the case at the very beginning of those technologies. On the other hand, some services/technologies can lose value because they get abandoned or can be replaced by other ones. Services with higher importance/impact can therefore be modeled by a larger  $p_0$

For those reasons, we investigate now the effect of the no-service cost  $p_0$  on the outcomes of the game. We assume that the variations of  $p_0$  are on a longer time scale than the game on prices and user behavior, so that we still compute the Nash equilibrium of the pricing game as described in the previous subsection.

Figure 8 plots the *Nash equilibrium prices*  $(p_1^*, p_2^*)$  versus  $p_0$ , while the corresponding user repartition at steady-state is plotted in Figure 9. We remark that the steady-state distribution (again, at Nash equilibrium depending on  $p_0$ ) is almost

constant when  $p_0$  varies, and equilibrium prices increase close to linearly with  $p_0$ . Therefore, if providers are aware of an increase in  $p_0$ , they should raise their prices correspondingly to benefit from the increased value of the service. Interestingly, this price increase compensates the service value increase from the point of view of the users, since the proportion of users choosing the service or not is unchanged.

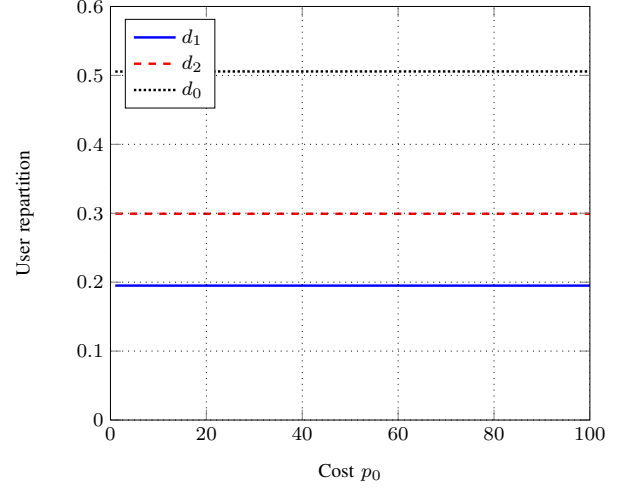


Figure 9: User repartition at Nash equilibrium when the no-service cost  $p_0$  varies.

#### E. Influence of the price sensitivity $\beta$

We study here the effect of the parameter  $\beta$ , that represented users' sensitivities to price differences between the different states of the Markov chain. When  $\beta$  increases, we expect providers to decrease their prices so as to attract more efficiently a maximum of customers. The Nash equilibrium prices  $(p_1^*, p_2^*)$  when  $\beta$  varies are shown in Figure 10. We give the corresponding user repartition and provider revenues in Figure 11 and 12, respectively. It appears that when the

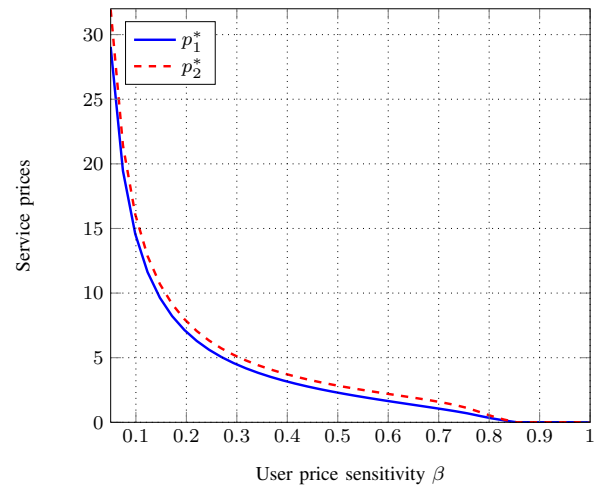


Figure 10: Nash equilibrium prices when  $\beta$  varies.

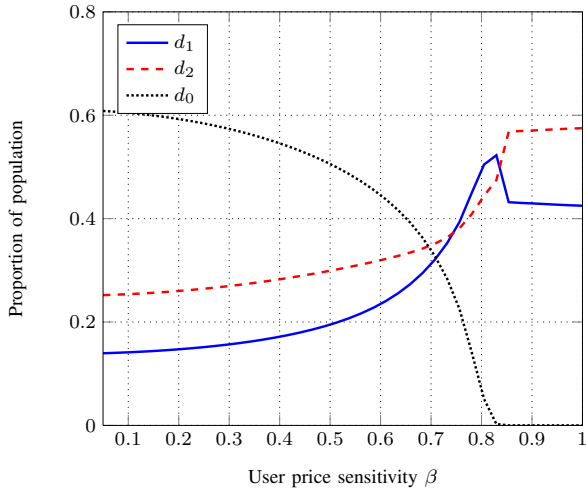


Figure 11: Steady-state user repartition at Nash equilibrium when  $\beta$  varies.

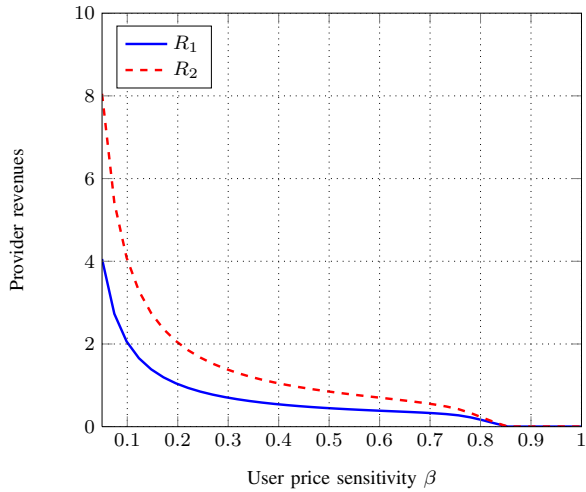


Figure 12: Provider revenues at Nash equilibrium when  $\beta$  varies.

price sensitivity exceeds a given threshold (that is around 0.85), providers engage in a price war that makes their price tend to 0, and they finally get no revenue from providing the service. This benefits users who all end up with one of the two providers. Below that threshold, an increased price sensitivity already implies a provider price reduction, and a revenue decrease for them. Remark also that provider 2, being more attractive than its opponent (since  $\gamma_2 > \gamma_1$  in our parameter values), takes benefit of that advantage by setting higher prices.

#### F. Influence of the asymmetry between providers

In our model, providers only differ for their parameter  $\gamma_i$ , that reflects the user likeliness to stay with him: from (8), a larger  $\gamma_i$  means lower transition rates to the other states of the Markov chain. That asymmetry between providers may for example come from users being more reluctant to leave

the incumbent operator than another one, because they trust less the newly-arrived operators in terms of honesty and/or QoS. Therefore, those parameters  $\gamma_i$  may vary due to word of mouth, advertisement, and are consequently difficult to evaluate.

We therefore investigate here the influence of the asymmetry in providers'  $\gamma_i$  value. To do so, we fix all values but  $\gamma_2$  from the parameter set  $S$ , and make  $\gamma_2$  vary. We assume that provider 2 is the one with an advantage, i.e.  $\gamma_2 \geq \gamma_1$ . In Figures 13, 14, and 15, we respectively plot the Nash prices, user repartition and provider revenues versus  $1/\gamma_2$  (therefore abscissa also gives the ratio  $\gamma_1/\gamma_2$ ). Notice that the curves are only given for the values of  $1/\gamma_2 > 0.18$ , because for values below that threshold the game may have several stable Nash equilibria (as in the case shown in Figure 7) and we cannot predict which one will be chosen. Nevertheless, those extreme values may seem unrealistic, since they mean an asymmetry magnitude larger than 5.

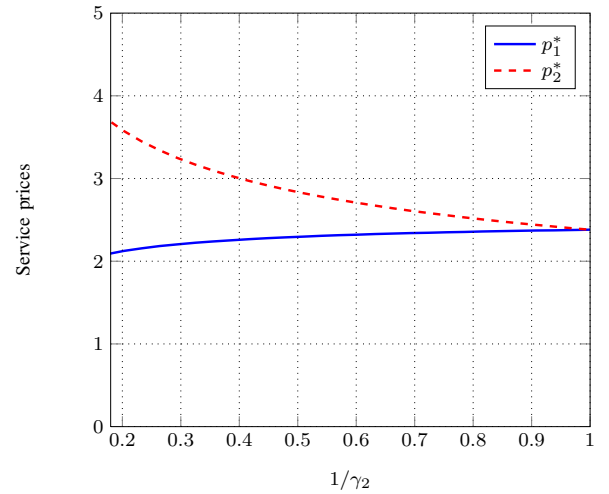


Figure 13: Nash equilibrium prices when  $1/\gamma_2$  varies.

We remark that provider 2 takes benefit from his advantage by setting a higher price than his opponent, while still having more customers. That difference in price and user repartition increases with the game asymmetry, and vanishes when the game becomes symmetric, i.e. when  $\gamma_2$  tends to 1.

## VI. CONCLUSIONS AND FUTURE WORK

In this paper, we have presented a competition game model, where providers take into account the user churn behavior to determine the price they fix for the service, so as to maximize their steady-state revenue. Through a numerical analysis, we remarked that the game has a Nash equilibrium, that might not be unique if the asymmetry between providers is very large. When providers are not too different in terms of attractivity or reputation, we investigate the effect of user sensitivity to prices (that is seen to exacerbate the price war), and the effect of an increase in the need for the service (that is observed to benefit only to providers).



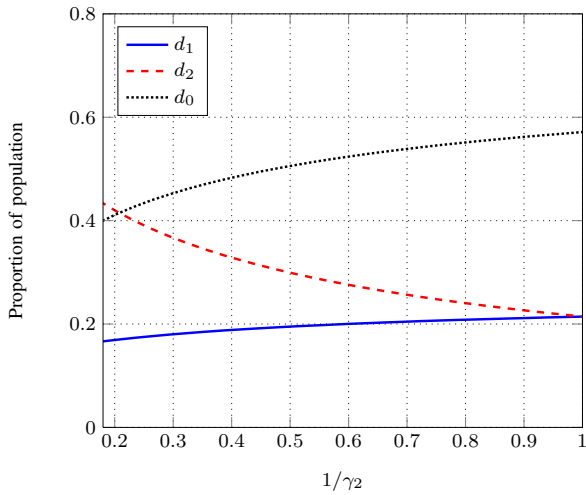


Figure 14: Steady-state user repartition at Nash equilibrium when  $1/\gamma_2$  varies.

One interesting direction for future work is to have a different approach as to the considered time scales. In this paper, we have considered several time scales: at the smallest time scale users are assumed to react to prices, while those prices are fixed at a larger time scale, reasoning on the user behavior steady-state outcome. Finally, the value of the service may vary at an even larger time scale. Those assumptions can be justified, but it would also be interesting to relax them, for example by considering the user dynamics within the pricing game: an incumbent provider may start the game with more customers than his opponent, and may therefore be better off beginning with a large price since not all users will immediately churn to the opponent. Likewise, the competitor may have an incentive to start with low prices so as to attract customers, before possibly raising its price.

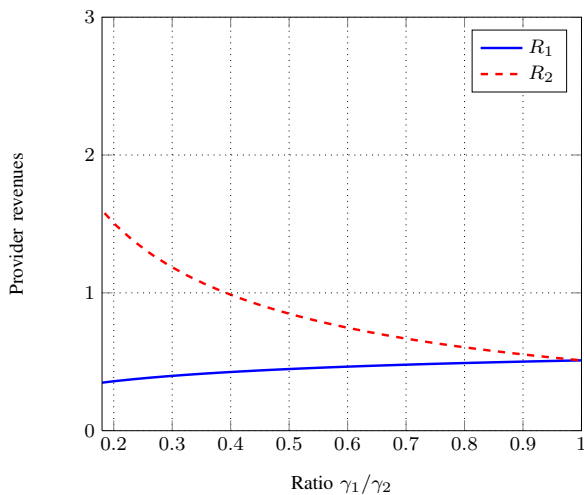


Figure 15: Provider revenues at Nash equilibrium when  $1/\gamma_2$  varies.

## ACKNOWLEDGMENTS

The authors would like to acknowledge the support of Euro-NF Network of Excellence through the specific research project PRECO, and the French *Agence Nationale pour la Recherche* through the CAPTURES project.

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