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# Generalized Identifiability Conditions for Blind Convolutive MIMO Separation

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**Abstract**—This paper deals with the problem of source separation in the case where the output of a multivariate convolutive mixture is observed: we propose novel and generalized conditions for the blind identifiability of a separating system. The results are based on higher-order statistics and are valid in the case of stationary but not necessarily i.i.d. signals. In particular, we extend recent results based on second-order statistics only. The approach relies on the use of so called reference signals. Our new results also show that only weak conditions are required on the reference signals: this is illustrated by simulations and opens up the possibility of developing new methods.

**Index Terms**—Higher order statistics, MIMO convolutive mixtures, Blind source separation, MIMO identification, Contrast functions, Independent Component Analysis, Reference system, Semi-blind methods

## I. INTRODUCTION

The problem of blind source separation has a very large scope of potential applications such as telecommunications, array processing, audio processing and biology. This explains a high interest in the field over the last decades. Numerous achievements have already been accomplished through the now well-recognized concept of Independent Component Analysis (ICA), see e.g. [8]. The first extensively studied mixture model was the instantaneous one. However, more recently, the more general convolutive model has been considered [1], [9], [14] and successful algorithms have been proposed [16], [18]. In particular, this has been done through the derivation of so called contrast functions. Such functions are very useful since they provide both identifiability conditions and criteria to be optimized. Classically they are based on statistics of order higher than or equal to three, which may lead to complicated optimization schemes. To simplify the optimization step, further improvements, which are based on the idea of reference signals, have been recently introduced [4], [6], [7], [12], [13]. In a semi-blind context, the reference signals can be interpreted as a partial knowledge on the sources: this has not been fully exploited up to now and it is done in our paper.

Two practical ways to consider the problem of blind identifiability can be found. The first one uses second order statistics

through matrix algebra decompositions, see e.g. [10], [11], [17]. The second one uses high order statistics (higher than three) through contrast functions, see e.g. [9], [14]–[16]. In all cases, questions about identifiability conditions remain unclear and particularly the link between the above two points of view. To a certain extent, the use of reference signals in conjunction with higher order statistics makes a bridge between different statistics. This is what we extensively use in the following.

In this paper, the following main novel points are introduced:

- We provide generalized conditions for the identifiability of a separating system, which include previous conditions as well as new ones (e.g. spatially dependent sources, . . .).
- We propose new results which make a clear link between the ones in [7], [13], which consider contrast functions, and the one in [11], which consider second order statistics.
- We show that the so-called reference signals can be used in a semi-blind context under weak condition. The corresponding simulation results legitimate the interest of our new approach.

The paper is organized as follows. Section II states the problem; it also introduces the notations and proves preliminary results. The main results are then given in Section III. Section IV details situations where the former results hold and the simulations illustrate the case of a semi-blind context. Further interpretations and links with former works are detailed in Section V. Finally, Section VI concludes the paper.

## II. PROBLEM STATEMENT AND PRELIMINARIES

### A. Mixing model and notations

In the whole paper, we consider discrete-time signals which by convention depend on a time index  $n \in \mathbb{Z}$  ( $n$  hence stands for a generic integer). For any positive integer  $N$ , we denote by  $\mathcal{I}_N$  the set  $\{1, \dots, N\}$ . We consider the following multichannel convolutive mixing model:

$$\mathbf{x}(n) = \sum_{k \in \mathbb{Z}} \mathbf{M}(k) \mathbf{s}(n-k), \quad (1)$$

where  $\mathbf{s}(n) \triangleq (s_1(n), \dots, s_N(n))^T$  is the  $N$ -dimensional random source vector,  $\mathbf{x}(n) \triangleq (x_1(n), \dots, x_Q(n))^T$  is the

$Q$ -dimensional output vector of observations, and  $\mathbf{M}(n) = (M_{ij}(n))_{(i,j) \in \mathcal{I}_Q \times \mathcal{I}_N}$  is the  $Q \times N$  matrix of impulse responses of the mixing MIMO channel. We also consider the so-called multichannel separating system whose action on the observations reads:

$$\mathbf{y}(n) = \sum_{k \in \mathbb{Z}} \mathbf{W}(k) \mathbf{x}(n-k). \quad (2)$$

In the latter relation  $\mathbf{y}(n) \triangleq (y_1(n), \dots, y_N(n))^T$  is the  $N$ -dimensional output of the separating system and  $\mathbf{W}(n) = (W_{ij}(n))_{(i,j) \in \mathcal{I}_N \times \mathcal{I}_Q}$  is the  $N \times Q$  matrix of impulse responses of the separating channel. For the sake of clarity, the signals, the mixing and the separating systems are assumed to be real-valued. The results can nevertheless be extended to the complex case. Additionally, we introduce the combined mixing-separating system defined by the impulse response  $\mathbf{G}(n) \triangleq \sum_{k \in \mathbb{Z}} \mathbf{W}(n-k) \mathbf{M}(k)$ . We thus have the following relation:

$$\mathbf{y}(n) = \sum_{k \in \mathbb{Z}} \mathbf{G}(k) \mathbf{s}(n-k). \quad (3)$$

The objective of source separation consists in recovering  $\mathbf{s}(n)$  from the observations  $\mathbf{x}(n)$ . In other words the separating system is searched for, such that  $\mathbf{y}(n)$  corresponds to an estimation of  $\mathbf{s}(n)$ . When only the observations are used, but no information on the mixing system, the problem is referred to as the *blind* source separation problem. Any situation where little and incomplete information is exploited in addition to  $\mathbf{x}(n)$  can be referred to as a *semi-blind* approach. The problem is tightly connected with the one of blind identification of the mixing system or of a separating system. In all cases, assumptions are required on the source signals and the mixing system. For the derivations in this paper more specifically, we need to introduce an  $(R-2)$ -dimensional vector  $\mathbf{r}(n)$  with components  $r_j(n)$ ,  $j \in \mathcal{I}_{R-2}$ , where  $R \geq 2$  is a given integer. These signals are called ‘‘reference signals’’ [4], [7], [13] and they can be used for the separation. As explained in [4], [7], these references may come from an additional information or may be constructed from the observations. We now sum up the necessary assumptions which hold throughout the paper:

- A0.** (i) The source signals  $s_i(n)$ ,  $i \in \mathcal{I}_N$  are zero mean and unit variance.  
 (ii) The mixing system is stable and left-invertible.  
 (iii) The signals  $r_j(n)$ ,  $j \in \mathcal{I}_{R-2}$  and  $s_j(n)$ ,  $j \in \mathcal{I}_N$  are jointly stationary up to the  $R$ -th order under consideration.

The first assumption can be made with no loss of generality, whereas the second one is obviously a necessary one to invert the mixing system. The third assumptions restrict ourselves to stationary signals and systems.

Surprisingly, we do not require the common assumption of spatial independence of the source signals. A corresponding, but weaker assumption will indeed appear later when necessary: thus, our general approach will include results which showed that independence is sometimes not required [2]. Finally, one should notice that the derivations in the present paper do not require that the signals be temporally independent and identically distributed (i.i.d.). They remain valid in the general non i.i.d. case.

## B. Higher order statistics and cumulant decompositions

The identifiability conditions derived in this paper specify the conditions to be met by the vector-valued reference signal  $\mathbf{r}(n)$  when it is used in higher order statistics. In addition, note that derivations similar to those in this section can be carried out in the case  $R = 2$  (see Section III-A). We consider higher order cumulants and define the following  $R$ -th order cross-cumulant:

$$C_{s_{i_1} s_{i_2}}^{\mathbf{r}}(\mathbf{n}) = \text{Cum} \{s_{i_1}(n), s_{i_2}(n-n_1), r_1(n-n_2), \dots, r_{(R-2)}(n-n_2)\} \quad (4)$$

Note that it exists and depends on  $\mathbf{n} = (n_1, n_2)$  only according to part (iii) of assumption **A0**. We will write in detail  $C_{s_{i_1} s_{i_2}}^{\mathbf{r}}(n_1, n_2)$  instead of  $C_{s_{i_1} s_{i_2}}^{\mathbf{r}}(\mathbf{n})$  only when necessary.

We consider a multidimensional  $z$ -transform of order two w.r.t. the variables  $\mathbf{n} = (n_1, n_2)$ . It is defined by:

$$C_{s_{i_1} s_{i_2}}^{\mathbf{r}}[\mathbf{z}] = \sum_{\mathbf{n} \in \mathbb{Z}^2} C_{s_{i_1} s_{i_2}}^{\mathbf{r}}(\mathbf{n}) z_1^{-n_1} z_2^{-n_2} \quad (5)$$

where  $\mathbf{z} = (z_1, z_2) \in (\mathbb{C}^*)^2$ . For the signals  $\mathbf{x}(n)$  and  $\mathbf{y}(n)$ , the cumulants  $C_{x_{i_1} x_{i_2}}^{\mathbf{r}}(\mathbf{n})$ ,  $C_{y_{i_1} y_{i_2}}^{\mathbf{r}}(\mathbf{n})$  and their respective  $z$ -transforms  $C_{x_{i_1} x_{i_2}}^{\mathbf{r}}[\mathbf{z}]$ ,  $C_{y_{i_1} y_{i_2}}^{\mathbf{r}}[\mathbf{z}]$  are defined similarly to the definitions given by (4) and (5). The matrix transfer function  $\mathbf{M}[z_1]$  of the mixing MIMO channel is given by the following  $z$ -transform

$$\mathbf{M}[z_1] = \sum_{n \in \mathbb{Z}} \mathbf{M}(n) z_1^{-n}. \quad (6)$$

A similar definition holds for the matrix transfer function  $\mathbf{W}[z_1]$  of the separator and it is known that the matrix transfer function of the global system corresponding to (3) reads  $\mathbf{G}[z_1] = \mathbf{W}[z_1] \mathbf{M}[z_1]$ . Let us introduce the matrices  $\mathbf{C}_x^{\mathbf{r}}[\mathbf{z}]$  and  $\mathbf{C}_s^{\mathbf{r}}[\mathbf{z}]$  whose  $(i, j)$  components are  $C_{x_i x_j}^{\mathbf{r}}[\mathbf{z}]$  and  $C_{s_i s_j}^{\mathbf{r}}[\mathbf{z}]$  respectively. The following proposition will be useful in section III.

*Proposition 1:* We have the following decomposition of matrix  $\mathbf{C}_x^{\mathbf{r}}[\mathbf{z}]$

$$\mathbf{C}_x^{\mathbf{r}}[\mathbf{z}] = \mathbf{M}[z_1 z_2] \mathbf{C}_s^{\mathbf{r}}[\mathbf{z}] \mathbf{M}[z_1^{-1}]^T. \quad (7)$$

*Proof:* Using (1), we have component-wise  $x_i(n) = \sum_{j \in \mathcal{I}_N} \sum_{k \in \mathbb{Z}} M_{ij}(k) s_j(n-k)$  and thus, using the multilinearity property of cumulants,

$$C_{x_{i_1} x_{i_2}}^{\mathbf{r}}(\mathbf{n}) = \sum_{j \in \mathcal{I}_N} \sum_{k \in \mathbb{Z}^2} M_{i_1 j_1}(k_1) M_{i_2 j_2}(k_2) C_{s_{j_1} s_{j_2}}^{\mathbf{r}}(n_1 + k_2 - k_1, n_2 - k_1). \quad (8)$$

Now using definition (5) for the signal  $\mathbf{x}(n)$ , we have

$$C_{x_{i_1} x_{i_2}}^{\mathbf{r}}[\mathbf{z}] = \sum_{\mathbf{n} \in \mathbb{Z}^2} \sum_{\mathbf{j} \in \mathcal{T}_N^2} \sum_{\mathbf{k} \in \mathbb{Z}^2} M_{i_1 j_1}(k_1) M_{i_2 j_2}(k_2) \quad (9)$$

$$C_{s_{j_1} s_{j_2}}^{\mathbf{r}}(n_1 + k_2 - k_1, n_2 - k_1) z_1^{-n_1} z_2^{-n_2} \\ = \sum_{\mathbf{n} \in \mathbb{Z}^2} \sum_{\mathbf{j} \in \mathcal{T}_N^2} \sum_{\mathbf{k} \in \mathbb{Z}^2} M_{i_1 j_1}(k_1) M_{i_2 j_2}(k_2) \quad (10)$$

$$C_{s_{j_1} s_{j_2}}^{\mathbf{r}}(n_1, n_2) z_1^{-n_1 + k_2 - k_1} z_2^{-n_2 - k_1} \\ = \sum_{\mathbf{j} \in \mathcal{T}_N^2} \sum_{\mathbf{k} \in \mathbb{Z}^2} M_{i_1 j_1}(k_1) M_{i_2 j_2}(k_2) \quad (11)$$

$$C_{s_{j_1} s_{j_2}}^{\mathbf{r}}[\mathbf{z}] z_1^{k_2 - k_1} z_2^{-k_1} \\ = \sum_{\mathbf{j} \in \mathcal{T}_N^2} M_{i_1 j_1}[z_1 z_2] M_{i_2 j_2}[z_1^{-1}] C_{s_{j_1} s_{j_2}}^{\mathbf{r}}[\mathbf{z}] \quad (12)$$

Component-wise, this is the result given by (7) in the proposition. ■

### III. HIGHER-ORDER IDENTIFIABILITY CONDITIONS

Based on the properties given in the previous section, we generalize in this section identifiability conditions of a FIR MIMO system. We first illustrate how second-order statistics appear as a specific case of the results stated above.

#### A. Second-order statistics

Second order statistics appear formally as a particular case of the preceding results when  $R = 2$ . Equation (4) where there is no reference signal simply corresponds to the correlation for which we adopt the specific notation:

$$\Gamma_{s_{i_1} s_{i_2}}(n_1) \triangleq \text{Cum} \{s_{i_1}(n), s_{i_2}(n - n_1)\} \\ \Gamma_{\mathbf{s}}(n_1) = (\Gamma_{s_{i_1} s_{i_2}}(n_1))_{(i_1, i_2) \in \mathcal{T}_N^2} \quad (13)$$

Corresponding to (5), we have the power spectral matrix of the sources defined by:

$$\Gamma_{\mathbf{s}}[z_1] \triangleq \sum_{n_2 \in \mathbb{Z}} \Gamma_{\mathbf{s}}(n_1) z_1^{-n_1} = (\Gamma_{s_{i_1} s_{i_2}}[z_1])_{(i_1, i_2) \in \mathcal{T}_N^2} \quad (14)$$

Note that contrary to (7),  $z_2$  does not appear here. Similar notations hold for the observations  $\mathbf{x}(n)$  ( $\Gamma_{\mathbf{x}}(n_1)$  and  $\Gamma_{\mathbf{x}}[z_1]$  respectively). Similarly to (7), we now have the well-known relation:

$$\Gamma_{\mathbf{x}}[z_1] = \mathbf{M}[z_1] \Gamma_{\mathbf{s}}[z_1] \mathbf{M}[z_1^{-1}]^{\text{T}} \quad (15)$$

#### B. Identifiability of one source

For a given vector-valued reference signal  $\mathbf{r}(n)$ , we first explicit a condition on  $\mathbf{C}_{\mathbf{s}}^{\mathbf{r}}[\mathbf{z}], i \in \{1, \dots, N\}$  to be able to identify one source of the mixture. The following proposition sums up our main result:

*Proposition 2:* Assume that

- A1. the matrices  $\Gamma_{\mathbf{s}}[z_1]$  and  $\mathbf{C}_{\mathbf{s}}^{\mathbf{r}}[\mathbf{z}]$  are diagonal,
- A2. there exist a diagonal element of  $\mathbf{C}_{\mathbf{s}}^{\mathbf{r}}[\mathbf{z}]$ , say the  $i_1$ th, which is a function distinct from all other diagonal elements.

Then, the  $i_1$ th source of the mixture is identifiable in the following sense:

For any  $N \times Q$  matrix  $\mathbf{W}[z_1]$  such that the  $N \times N$  combined channel-equalizer  $z$ -transform matrix  $\mathbf{G}[z_1] = \mathbf{W}[z_1] \mathbf{M}[z_1]$  is irreducible, if  $\mathbf{W}[z_1] \Gamma_{\mathbf{x}}[z_1] \mathbf{W}[z_1^{-1}]^{\text{T}}$  and  $\mathbf{W}[z_1 z_2] \mathbf{C}_{\mathbf{x}}^{\mathbf{r}}[\mathbf{z}] \mathbf{W}[z_1^{-1}]^{\text{T}}$  are both diagonal, then:

$$\mathbf{G}[z_1] = \mathbf{P} \begin{pmatrix} & 0 & \\ \tilde{\mathbf{Q}}_{11} & \vdots & \tilde{\mathbf{Q}}_{12} \\ 0 \dots 0 & \lambda_1 & 0 \dots 0 \\ & \vdots & \\ \tilde{\mathbf{Q}}_{21} & \vdots & \tilde{\mathbf{Q}}_{22} \\ & 0 & \end{pmatrix} \quad (16)$$

where  $\lambda_1 \neq 0$  is situated at the  $i_1$ th row and  $i_1$ th column and  $\mathbf{P}$  is a permutation matrix.

*Proof:* One can write the assumptions of the above proposition as follows:

$$\mathbf{G}[z_1] \Gamma_{\mathbf{s}}[z_1] \mathbf{G}[z_1^{-1}]^{\text{T}} = \hat{\Gamma}_{\mathbf{s}}[z_1] \quad (17)$$

$$\mathbf{G}[z_1 z_2] \mathbf{C}_{\mathbf{s}}^{\mathbf{r}}[\mathbf{z}] \mathbf{G}[z_1^{-1}]^{\text{T}} = \hat{\mathbf{C}}_{\mathbf{s}}^{\mathbf{r}}[\mathbf{z}] \quad (18)$$

where  $\hat{\Gamma}_{\mathbf{s}}[z_1]$  and  $\hat{\mathbf{C}}_{\mathbf{s}}^{\mathbf{r}}[\mathbf{z}]$  are diagonal matrices and  $\hat{\Gamma}_{\mathbf{s}}[z_1]$  satisfies the conditions associated to a power spectral  $z$ -transform matrix (that is  $\hat{\Gamma}_{\mathbf{s}}[z_1]^{\text{T}} = \hat{\Gamma}_{\mathbf{s}}[z_1^{-1}]$  and  $\hat{\Gamma}_{\mathbf{s}}[e^{j\omega}] \geq 0, \forall \omega \in [0, 2\pi]$ ). Hence, following the proof in [11], one deduces from (17) that  $\mathbf{G}[z_1]$  is a constant orthogonal matrix, say  $\mathbf{G}[z_1] = \mathbf{Q}$  and (18) reads:

$$\mathbf{Q} \mathbf{C}_{\mathbf{s}}^{\mathbf{r}}[\mathbf{z}] = \hat{\mathbf{C}}_{\mathbf{s}}^{\mathbf{r}}[\mathbf{z}] \mathbf{Q} \quad (19)$$

Denote by  $Q_{mn}$  the  $(m, n)$ th element of  $\mathbf{Q}$  and by  $C_n[\mathbf{z}]$  (resp.  $\hat{C}_n[\mathbf{z}]$ ) the  $n$ th element of the diagonal matrix  $\mathbf{C}_{\mathbf{s}}^{\mathbf{r}}[\mathbf{z}]$  (resp.  $\hat{\mathbf{C}}_{\mathbf{s}}^{\mathbf{r}}[\mathbf{z}]$ ). In the above equation, the term at the  $(m, n)$ th position reads  $Q_{mn} C_n[\mathbf{z}] = \hat{C}_m[\mathbf{z}] Q_{mn}$ . Because of the orthogonality, there is a non-zero element  $Q_{\sigma(i_1) i_1}$  in the  $i_1$ th column. Based on the assumption that  $C_{i_1}[\mathbf{z}]$  is distinct from all other diagonal elements  $C_n[\mathbf{z}], n \neq i_1$ , one can obtain that on the same row  $Q_{\sigma(i_1) n} = 0$  if  $n \neq i_1$ . The orthogonality of  $\mathbf{Q}$  then implies that in the  $i_1$ th column,  $Q_{n i_1} = 0$  for  $n \neq \sigma(i_1)$ . This shows that the matrix  $\mathbf{G}[z_1]$  has the desired structure. ■

One can notice that in Proposition 2, we have also that the matrix  $\begin{pmatrix} \tilde{\mathbf{Q}}_{11} & \tilde{\mathbf{Q}}_{12} \\ \tilde{\mathbf{Q}}_{21} & \tilde{\mathbf{Q}}_{22} \end{pmatrix}$  is orthogonal. Additionally, two important comments should be made to link the above proposition with usual assumptions.

First, note that the assumption that  $\Gamma_{\mathbf{s}}[z_1]$  be diagonal is equivalent to the assumptions that the sources be uncorrelated. Similarly, the assumption that  $\mathbf{C}_{\mathbf{s}}^{\mathbf{r}}[\mathbf{z}]$  is diagonal corresponds to the classical assumption that the sources be independent, although both are not equivalent. It has indeed been illustrated in [2] through several examples that mutual independence is a sufficient condition for separation, but is not always necessary. In the present formulation, we assume a weaker condition than independence. This allows us to deal with cases such where the sources are possibly spatially mutually dependent, such as in [2].

Consider also the situation where one or more diagonal elements of  $\mathbf{C}_{\mathbf{s}}^{\mathbf{r}}[\mathbf{z}]$  vanish, say for example  $C_{s_{i_1} s_{i_1}}^{\mathbf{r}}[\mathbf{z}] = 0$ : in this case, and according to Assumption (ii) in Proposition 2, the  $i_1$ -th source is identifiable only if there is no other zero element on the diagonal of  $\mathbf{C}_{\mathbf{s}}^{\mathbf{r}}[\mathbf{z}]$ . In Section V-B,

<sup>1</sup>Similarly to [11], only rational spectra are considered in the paper, that is  $\Gamma_{\mathbf{s}}[z_1]$  and  $\hat{\Gamma}_{\mathbf{s}}[z_1]$  have rational elements.

it will be illustrated how in the case of i.i.d. signals, this condition implies the well-known necessary condition that at most one source may have vanishing  $R$ -th order cumulants. The condition given in Proposition 2 is hence more widely satisfied and weaker than the usual one: indeed an example has been given in [5] of non i.i.d. sources, which auto-cumulants with no delay vanish whereas the cross-cumulants at different times are non zero: our result confirm that a mixture of these sources can be separated.

### C. Identifiability of all sources

We have shown in the previous section that it is possible to identify one source under a condition on the reference signal  $\mathbf{r}(n)$ . We now extend the result and explicit a condition to be satisfied so as to ensure that all sources can be identified. For that, we consider  $N$  reference signals  $\mathbf{r}_i(n)$ ,  $i = 1, \dots, N$  which may possibly be the same ones.

*Proposition 3:* Assume that there exist  $N$  vector-valued reference signals  $\mathbf{r}_i(n)$ ,  $i = 1, \dots, N$  and a permutation  $\sigma$  of the set  $\mathcal{I}_N$  such that the following condition hold:

- A3. the matrices  $\Gamma_{\mathbf{s}}[z_1]$  and  $\mathbf{C}_{\mathbf{s}}^{\mathbf{r}_i}[\mathbf{z}]$  for all  $i \in \mathcal{I}_N$  are all diagonal,
- A4. for all  $i \in \mathcal{I}_N$  the  $\sigma(i)$ th diagonal element of  $\mathbf{C}_{\mathbf{s}}^{\mathbf{r}_i}[\mathbf{z}]$  is a function distinct from all other diagonal elements.

Then, the  $N$  sources are identifiable up to a permutation and scaling factor in the following sense:

For any  $N \times Q$  matrix  $\mathbf{W}[z_1]$  such that the  $N \times N$  combined channel-equalizer  $z$ -transform matrix  $\mathbf{G}[z_1] = \mathbf{W}[z_1]\mathbf{M}[z_1]$  is irreducible, if  $\mathbf{W}[z_1]\Gamma_{\mathbf{x}}[z_1]\mathbf{W}[z_1^{-1}]^T$  and  $\mathbf{W}[z_1 z_2]\mathbf{C}_{\mathbf{x}}^{\mathbf{r}_j}[\mathbf{z}]\mathbf{W}[z_1^{-1}]^T$ ,  $j \in \{1, \dots, N\}$  are all diagonal, then:

$$\mathbf{G}[z_1] = \mathbf{P}\mathbf{A} \quad (20)$$

where  $\mathbf{P}$  is a permutation matrix and  $\mathbf{A}$  is a diagonal matrix with non zero elements on the diagonal.

*Proof:* Using the same arguments as in the proof of Proposition 2 one obtains that  $\mathbf{G}[z_1] = \mathbf{Q}$  is a constant orthogonal matrix. Under the assumptions of the proposition, we have in addition:

$$\forall j \in \{1, \dots, N\} \quad \mathbf{Q}\mathbf{C}_{\mathbf{x}}^{\mathbf{r}_j}[\mathbf{z}] = \hat{\mathbf{C}}_{\mathbf{x}}^{\mathbf{r}_j}[\mathbf{z}]\mathbf{Q} \quad (21)$$

Using arguments similar to the previous ones in the proof of Proposition 2, one obtains by induction that  $\mathbf{G}[z_1]$  has the expected form:  $\mathbf{G}[z_1] = \mathbf{P}\mathbf{A}$ . ■

Let us make two remarks.

**Remark 1:** The reference signals  $\mathbf{r}_1(n), \dots, \mathbf{r}_N(n)$  may possibly be the same. In particular, if  $\mathbf{r}_1(n)$  is such that the elements of  $\mathbf{C}_{\mathbf{x}}^{\mathbf{r}_1}[\mathbf{z}]$  are mutually different, then one can consider  $\mathbf{r}_1(n) = \mathbf{r}_2(n) = \dots = \mathbf{r}_N(n)$  and the reference signals  $\{\mathbf{r}_1(n), \dots, \mathbf{r}_N(n)\}$  satisfy the condition of the proposition. In this case,  $\mathbf{C}_{\mathbf{x}}^{\mathbf{r}_1}[\mathbf{z}]$  and  $\Gamma_{\mathbf{x}}[z_1]$  are enough to separate the sources. Similarly, each vector-valued reference signal  $\mathbf{r}_j(n)$  (for  $j$  fixed) is constituted of  $R-2$  components, some or all of which may possibly be identical.

**Remark 2:** One could state a weaker condition instead of A4 and replace it with:

- A4<sup>o</sup>. for all  $i \in \mathcal{I}_N$  the  $\sigma(i)$ th element of the diagonal

matrix  $\mathbf{C}_{\mathbf{x}}^{\mathbf{r}_i}[\mathbf{z}]$  is a function distinct from the diagonal elements which indices are not in  $\{\sigma(1), \dots, \sigma(i-1)\}$ . The condition A4<sup>o</sup> can be easily interpreted: it says merely that the  $\sigma(i)$ th source can be identified by using the reference  $\mathbf{r}_i(n)$ , and that the  $\sigma(i)$ th source has not been identified previously by either one of the references  $\mathbf{r}_1(n), \dots, \mathbf{r}_{i-1}(n)$ .

Finally, let us stress that Propositions 2 and 3 hold under the condition that  $\mathbf{C}_{\mathbf{s}}^{\mathbf{r}}[\mathbf{z}]$  is diagonal: this corresponds in a weaker form to the independence of the sources. An interesting situation where this happens is detailed in the next Section.

## IV. SEMI-BLIND REFERENCES

The semi-blind context seems one of the most interesting and promising where a reference based approach can be used. Indeed, if a reference signal is available, one can obtain an effective separation method by using existing algorithms [7], [13]. A challenging issue consists in dealing with the validity of a given reference signal and in finding sufficient condition which can be used in a practical situation. We give here some elements only. Note also that in a blind context, reference signals can be constructed from the observations only [7], [13].

### A. Diagonal matrices of reference cumulants

The conditions presented in section III require the sources be uncorrelated and  $\mathbf{C}_{\mathbf{s}}^{\mathbf{r}}[\mathbf{z}]$  be diagonal. We now give sufficient conditions on the vector-valued reference signal under which it may be used for separation. This gives a more accurate view of the results presented in [7]. We have the following proposition:

*Proposition 4:* For  $i \in \mathcal{I}_N$ , consider  $N$  signals  $\tilde{s}_i(n)$  and denote  $\tilde{s}_i(n) \triangleq (s_i(n), \tilde{s}_i(n))$ . Assume that:

- A5. there exist a  $(R-2) \times N$  filter with impulse response  $\mathbf{T}(n)$  such that:

$$\mathbf{r}(n) = \sum_{k \in \mathbb{Z}} \mathbf{T}(k)\tilde{\mathbf{s}}(n-k) \quad (22)$$

- A6. the processes  $\tilde{s}_i(n)$  are mutually independent for  $i \in \mathcal{I}_N$ .

Then, the matrix  $\mathbf{C}_{\mathbf{s}}^{\mathbf{r}}[\mathbf{z}]$  is diagonal.

*Proof:* For all  $j \in \mathcal{I}_{R-2}$ , we have  $r_j(n) = \sum_{p \in \mathcal{I}_N} \sum_{k \in \mathbb{Z}} T_{jp}(k)\tilde{s}_p(n-k)$ . Replacing  $r_j(n-n_2)$  for all  $j$  in (4) and then using multilinearity, one can write:

$$\begin{aligned} C_{s_{i_1} s_{i_2}}^{\mathbf{r}}(\mathbf{n}) &= \sum_{\mathbf{p} \in \mathcal{I}_N^{R-2}} \sum_{\mathbf{k} \in \mathbb{Z}^{R-2}} \left( \prod_{j=1}^{R-2} T_{jp_j}(k_j) \right) \times \\ &\quad \text{Cum}\{s_{i_1}(n), s_{i_2}(n-n_1), \tilde{s}_{p_1}(n-n_2-k_1), \dots, \\ &\quad \tilde{s}_{p_{R-2}}(n-n_2-k_{R-2})\} \end{aligned}$$

By the independence assumptions on the sources A6 and on the signals  $\tilde{s}_i(n)$ ,  $i \in \mathcal{I}_N$ , the above cumulant expressions systematically vanish when  $i_1 \neq i_2$ . It follows by (5) that  $\mathbf{C}_{\mathbf{s}}^{\mathbf{r}}[\mathbf{z}]$  is diagonal. ■

A specific but interesting situation where the condition of the above Proposition is satisfied is when for all  $i$ ,  $\tilde{s}_i(n) = s_i(n)$ . In other words, the references are obtained from the sources by a MIMO filtering operation. This is the case

considered in [7], and also in [13]. In particular, the observations being themselves obtained by a MIMO filtering of the sources, they can serve as reference signal upon the additional assumptions which are specified before in Propositions 2 and 3.

Another specific situation where the condition of the above Proposition are satisfied is when  $\mathbf{r}(n)$  depends on one source (say the  $i_1$ -th) and is independent of the other ones. In this case, one can observe additionally that  $\mathbf{C}_s^r[\mathbf{z}]$  contains a single non zero element: this seemingly very favorable assumption could more likely correspond to the situation where additional extra information is available which can be used to extract the  $i_1$ -th source. This semi-blind context is considered in simulations to illustrate the potentialities of our results.

### B. Simulations

We now illustrate by simulations the validity of the results in the previous section. We assume that some information is partially available on the sources through the reference signal: we assume the reference is a binary signal which depends on the sign of one source signal in a non deterministic way: such a situation could be of interest (e.g. in a telecommunication context) and, to the best of our knowledge, it has not been considered anywhere before. Another example has been treated in [3] where a partial information is available on the phase or the modulus of a complex-valued source signal.

More precisely, we deal with the case where  $\tilde{s}_i(n) = \text{sign}(s_i(n))\epsilon_i(n)$  where for all  $i$ ,  $\epsilon_i(n)$  is a binary i.i.d. Bernoulli process with  $P(\epsilon_i(n) = 1) = p$  and  $P(\epsilon_i(n) = -1) = 1 - p$ . The processes  $\epsilon_i(n)$  are mutually independent and independent of the sources. The signals  $\tilde{s}_i(n)$  then satisfy the conditions of Proposition 4 and each  $\tilde{s}_i(n)$  contains information on the sign of  $s_i(n)$ :  $s_i(n)$  and  $\tilde{s}_i(n)$  indeed have the same sign with probability given by the parameter  $p$ .

In our simulations, we tried to extract one particular source (say the  $i_1$ th) and we considered the reference signal defined by  $r(n) = \sum_k t_{i_1}(k)\tilde{s}_{i_1}(n-k)$  where  $(t_{i_1}(k)), k \in \mathbb{Z}$  is the impulse response of a scalar filter. We considered for  $(t_{i_1}(k)), k \in \mathbb{Z}$  both the identity filter and a randomly driven FIR filter of length 3.  $r(n)$  thus contains directly or indirectly information on the sign of  $s_i(n)$  and it can be seen that only the  $i_1$ th diagonal element of the corresponding matrix  $\mathbf{C}_s^r[\mathbf{z}]$  is non zero.

A set of  $N$  mutually independent and temporally i.i.d. uniformly distributed sources have been generated. They have been mixed by a  $Q \times N$  randomly chosen FIR filter of length 3. We then used the real valued fourth order cumulant-based contrast proposed in [7] and the associated algorithm to test the effectiveness of our result. Different values of the parameter  $p$  and different sample sizes have been considered. Results obtained with 1000 Monte-Carlo realizations are reported in Table I for the values  $N = 3, Q = 4$  and in Table II for  $N = 4, Q = 5$ : the averaged value and the 90% quantile value of the the mean square error on the estimated source are reported.

One can observe the effectiveness of the separation when  $p \neq 0.5$ : it should indeed be no surprise that no separation is obtained for  $p = 0.5$  since in this case,  $\tilde{s}_i(n)$  is independent

of  $s_i(n)$ , implying that the matrix  $\mathbf{C}_s^r[\mathbf{z}]$  is identically zero. On the contrary,  $p \neq 0.5$  implies that  $(s_{i_1}(n), \tilde{s}_{i_1}(n))$  are dependent, but independent of all other sources. In such considered cases, this yields a sufficient condition on the reference signals so that the methods [7], [13] can be used.

## V. LINK WITH FORMER RESULTS

### A. Second-order identifiability condition

The results presented in the previous sections are connected with the results in [11] on second-order identifiability of FIR-MIMO systems. In the case where  $R = 2$  and no reference signal is considered, the Propositions 2 and 3 lead to the result in [11]. The role played by  $\Gamma[z_1]$  and  $\mathbf{C}_s^r[\mathbf{z}]$  in Proposition 3 is played simultaneously by the power spectral matrix  $\Gamma[z_1]$  in [11]. More precisely, it is seen from the proof that using the second-order statistics only (that is the spectral matrix), the FIR-MIMO system may be identified *up to an orthogonal transform*. The remaining orthogonal transform is identified thanks to a diversity which can be satisfied either at second order (as in [11]) or at  $R$ -th order for a given set of reference signals (as in our Propositions 2 and 3). In other words, the use of reference signals allows one to consider another interesting signal diversity. It follows that our identifiability condition is actually much weaker than the usual known one, since the diversity condition may be satisfied at any given statistic order.

### B. i.i.d. sources

We now show that the identifiability condition in Proposition 2 is necessarily satisfied in the results given by [7] and in [13]. The case of non i.i.d. sources, which is treated in [7] but not in [13], is addressed in Section V-C. Both works focus mainly on the problem of extracting one source in the context where the reference signals are implicitly obtained by a filtering operation on the sources. In particular, the conditions of Proposition 4 hold true, which implies  $C_{s_{i_1} s_{i_2}}^r(\mathbf{n}) = 0$  when  $i_1 \neq i_2$ . More precisely, we have  $\tilde{\mathbf{s}}(n) = \mathbf{s}(n)$  and hence we can write:

$$\forall j \in \mathcal{I}_{R-2} \quad r_j(n) = \sum_{p \in \mathcal{I}_N} \sum_{l \in \mathbb{Z}} T_{jp}(l) s_p(n-l) \quad (23)$$

Assuming in addition that the sources are temporally i.i.d. we obtain:

$$C_{s_i s_i}^r(\mathbf{n}) = \begin{cases} 0 & \text{if } n_1 \neq 0, \\ \left( \prod_{j=1}^{R-2} T_{ji}(-n_2) \right) \mathcal{K}_{s_i}^{(R)} & \text{otherwise.} \end{cases} \quad (24)$$

where  $\mathcal{K}_{s_i}^{(R)}$  denotes the  $R$ -th order auto-cumulant  $\text{Cum}\{s_i(n), \dots, s_i(n)\}$ , which does not depend on  $n$  because of stationarity (assumption A0). It follows that

$$\begin{aligned} C_{s_i s_i}^r[\mathbf{z}] &= \sum_{n_2 \in \mathbb{Z}} C_{s_i s_i}^r(0, n_2) z_2^{-n_2} \\ &= \mathcal{K}_{s_i}^{(R)} \sum_{n_2 \in \mathbb{Z}} \left[ \left( \prod_{j=1}^{R-2} T_{ji}(-n_2) \right) z_2^{-n_2} \right]. \end{aligned} \quad (25)$$

|               | Number of samples | $(t_{i_1}(n)) \leftrightarrow \text{identity}$ |           |           |           | $(t_{i_1}(n)) \leftrightarrow \text{FIR random}$ |           |           |           |
|---------------|-------------------|--|-----------|-----------|-----------|--|-----------|-----------|-----------|
|               |                   | $p = 1$  | $p = 0.9$ | $p = 0.7$ | $p = 0.5$ | $p = 1$  | $p = 0.9$ | $p = 0.7$ | $p = 0.5$ |
| Average value | 1000              | 0.0078   | 0.0249    | 0.1548    | 1.0780    | 0.0379   | 0.0945    | 0.7079    | 1.1340    |
|               | 5000              | 0.0015   | 0.0050    | 0.0331    | 1.0799    | 0.0130   | 0.0248    | 0.2470    | 1.1303    |
|               | 10000             | 7.57e-4  | 0.0025    | 0.0168    | 1.0762    | 0.0073   | 0.0142    | 0.1383    | 1.1335    |
| 90% quantile  | 1000              | 0.0109   | 0.0345    | 0.2197    | 1.2415    | 0.0770   | 0.2186    | 1.1956    | 1.2653    |
|               | 5000              | 0.0022   | 0.0069    | 0.0466    | 1.2393    | 0.0151   | 0.0424    | 0.5853    | 1.2585    |
|               | 10000             | 0.0010   | 0.0035    | 0.0238    | 1.2400    | 0.0062   | 0.0190    | 0.3369    | 1.2647    |

TABLE I  
AVERAGE AND 90% QUANTILE VALUE (1000 REALIZATIONS) OF THE MSE ON THE RECONSTRUCTED SOURCE (N=3,Q=4).

|               | Number of samples | $(t_{i_1}(n)) \leftrightarrow \text{identity}$ |           |           |           | $(t_{i_1}(n)) \leftrightarrow \text{FIR random}$ |           |           |           |
|---------------|-------------------|--|-----------|-----------|-----------|--|-----------|-----------|-----------|
|               |                   | $p = 1$  | $p = 0.9$ | $p = 0.7$ | $p = 0.5$ | $p = 1$  | $p = 0.9$ | $p = 0.7$ | $p = 0.5$ |
| Average value | 1000              | 0.0130   | 0.0416    | 0.2403    | 1.2262    | 0.0498   | 0.1338    | 0.9502    | 1.2206    |
|               | 5000              | 0.0026   | 0.0084    | 0.0545    | 1.2290    | 0.0148   | 0.0330    | 0.3938    | 1.1077    |
|               | 10000             | 0.0013   | 0.0042    | 0.0280    | 1.2298    | 0.0104   | 0.0234    | 0.2285    | 1.0544    |
| 90% quantile  | 1000              | 0.0169   | 0.0547    | 0.3181    | 1.3586    | 0.0859   | 0.2562    | 1.3550    | 1.3637    |
|               | 5000              | 0.0034   | 0.0110    | 0.0717    | 1.3548    | 0.0156   | 0.0512    | 0.9503    | 1.3513    |
|               | 10000             | 0.0017   | 0.0055    | 0.0365    | 1.3595    | 0.0093   | 0.0333    | 0.5211    | 1.3622    |

TABLE II  
AVERAGE AND 90% QUANTILE VALUE (1000 REALIZATIONS) OF THE MSE ON THE RECONSTRUCTED SOURCE (N=4,Q=5).

a) *Link with [7]:* Define

$$\mathcal{C}_R^{\max} \triangleq \max_{i=1}^N \sup_{k \in \mathbb{Z}} |C_{s_i s_i}^{\mathbf{r}}(0, k)| \quad (26)$$

Assume as in [7] that the above supremum is finite and uniquely reached, that is

$$|C_{s_{i_1} s_{i_1}}^{\mathbf{r}}(0, k_1)| = \mathcal{C}_R^{\max} < \infty \text{ and:} \\ \forall (i, k) \neq (i_1, k_1) \quad |C_{s_i s_i}^{\mathbf{r}}(0, k)| < \mathcal{C}_R^{\max}. \quad (27)$$

Considering (25) one can then see that for  $i \neq i_1$ ,  $C_{s_i s_i}^{\mathbf{r}}[\mathbf{z}]$  and  $C_{s_{i_1} s_{i_1}}^{\mathbf{r}}[\mathbf{z}]$  are distinct functions. The source  $s_{i_1}(n)$  is hence identifiable according to Proposition 2. This is confirmed by the results in [7] which show the validity of a particular contrast function. The validity condition for the contrast function in [7] is hence a much stronger condition than the identifiability condition which is given here.

Having a closer look at Equation (25), one can distinguish the importance of the identifiability conditions at two distinct levels. On the one hand, the cumulant values play a fundamental role: indeed, the source  $i_1$  can be extracted if and only if the diagonal element  $C_{s_{i_1} s_{i_1}}^{\mathbf{r}}[\mathbf{z}]$  of  $\mathbf{C}_s^{\mathbf{r}}[\mathbf{z}]$  is distinct from the others. From (25), this necessarily implies that either  $\mathcal{K}_{s_{i_1}}^{(R)}$  be non zero, or that source  $i_1$  be the only source with vanishing  $R$ -th order cumulant. This corresponds to the well-known condition that at most one source may be Gaussian. On the other hand, as soon as  $\mathcal{K}_{s_{i_1}}^{(R)} \neq 0$ , there always exist an appropriate choice of the reference system in (25) which makes possible the separation of the corresponding source.

b) *Link with [13]:* In [13], fourth-order cumulants ( $R = 4$ ) are considered and it is assumed that the reference  $\mathbf{r}(n)$  consists of two identical components  $r_1(n) = r_2(n) \triangleq r(n)$ , where  $r(n)$  is written as

$$r(n) = \sum_{p \in \mathcal{I}_N} \sum_{l \in \mathbb{Z}} t_p(l) s_p(n-l) \quad (28)$$

Then, one can easily obtain:

$$C_{s_i s_i}^{\mathbf{r}}[\mathbf{z}] = \mathcal{K}_{s_i}^{(4)} \sum_{n_2 \in \mathbb{Z}} t_i(-n_2)^2 z_2^{-n_2} \quad (29)$$

It is shown in [13] that if all the values  $\lambda_i(k) \triangleq |t_i(k)|^2 \mathcal{K}_{s_i}^{(4)}$  are distinct for  $i \in \mathcal{I}_N$  and  $k \in \mathbb{Z}$ , then the corresponding source can be recovered. Clearly, distinct values of  $\lambda_i(k)$  correspond to distinct rational functions  $C_{s_i s_i}^{\mathbf{r}}[\mathbf{z}]$ . Proposition 2 and 3 hence show that the separating system can be identified under conditions weaker than those in [13]. It is sufficient that there exist  $N$  choices of  $\mathbf{r}(n)$  such that the functions in (25) (or (29)) are different for different  $i$ . This seems quite a weak condition on the reference system  $(\mathbf{T}(k))_{k \in \mathbb{Z}}$ . From (29), it is seen in addition that the condition on the reference system does not depend on the particular values of the auto-cumulants of the sources, if different from zero.

C. *non i.i.d. sources*

We now move to the case of non i.i.d. sources. This case has been considered in [7]. Let us show that the validity conditions in [7] imply that the identifiability condition given in this paper is satisfied. Define  $y_i(n)$  as the result of a scalar filter applied on  $s_i(n)$ . If  $g_i[z_1]$  is the corresponding  $z$ -transform of the scalar filter, we have according to (7):

$$C_{y_i y_i}^{\mathbf{r}}[\mathbf{z}] = g_i[z_1^{-1}] g_i[z_1 z_2] C_{s_i s_i}^{\mathbf{r}}[\mathbf{z}] \quad (30)$$

In [7], the quantity  $|C_{y_i y_i}^{\mathbf{r}}(0, 0)|$  is considered when  $y_i(n)$  is the particular output corresponding to the maximum value of this quantity over a set of unit-norm filters (the supremum are assumed to be reached and finite). The validity condition in [7] expresses that there should be an index (say the index  $i_1$ ) such that  $|C_{y_{i_1} y_{i_1}}^{\mathbf{r}}(0, 0)| > |C_{y_i y_i}^{\mathbf{r}}(0, 0)|$  for all  $i \neq i_1$ . In this case precisely, we must have  $C_{y_{i_1} y_{i_1}}^{\mathbf{r}}[\mathbf{z}] \neq C_{y_i y_i}^{\mathbf{r}}[\mathbf{z}]$  and thus  $C_{s_{i_1} s_{i_1}}^{\mathbf{r}}[\mathbf{z}] \neq C_{s_i s_i}^{\mathbf{r}}[\mathbf{z}]$  according to (30). Again, Proposition 2 hence provides weaker identifiability conditions than the results in [7].

## VI. CONCLUSION

General identifiability conditions of a separating system have been given by considering reference signals in higher order statistics. In particular, our new results include former

existing ones. Known results based on second-order statistics have been generalized to higher-order statistics and we have shown that the given identifiability conditions are weaker than the assumptions required in former papers in which reference-based approaches have been considered. The interest of our approach has been clearly illustrated in a semi-blind context. These new results, although theoretical ones, open up the possibility of developing novel algorithms.

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