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To cite this version:
Ahmed Khelassi, Didier Theilliol, Philippe Weber. Reconfigurability analysis for reliable fault-tolerant control design. 7th Workshop on Advanced Control and Diagnosis, ACD’2009, Nov 2009, Zielona Gora, Poland. pp.CDROM. hal-00443041

HAL Id: hal-00443041
https://hal.science/hal-00443041
Submitted on 27 Dec 2009

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Reconfigurability Analysis for Reliable Fault-Tolerant Control Design

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Abstract: This paper interests to the reconfigurability of fault-tolerant control system based on the reliability analysis of components. The aim of this work is to present the need of reliability analysis in fault-tolerant control design. The admissibility of system reconfigurability with respect to the reliability constraints is considered in this research. This analysis proves that the control reconfigurability is a system property that not only related to the Gramian controllability but also to the overall system reliability. An admissible solution for reconfigurability indicator generation is proposed according to the reliability evaluation in degraded mode. A linearized aircraft model is considered as an example to illustrate this approach.

Keywords: Fault-tolerant control system, Reconfigurability, Reliability, Actuator faults.

1. INTRODUCTION

Manufacturing systems consist of many different components, which ensure their operation and high-quality production. In order to respect the growing of economic demand for high plan availability, and system safety, dependability is becoming an essential need in industrial automation. In this context and to satisfy these requirements, fault-tolerant control (FTC) is introduced. The aim of FTC systems is to keep plan available by the ability to achieve the objectives that have been assigned to the system in the faulty behavior and accept reduced performance when critical faults occur (Blanke et al. (2006)). Thus, increasing systems autonomy involves the capability of critical system safety to compensate the impact of component faults on the system behavior (H. Noura et al. (2009)). Within this framework, the main goal of FTC is to improve the reliability of the system, which is rarely associated with an objective criterion that guides a design. However, it is difficult to establish a functional linkage between the overall system reliability and control performance requirement.

In fault-tolerant control, the information obtained from the fault diagnosis module is used in the controller re-design. In fact, process diagnosis should not only indicate the fault occurrence but also identify the fault location and magnitudes. This assumption will make it possible the controller re-design. In this context, fault accommodation can be a means to maintain the overall system operational after fault occurrence by adapting the controller parameters, or by the generation of an additional control law. (Blanke et al. (2001)). Moreover, if fault accommodation cannot be achieved, the complete control loop has to be reconfigured. Then, a new control law has to be designed and the controller structure has to be changed on line. In this case, the original control objective is achieved, although degraded performance can be accepted.

Even though, the study of the system property is necessary to determine, which failure modes could severely affect the plant safety. However, only few attempts are now focusing on the fundamental FTC property analysis, some of studies are often defined as the fault detectability, fault isolability (Patton (1997)). In this context, the concept of reconfigurability was introduced as the control system property in Wu et al. (2000). The reconfigurability of LTI system can be also evaluated using the controllability and observability Gramians (Frei et al. (1999)). Moreover, a reconfigurability measures have been proposed to evaluate the size of fault-tolerant situations set named number of recoverable failures, under possible energy limitations (Staroswiecki (2002)), and also as general quadratic control problem (Staroswiecki (2003)). As shown in Yang (2006), the reconfigurability measures can be viewed like an intrinsic reconfigurability property or as a reconfigurability property performance. All these approaches have been considered off-line. As recently introduced by Gonzalez-Contreras et al. (2009), the reconfigurability analysis can be evaluated on-line using input/output data.

The aim of this paper is to study the fault tolerance property in the presence of actuator faults. Our interest is to improve the availability and the safety of system after the fault occurrence in order to achieve the functional objectives until the end of the mission. Then, the fault-tolerant control design requires to ensure the commandability, the observability and the capability of a new control law to recover the faulty behavior with respect to the industrial demand as overall system reliability requirement. In this framework, the reliability analysis is introduced to establish an admissible solution of reconfigurability property based on the energy consumption. The relationship be-
between energy required in degraded mode and reliability constraint is illustrated.

This paper is organized as follows: section 2 formulates the fault-tolerant control problem and defines the reconfigurability concept. Reconfigurability statement is examined under actuator faults occurrence. The notion of admissibility for fault tolerance is defined. In section 3, the computation of reliability is introduced. The impact of actuator faults in degraded mode on the reliability evaluation is determined in order to reformulate the reconfigurability problem under reliability requirement. An index for reconfigurability property, and a solution for admissibility under reliability requirement is proposed to evaluate the reconfigurability capability of the faulty system. Section 4 is devoted to illustrate this analysis. An aircraft model is taken as application. Finally, conclusion is given in the last section.

2. CONTROL RECONFIGURABILITY EVALUATION

2.1 Problem statement

Consider the system in fault-free case modeled by a linear state-space representation:

\[
\begin{align*}
\dot{x}(t) &= Ax(t) + Bu(t) \\
y(t) &= Cx(t)
\end{align*}
\]  

(1)

where the state vector \( x(t) \in \mathbb{R}^n \), the control vector \( u(t) \in \mathbb{R}^m \), the output vector \( y(t) \in \mathbb{R}^r \) and matrices \( A \in \mathbb{R}^{n \times n}, B \in \mathbb{R}^{n \times m}, C \in \mathbb{R}^{r \times n} \).

Indeed, for the actuator faults, the control law applied to the plant is interrupted or modified. For this study, the loss of control effectiveness is considered (Wu et al. (2000), Zhang et al. (2008)), and the system (1) can be represented in the faulty case as follows:

\[
\begin{align*}
\dot{x}(t) &= Ax(t) + B_f u_f(t) \\
y(t) &= Cx(t)
\end{align*}
\]  

(2)

Where \( B_f = B(I_m - \Gamma) \) and \( \Gamma = \text{diag}\{\gamma\} \). In fact, \( \gamma = \{\gamma_1, \gamma_2, \ldots, \gamma_m\} \) represents a set of loss effectiveness factors \( \gamma_i \), which define a values of actuator faults magnitudes, with \( \gamma_i \in [0 \ldots 1] \). Indeed, if \( \gamma_i = 0 \), then the actuator \( i \) is considered under normal operation. Nevertheless, when \( 0 < \gamma_i < 1 \), a fault is considered to represent a partial loss of control effectiveness. Moreover, when \( \gamma_i = 1 \) failure is considered and the actuator is out of order. Indeed, the reconfigurability property can be discussed as the ability of the considered system to recover some admissible performance taken into account fault occurrence. According to Frei et al. (1999), Wu et al. (2000), Staroswiecki (2003), Yang (2006), the reconfigurability can be defined as follows:

**Definition 1.** Generally, the system (1) is called (completely) reconfigurable if and only if the controllability property of the nominal system is kept by the faulty system until the end of the mission.

The reconfigurability analysis is based on the limitation of energy consumption, which defines an admissible solution. However, to keep the system performance requirement in the degraded mode until the end of the mission, the actuators reliability evaluation can be introduced. Indeed, the reliable fault-tolerant control design can be defined based on energy, by the reliability analysis of the system.

2.2 Reconfigurability based on controllability Gramian

As proposed by (Staroswiecki (2002)), the controllability Gramian appears to be useful for the following meanings: (1) to guarantee the controllability condition of the system proving the existence of a solution; (2) there exists at least one admissible solution with respect to some specific energy limitation. The aim is to take the state of system from \( x(0) = x_0 \) where \( x_0 \in \mathbb{R}^n \) to the origin state \( x(\infty) = 0 \) and to guarantee the existence of an admissible solution.

This problem amounts minimize the energy consumed by the system. The criterion used is represented as follows:

**Criterion 2.** Minimize the functional:

\[
J(x_0) = \frac{1}{2} \int_0^{\infty} \|u(t)\|^2 dt,
\]

(3)

to transfer \( x(0) = x_0 \) to \( x(\infty) = 0 \), where \( x_0 \in \mathbb{R}^n \), and \( x(\infty) \) stands for \( \lim_{t \to \infty} x(t) \).

Such that the system is modeled by (1), the solution of (3) is obtained by the Hamiltonian equation from the theory of optimal control.

\[
u(t) = B^T P x(t)
\]

(4)

where \( P \) is the unique solution of the Lyapunov equation defined as:

\[
A^T P + P A = -B B^T
\]

(5)

For the criterion (3), the Matrix \( P^{-1} \) is the controllability Gramian \( W_c \) of the control law \( u(t) \). In fact, \( W_c \) defines the energy consumption required to transfer the system state to the origin. Moreover, \( W_c \) is invertible since the pair \( (A, B) \) is controllable, it can be defined analytically as follows:

\[
W_c = \int_0^{\infty} e^{At} BB^T e^{A^T t} dt
\]

(6)

The optimal value of the criteria (3) is obtained in this case as follows:

\[
* J(x_0) = x_0^T W_c^{-1} x_0
\]

(7)

As illustrated in (Staroswiecki (2002)), eq. (7) shows that the performance of actuators depends on the control objective \( x_0 \). The functional performance can be characterized independently of the control objective, leads to consider the worst case in term of energy consumption.

**Lemma 3.** In degraded mode, the actuators performance are characterized independently to the control objective by the maximum eigenvalue of \( W_c^{-1}(\gamma) \), which is interpreted as the maximum energy required to transfer the system state to the origin. This value of energy correspond to
the worst case, which can be occurred in degraded functional (Blanke et al. (2006)):

\[ \sigma(\gamma) = \max \Lambda(W_c^{-1}(\gamma)) \]  

where \( \Lambda(W_c^{-1}) \) is the set of the \( W_c^{-1} \) eigenvalues.

In fact, \( W_c(\gamma) \) is invertible and positive matrix since \( A \) is stable and the pair \((A, B_f(\gamma))\) keeps controllable. It is evaluated as follows:

\[ AW_c(\gamma) + W_c(\gamma)A^T = -B_f(\gamma)B_f^T(\gamma), \]  

As illustrated in Khelassi et al. (2009), the fault-tolerance is evaluated by means of the energy cost of the worst situation in which, the system is still controllable for an admissible solution. An index of reconfigurability based on the maximum energy consumed after fault occurrence (8) is proposed by normalization as:

\[ \rho(\gamma) = \frac{\sigma(\gamma) - \sigma_{\min}}{\sigma_{\max} - \sigma_{\min}} \]  

Where \( \sigma_{\max} \) is the upper value of energy in the worst case, which corresponds to the most degraded mode. \( \sigma_{\min} \) is the smallest value corresponding to the nominal case for \( \gamma = 0 \). Due to the normalization of reconfigurability measure (8), the index (10) values vary between 0 and 100\%. The index (10) can be seen like an image on the system behavior according to loss of performance. It is evaluated in terms of the energy consumption value in the degraded mode.

**Definition 4.** In the degraded behavior, the solution of FTC problem is admissible with respect to a control objective if:

\[ \rho(\gamma) \leq \rho_{\text{pth}} \]  

Where \( \rho_{\text{pth}} \) is a predefined threshold, which represents the maximal functional degradation that can be admitted and accepted when a control solution is used.

Based on the definition of admissibility, the admissible values for the solution are established in order to limit the worst system operation in term of fault and its magnitude. However, the problem is to how define the value of the threshold \( \rho_{\text{pth}} \) based on a specified requirement. In the following section, the solution for admissibility problem based on reliability requirement is proposed.

### 3. ADMISSIBILITY SOLUTION BASED ON RELIABILITY ANALYSIS

As presented previously, the reconfigurability based on the controllability Gramian is applied to evaluate the quality of the control, which can be achieved by a fault-tolerant control scheme. However, it is crucial to ensure that mean operating time of the novel configuration selected for the recovery problem is sufficient to achieve the system’s performance until the end of the mission. In this context, the problem of minimization (3) becomes a minimization problem of energy with respect to a reliability constraint such as:

\[ J(x_0) = \int_0^\infty \|u(t)\|^2 \, dt \]  

s.t. \( R(t) \geq R_{\text{pth}} \)

Where \( R(t) \) is the overall system reliability. \( R_{\text{pth}} \) is a predefined threshold, which define the minimal value of the acceptable reliability value in the degraded mode. The aim of this section is to establish a solution for the choice of the energy threshold \( \rho_{\text{pth}} \) based on reliability analysis. In fact, \( \rho_{\text{pth}} \) is the normalization of a predefined energy consumption noted \( \sigma_{\text{pth}} \), which defines the maximum value of the acceptable energy consumption in the degraded mode.

#### 3.1 Reliability computation

In general, the reliability is defined as the probability that units, components, equipments and systems will accomplish its intended function for a specified period of time under some stated conditions and specific environments (Gertsbakh (2000)).

In many situations and especially in the considered study, failure rates are obtained from components under different levels of loads. Several mathematical models have been developed to define the failure level in order to estimate the failure rate \( \lambda \) (Martorell et al. (1999)). Proportional hazard model introduced by Cox (1972) is used in this paper.

**Definition 5.** The failure rate is modeled as follows:

\[ \lambda_i = \lambda_0^i \times g(\ell, \vartheta) \]

where \( \lambda_0^i \) represents the baseline failure rate (nominal failure rate) for the \( i^{th} \) subsystem or component and \( g(\ell, \vartheta) \) is a function (independent of time) taking into account the effects of applied loads with \( \ell \) presenting an image of the load and \( \vartheta \) defining some parameters of the subsystem or component.

Different definitions of \( g(\ell, \vartheta) \) exist in the literature. However, the exponential form is commonly used. Moreover, the failure rate functions for the exponential distribution change according to the load level, which are assumed to be directly associated to the control input of the actuators. For the nominal behavior, the equation (12) can be written as follows:

\[ \lambda_i = \lambda_i^0 \times e^{u_{i,\text{nom}}} \]

where \( u_{i,\text{nom}} \) is the nominal control law delivered by the \( i^{th} \) actuator in the fault-free case. However, the reliability of a component can be evaluated as follows:

\[ R_i(t) = e^{-\lambda_i x t} \]

#### 3.2 Reliability evaluation in degraded mode

As explained in Guenab et al. (2006), the value of failure rate change according to the intensity of control law on actuators due to fault occurrence. It is noted that when even actuator faults occurs, loss of control effectiveness presented by \( \gamma \) changes, and a new failure rate value is obtained due to the increasing of load level. In the context of FTC, the relationship between the increasing of the energy required and reliability can be established. Indeed, when a fault occurs, the control law changes in order to recover the impact of fault on system performance. In this case, the energy consumption increase and the value of
Based on (1) and (2), the failure rate in degraded load function becomes higher due to the increasing of applied load function.

Based on (1) and (2), the failure rate in degraded mode can be established according to the loss of control effectiveness factors $\gamma$ and $u_{\text{nom}}$ as following:

$$u_f = (I - F)^{-1} u_{\text{nom}}$$

(16)

where $u_f$ is the control law delivered fault occurrence and required to transfer the system state to the nominal case. Based on this aspect, the failure rate can be modeled as follows:

$$\lambda_i(\gamma) = \lambda_i^0 \times e^{(1-\gamma_i)^{-1} u_{\text{nom}}^i}$$

(17)

Lemma 6. In degraded mode, overall system reliability decrease due to the evaluation of the failure and the intensity of energy required to recover the actuators fault.

$$R_i(t, \gamma) = e^{-\lambda_i(\gamma) \times t}$$

(18)

The reliability of overall system, which composed on different components or sub-systems depends on the way that these components are connected. In this context, for a system with $q$ series sub-systems, reliability is given by:

$$R_q(t) = \prod_{i=1}^{q} R_i(t, \gamma)$$

(19)

and with $q$ parallel sub-systems, is calculated as follows:

$$R_q(t) = 1 - \prod_{i=1}^{q} (1 - R_i(t, \gamma))$$

(20)

In general case, the system reliability is computed from a combination of the elementary function (19) and (20).

### 3.3 Admissibility and reliability for Reconfigurability

In order to improve fault-tolerant control design, the measures of reconfigurability presented in section 2 is not adequate. In fact, the system is considered reconﬁgurable if the availability of the novel conﬁguration in degraded functional conditions is assured not only after fault occurrence (see section 2), but also until the final time of the mission in term of reliability. To satisfy the reliability constraint $R(t) \geq R_{\text{pth}}$, $\forall t$ during the mission, it is sufficient to respect this constraint a priori at $t = t_m$, which represents the reliability at the end of the mission.

In order to compute the value of the maximum admissible energy consumed in degraded mode $\sigma_{\text{pth}}$, the set of the acceptable degraded modes, which respects the reliability constraint can be defined as follows:

$$\gamma^* = \{ \gamma \in \mathbb{R}^m \mid R(t_m, \gamma) \geq R_{\text{pth}} \}$$

(21)

where, $\gamma^*$ represents the set of effectiveness control correspond to the possible degraded modes, which respect the reliability constraint.

Based on (20) and (8), the choice of the maximum acceptable energy $\sigma_{\text{pth}}$ can be defined as follows:

$$\sigma_{\text{pth}} = \max(\sigma(\gamma^*))$$

(22)

In fact, by normalization (10), the reconﬁgurability index based on energy and under reliability constraint can be found.

### 4. Example

To illustrate the different steps of the proposed approach, the model of an aircraft drawn from the $\mu - \text{Toolbox}$ and used in Wu et al. (2000) is proposed. The plant model has two inputs: elevon command and canard command; two output: angle of attack, pitch rate and pitch angle. This example is considered with two actuators in order to simplify the presentation of results. The values of the nominal failure rates associated to the actuators are presented in the following table:

<table>
<thead>
<tr>
<th>Failure rates of elementary components</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda_1$</td>
</tr>
<tr>
<td>$\lambda_2$</td>
</tr>
</tbody>
</table>

The Control objectives were originally specified on vertical transition, pitch pointing and direct lift. The state-space description of the plan model is given by (1) with:

$$A = \begin{bmatrix} -0.0226 & -36.6 & -18.9 & -32.1 \\ 0 & -1.9 & 0.983 & 0 \\ 0.0123 & -11.7 & -2.63 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix},$$

$$B = \begin{bmatrix} 0 & 0.414 & 0 & -77.8 \\ 0 & 22.4 & 0 & 0 \end{bmatrix}, C = \begin{bmatrix} 0 & 5.73 & 0 & 0 \\ 0 & 0 & 0 & 5.73 \end{bmatrix}.$$

The matrix $B$ is viewed as $B = [b_1 b_2]$. The actuator loss of effectiveness factors $\gamma_1$ and $\gamma_2$ are introduced for each of these vectors by (2). Matrices $A$ and $B$ remain constant. The elevons are regarded as the primary control effectors, and the canards as the secondary, which could also produce secondary effects to the vehicle’s lateral and directional motion when used differentially. First, the controllability Gramian is calculated by using the Lyapunov equation (9) for each degraded mode, which defined according to the different values of $(\gamma_1, \gamma_2)$ where $0 \leq \gamma_i < 1$. In order to study the control reconﬁgurability of the plant, the index based on the normalization of energy consumption is calculated from (10). After reliability evaluation, this index is compared to the energy threshold $R_{\text{pth}}$, which defines the worst acceptable degraded performance. Indeed, for this application, the overall system reliability presented in (1) is evaluated from (19) for each degraded mode as follows:

$$R_q(t_m) = 1 - \prod_{i=1}^{2} (1 - R_i(t_m, \gamma_i))$$

(23)

The predefined reliability threshold $R_{\text{pth}} = 95\%$, which means that for all reconﬁgurable degraded modes and after fault occurrence, the probability that the system accomplishes its intended until the end of the mission must be higher than 0.95.

Taken into account the reliability constraint, the energy threshold (22) is obtained: $\sigma_{\text{pth}} \simeq 2\%$. Fig.2 shows the evaluation of the reconﬁgurability index based on energy and compared to the value of $\sigma_{\text{pth}}$. The axes $x$ and $y$ represent respectively the variation of the set $(\gamma_1, \gamma_2)$. 

**Table 1. Failure rates of elementary components**

<table>
<thead>
<tr>
<th>Failure rates</th>
<th>$\lambda_1$</th>
<th>$9 \times 10^{-6}$ h$^{-1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda_2$</td>
<td>$5 \times 10^{-6}$ h$^{-1}$</td>
<td></td>
</tr>
</tbody>
</table>
The considered reconfigurable modes based on the energy under reliability requirement are shown in Fig.4. This modes are presented according to the values of the set \((\gamma_1, \gamma_2)\), where the axis z is an indicator of control reconfigurability based on the energy threshold obtained by (22). However, Fig.3 shows the degraded modes, which respect to the reliability constraint, named: the acceptable degraded modes.

These results show the need of reliability evaluation for a reliable fault-tolerant control design. In fact, as it can be shown, the control reconfigurability is established based on energy consumption by the evaluation of actuators reliability. The considered reconfigurable modes, which respect the obtained energy threshold, minimize the energy consumption in the degraded functional, and maintain the performance requirement until the predefined final time of the mission. All this admissible modes minimize the energy consumption and guarantee that overall system reliability is upper then \(R_{pth}\).

5. CONCLUSION

In this paper, a reconfigurability index based on the energy consumption and the reliability analysis has been proposed. The Results obtained in this study prove that the solution for the admissibility and the control reconfigurability can be established versus overall system reliability evaluation, in addition to the energy criteria. Indeed, an admissible solution for the control reconfigurability based on the reliability analysis is proposed. This relation characterizes those states are reachable - by an acceptable degraded functional - in term of energy based on the constraint. For the proposed approach, the on-line reliability computation of the system is not necessary. However, for an admissible solution characterized by the reconfigurability index found, the decision for reconfiguration can be taken on-line. The aim is to guarantee the control objectives after reconfiguration by energy minimization until the end of the mission.

REFERENCES

