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FOCUSING WAVE GROUP IN SHALLOW WATER IN THE PRESENCE OF WIND

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ABSTRACT. The influence of wind on extreme wave events in shallow water is investigated numerically. A series of numerical simulations using a pressure distribution over the steep crests given by the modified Jeffreys' sheltering theory shows that wind blowing over a strongly modulated wave group due to the dispersive focusing of a chirped long wave packet increases the time duration and maximal amplitude of the extreme wave event. These results are coherent with those obtained within the framework of deep water. However, steep wave events are less unstable to wind perturbation in shallow water than in deep water.

1. Introduction. Extreme wave events are due to the focusing of wave energy into a small area. The main physical mechanisms which may produce extreme wave events are described and discussed in detail in Kharif et al 2009 [1]. Extreme wave occurrence on currents is a well-understood problem that can explain the formation of rogue waves when wind waves or swells are propagating against a current. Geometrical or spatial focusing may result in large amplitude waves. The spatio-temporal wave focusing due to the dispersive nature of water waves is a classic mechanism yielding wave energy concentration in a small area. Nonlinear uniform wave trains suffer an instability known as the Benjamin-Feir instability, which produces growing modulations of the envelope that evolve into short groups of steep waves. A uniform wave train under modulational instability transforms into a system of envelope that may collide to give rise to huge wave events.

Most of the studies have considered the extreme wave occurrence without including the direct effect of the wind on their dynamics. To our knowledge, there are few published results about this coupling. Among the recent papers on wind forcing on large amplitude deep water waves, one can cite Banner and Song 2002 [2], Touboul et al 2006 [3] and Kharif et al 2008 [4]. Within the framework of extreme wave events due to the spatio-temporal focusing and Benjamin-Feir instability, the two latter papers have shown experimentally and numerically that wind sustains steep waves which then evolve into breaking waves. More specifically, wind increases both the amplitude and life-time of steep wave events. Wind effect play a significant role in the persistence of extreme wave events. Until now there is no investigation on wind effect on the formation of extreme wave events in shallow water. Within

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the framework of one-dimensional propagation it is well known that the Benjamin-Feir instability stabilizes when kh becomes less than 1.363, where k and h are the wavenumber and depth respectively. Hence we shall use the spatio-temporal wave focusing (or dispersive focusing) for generating extreme wave events. The purpose of the present study is to extend the results of Touboul et al 2006 [3] and Kharif et al 2008 [4] obtained in deep water to extreme wave events with wind forcing in shallow water. Note that rogue occurrence in shallow water is discussed in Kharif et al 2009 [1], but without considering wind/wave coupling.

In Sect. 2 the mathematical statement of the water wave problem is presented as well as the wind modelling based on the sheltering mechanism. The numerical method is briefly described too. An inverse method is used to determine the initial conditions. Sect. 3 reports on numerical simulations of extreme wave events due to dispersive focusing in the presence of wind and a discussion is given.

2. Mathematical formulation. The problem is solved by assuming that the fluid is inviscid, incompressible, and the motion irrotational. Hence the velocity field is given by $\mathbf{u} = \nabla\phi$ where the velocity potential $\phi(x, z, t)$ satisfies the Laplace's equation which is solved in a domain bounded by the free surface, a horizontal solid bottom and two vertical solid walls located at the ends of the numerical domain. The horizontal and vertical coordinates are x and z respectively whereas t is time. The still-water level lies at $z = 0$, and the horizontal impermeable bed lies at $z = -h$. The dynamic free surface condition states that the pressure at the surface, $z = \eta(x, t)$, is equal to the atmospheric pressure p_a . Assuming the free surface to be impermeable, the problem to be solved is the Laplace equation with the kinematic, dynamic and bottom conditions.

$$\left\{ \begin{array}{ll} \Delta\phi = 0 & \text{for } -h < z < \eta(x, t), \\ \frac{\partial\eta}{\partial t} + \frac{\partial\eta}{\partial x} \frac{\partial\phi}{\partial x} - \frac{\partial\phi}{\partial z} = 0 & \text{on } z = \eta(x, t), \\ \frac{\partial\phi}{\partial t} + \frac{1}{2}(\nabla\phi)^2 + g\eta = -\frac{p_a}{\rho_\omega} & \text{on } z = \eta(x, t), \\ \frac{\partial\phi}{\partial n} = 0 & \text{on } z = -h, \end{array} \right. \quad (1)$$

where g is the acceleration due to gravity and ρ_ω is the water density.

A Boundary Integral Equation Method (BIEM) is used to solve the system of equations (1) with a mixed Euler Lagrange (MEL) time marching scheme. For more details about this numerical method see the paper by Touboul et al 2006 [3].

The Green's second identity is used to solve Laplace's equation for the velocity potential

$$\begin{aligned} c(Q)\phi(Q) &= \int_{\partial\Omega} \phi(P) \frac{\partial G}{\partial n}(P, Q) dl \\ &\quad - \int_{\partial\Omega} \frac{\partial\phi}{\partial n}(P) G(P, Q) dl, \end{aligned} \quad (2)$$

where G is the free space Green's function. The fluid domain boundary $\partial\Omega$ is $\partial\Omega_F \cup \partial\Omega_B$, the union of the free surface $\partial\Omega_F$ and solid boundaries $\partial\Omega_B$. The unit normal vector \mathbf{n} points outside the fluid domain. The unknowns are $\partial\phi/\partial n$ on $\partial\Omega_F$ and ϕ on $\partial\Omega_B$. Let P , Q and $c(Q)$ denote two points of the domain and the angle

between two consecutive panels respectively. The angle $c(Q)$ is defined as follows

$$c(Q) = \begin{cases} 0 & \text{if } Q \text{ is outside the fluid domain,} \\ \alpha & \text{if } Q \text{ is on the boundary,} \\ -2\pi & \text{if } Q \text{ is inside the fluid domain,} \end{cases} \quad (3)$$

where α is the inner angle relative to the fluid domain at point Q along the boundary.

Time stepping is performed using a fourth order Runge & Kutta scheme, with a constant time step.

Extreme wave events that are due to spatio-temporal focusing phenomenon can be described as follows. If initially wave packets are located in front of longer wave packets having larger group velocities, then during the stage of evolution, longer waves will overtake shorter waves. A large-amplitude wave can occur at some fixed time because of superposition of all the waves merging at a given location (the focus point). Afterwards, the longer waves will be in front of the shorter waves, and the amplitude of the wave train will decrease. In shallow water the waves are weakly dispersive, hence to obtain a steep wave event during a relatively short period of time it is necessary to use amplitude modulated wave train too. Longer wave packets should have larger amplitude (see figure 2). This focusing-defocusing cycle was described by Pelinovsky et al 2000 [5] within the framework of the shallow water theory. Later, Slunyaev et al 2002 [6] used the Davey-Stewartson system for three-dimensional water waves propagating in finite depth. More recently, this technique was also used in the experiments on extreme waves conducted by Touboul et al 2006 [3] and Kharif et al 2008 [4].

The initial conditions $(\eta(x, 0), \phi(x, \eta, 0))$ of the system of equations (1) evolving into an extreme wave are obtained by using the KdV equation and an inverse method described in the next section.

2.1. The Korteweg de Vries equation and inverse method. The Korteweg-de Vries equation is an approximate equation which may be derived from the fully nonlinear equations (1) by using a multiple scale method. The dependent variables η and ϕ are expanded as a power series of a/h with $(kh)^2 = \mathcal{O}(a/h)$, where a is a measure of the wave amplitude, k the wavenumber and h the water depth. The KdV equation which describes the leading-order contribution to the surface wave is

$$\frac{\partial \eta}{\partial t} + c_0 \left(1 + \frac{3\eta}{2h}\right) \frac{\partial \eta}{\partial x} + \frac{c_0 h^2}{6} \frac{\partial^3 \eta}{\partial x^3} = 0, \quad (4)$$

where $c_0 = \sqrt{gh}$.

This universal equation describes the spatio-temporal evolution of weakly nonlinear and weakly dispersive waves. This equation is invariant under the transformation $x \rightarrow -x$, $t \rightarrow -t$. Hence, we can choose the expected extreme wave event as the initial condition for equation (4) and calculate $\eta(x, t)$ for any time T . Due to invariance of equation (4) by reversal of abscissa, the initial condition $\eta(x, T)$ evolves to the expected steep wave event. Pelinovsky et al 2000 [5] used this inverse method with KdV equation to study the rogue wave formation in shallow water. Following Pelinovsky et al 2000 [5] we choose as initial condition representing the expected steep wave event the following Gaussian impulse

$$\eta(x, 0) = A \exp(-K^2 x^2), \quad (5)$$

where A is the amplitude and K^{-1} is the initial width of the wave.

The evolution of this initial condition for equation (4) is given in figure 1 with $A = 0.20 m$, $K^{-1} = 1.8 m$ and the depth $h = 1 m$. A pseudo-spectral method is

used to compute numerically the surface elevation at different instants of time. At time T , a soliton is formed ahead of dispersive wave train. As expected, the soliton with the modulated wave train ahead turns into the Gaussian impulse under the inverse transformation.

The profile $\eta(x, T)$ obtained from the KdV equation with the corresponding potential $\phi(x, T)$ is then used as initial condition for numerical simulations of the fully nonlinear equations (1) (see figure 2).

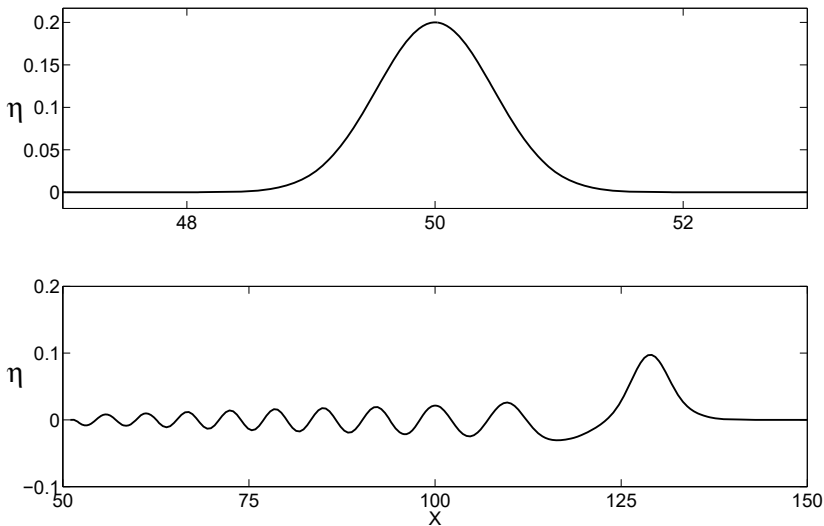


FIGURE 1. Evolution of an initial Gaussian impulse into soliton and dispersive wave train within the framework of the KdV equation (η and x are given in m)

2.2. Wind modelling: The modified Jeffreys' sheltering theory. Kharif et al 2008 [4] demonstrated experimentally for a wind velocity $U = 4m/s$ that steep wave events occurring in water wave groups are accompanied by air flow separation. Furthermore, it was suggested that a significant wind effect takes place when the steep wave event occurs. The focusing stage was almost independent of the wind velocity. Deviations were observed only in the vicinity of the focus point where the waves become steep. This observation reinforced the idea that separation of the air flow in the lee of the wave crests is responsible for the growth and persistence of steep waves. The Jeffreys' sheltering mechanism which was introduced by Jeffreys 1925 [7] could be modified and used as wind modelling. Since air flow separation occurs only over steep waves, the Jeffreys' sheltering mechanism has to be applied locally in time and space and not permanently over the whole wave field. It is well known that this mechanism cannot be applied continuously over water waves. This mechanism is working only when air flow separation occurs over steep waves (Banner and Melville 1976 [8], Kawai 1982 [9]).

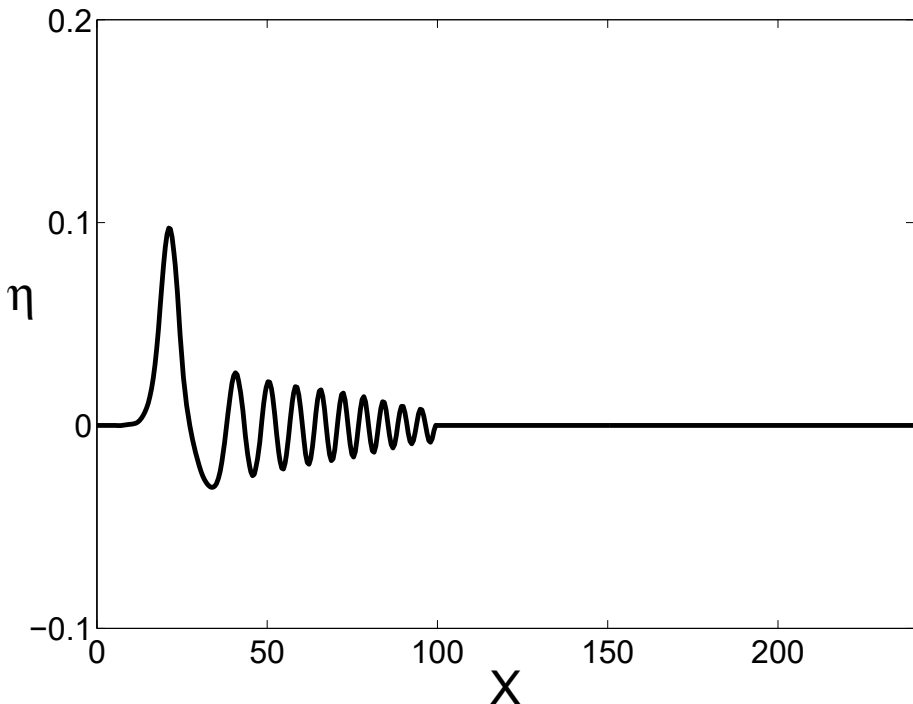


FIGURE 2. Initial surface elevation for system of equations (1) (η and x are given in m)

Following Jeffreys 1925 [7], the pressure at the interface $z = \eta(x, t)$ is related to the local wave slope according to the following expression

$$p_a = \rho_a s (U - c)^2 \frac{\partial \eta}{\partial x}, \quad (6)$$

where the constant s is termed the sheltering coefficient, U is the wind speed, c is the wave phase velocity and ρ_a is atmospheric density. The sheltering coefficient is $s = 0.5$. We assume that this value which was obtained by Kharif et al 2008 [4] from experimental data in deep water, holds in shallow water too. In order to apply the relation (6) for only steep waves we introduce a threshold value for the slope $(\partial \eta / \partial x)_c$. When the local slope of the waves becomes larger than this critical value, the pressure is given by equation (6) otherwise the pressure at the interface, p_a , is taken equal to a constant which is chosen equal to zero without loss of generality. This means that wind forcing is applied locally in time and space. When the critical value is low, the transfer of energy from the wind to the waves yields to wave breaking and when it is too high this transfer becomes negligible to influence the wave dynamics. Our main purpose is to show that the application of the modified Jeffreys' mechanism could explain simply some features of the interaction between wind and strongly modulated water wave groups in shallow water.

Figure 3 shows the pressure distribution at the interface in the vicinity of the crest, given by equation (6) during the occurrence of the steep wave event.

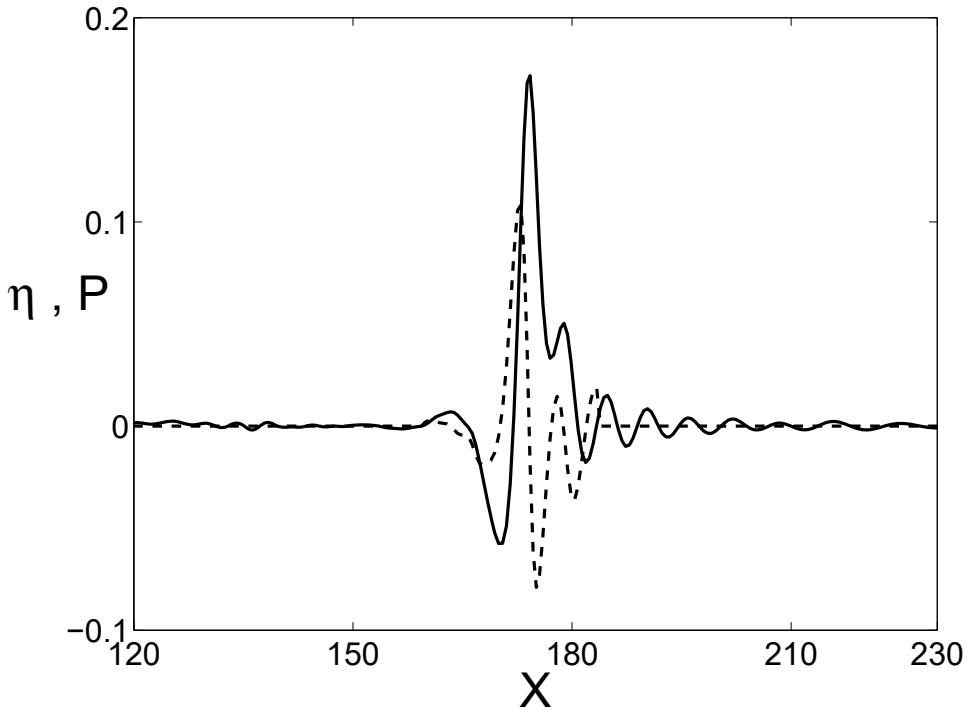


FIGURE 3. Pressure at the interface (dashed line) and surface elevation (solid line) as a function of x during the occurrence of the steep wave event (η and x are given in m and $P = p_a$ in $10^{-1}Pa$)

3. Numerical simulations, validation, results and discussion. In this section we consider numerical simulations of the fully nonlinear equations by using the Boundary Integral Equation Method.

3.1. Validation: Comparison with experiment. To demonstrate the efficiency of the present numerical method (BIEM) used to solve the system of equations (1), a comparison between numerical results and experimental data has been done in the absence of wind.

The number of meshes to discretize the boundary is 1100, 600 of them on the water surface. The time integration is performed using a RK4 scheme, with a constant time step of 0.001 s . To avoid numerical instability, the grid spacing Δx and time increment Δt have been chosen to satisfy the following Courant criterion derived from the linearized surface conditions

$$(\Delta t)^2 \leq \frac{8\Delta x}{\pi g}. \quad (7)$$

The experimental simulation on the focusing phenomenon has been conducted in the wave tank of Ecole Centrale Marseille. The tank is 17 m long and 0.65 m width. The water depth in front of the wavemaker was about 0.35 m . The generation of the focalized wave follows from a given focal point and a given spectrum. The chosen

spectrum was a Ricker (the second derivative of a Gaussian function):

$$R(\omega) = A_r \sqrt{\pi} T_r e^{-\omega^2 T_r^2 / 4} \left(1 - a_r \left(\frac{1}{4} \omega^2 T_r^2 - \frac{1}{2} \right) \right)$$

with A_r the amplitude of the focalized wave and a_r and T_r parameters to adjust the peak pulsation and the narrowness of the signal. To characterize the wave field, water elevation was measured with four resistive wave gauges. The setup of the Ricker and the distance of gauges from the wavemaker are given in the following tables:

A_r (m)	a_r	T_r	focal point (m)
0.1	-0.7	0.23	7.75

Table 1 : Ricker setup

wave gauge n^o	1	2	3	4
distance (m)	1.7	5.73	7	7.46

Table 2 : location of gauges

Figure 4 shows the experimental and computed surface elevation $\eta(t)$ at $x = 1, 7 m$, $x = 5.73 m$, $x = 7 m$ and $x = 7.46 m$. The agreement between the experimental data and the numerical results is satisfactory. We observe a very good agreement for the phase while some deviations occur for the elevation. This comparison demonstrates the efficiency of the numerical model to simulate realistically the formation of steep wave event in shallow water.

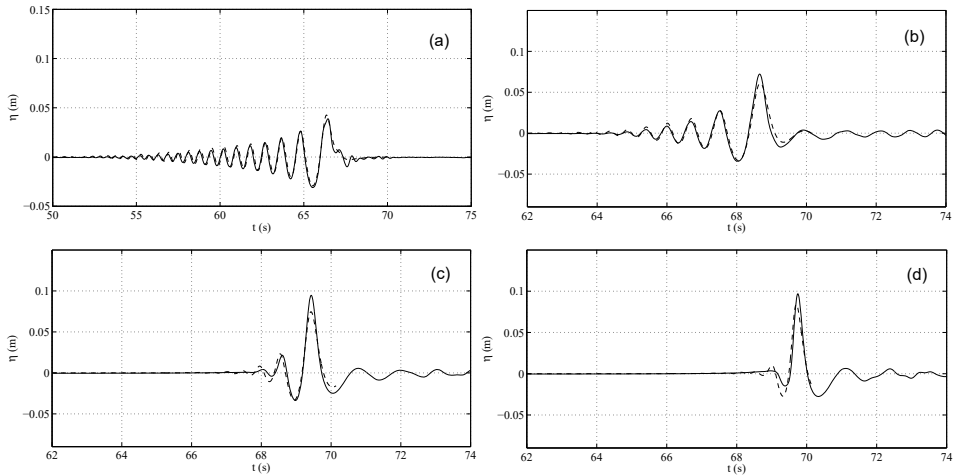


FIGURE 4. Time evolution of the experimental (solid line) and numerical (dashed line) surface elevation at several fetches:(a) $x = 1, 7 m$, (b) $x = 5.73 m$, (c) $x = 7 m$ and (d) $x = 7.46 m$

3.2. Extreme wave event in the presence of wind. We consider now the time evolution of the initial wave train plotted in figure 2. The number of meshes to discretize the boundary is 2000, 1000 of them on the water surface and the time step is $0.05 s$. The length of the numerical tank is $800 m$ and depth is $h = 1 m$. The focusing and defocusing stages of the wave train without wind effect ($U = 0 m/s$) is shown in figure 6 in the plane (x, t) . At the maximum of modulation the profile of

the steep wave event does not fit exactly with the expected Gaussian impulse (see figure 5). The theoretical value of the normalized amplitude is 0.20 whereas the maximal value of the normalized amplitude of the numerical simulation is 0.175. This disagreement is easily explained. We have used as initial condition for the fully nonlinear system (1) the profile resulting from the KdV equation which is a weakly nonlinear model. The deviation between fully nonlinear and weakly nonlinear is due to higher-order terms which are not considered in the KdV equation. Our main purpose was to obtain a large wave due the dispersive focusing of an initial wave group frequency and amplitude modulated.

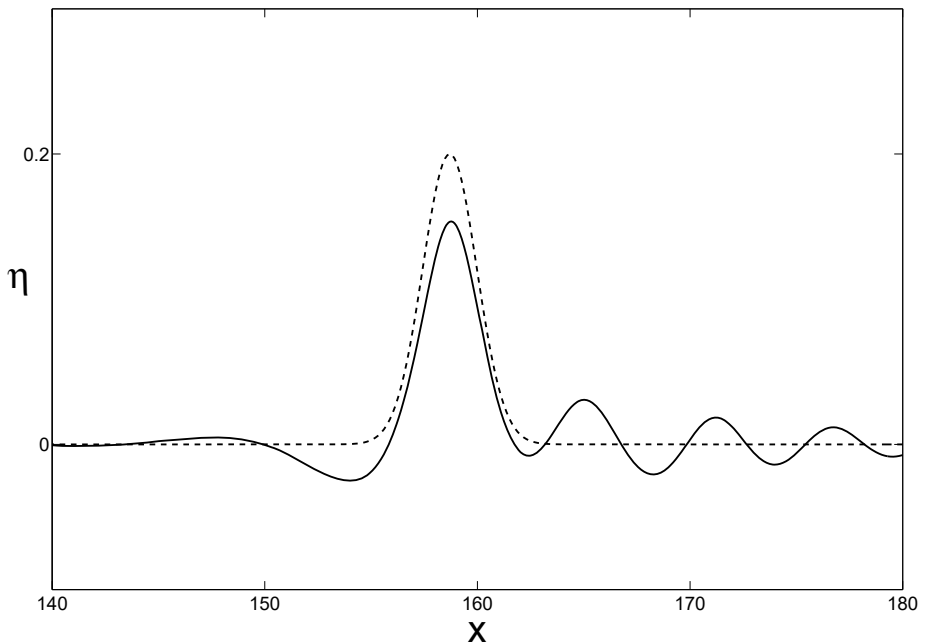


FIGURE 5. Comparison between the expected Gaussian impulse of KdV model (dashed line) with the steep wave event from numerical simulation (solid line)

The question is how do extreme wave events due to dispersive focusing under wind action evolve? How are the amplification and time duration of these waves under wind effect modified? Are these effects similar or different from those observed in the case of extreme wave events due to the spatio-temporal focusing in deep water?

For each value of the wind velocity, the amplification factor $A(t, U)$ of the group of waves between initial instant of time and time t is defined as follows

$$A(t, U) = \frac{\eta_{\max}(t, U)}{\eta_{\text{ref}}}, \quad (8)$$

where η_{ref} is the maximal wave amplitude of the initial wave group.

Figure 7 gives the amplification factor as a function of time for various values of the wind velocity, equal to 0 m/s , 8 m/s , 16 m/s , 20 m/s and 30 m/s . The critical slope of the modified Jeffreys mechanism is $(\partial\eta/\partial x)_c = 0.25$. The curves corresponding to $U = 0 \text{ m/s}$ and $U = 8 \text{ m/s}$ are almost identical. Larger wind

velocities are needed to influence the dynamics of the steep wave events. In deep water Touboul et al 2006 [3] and Kharif et al 2008 [4] observed significant influence of wind on extreme wave events for weaker values of the wind speed. In relation to deep water, larger values of the wind velocity are needed in shallow water to influence significantly the behaviour of the steep wave events. This means that steep wave events are more stable to wind perturbation in shallow water than in deep water. Figure 7 shows as well that the effect of the wind is threefold: it increases the amplification factor and lifetime of the steep wave event, and shifts downstream the focus point which corresponds to the maximum value of $A(t, U)$. For $U = 30 \text{ m/s}$, the defocusing is stopped by the breaking of the wave. Before the focus point, the slope of the waves is less than the threshold value imposed to produce air flow separation on leeward side of the crests. This feature was observed experimentally by Kharif et al 2008 [4] (see their figure 5) in deep water. The effect of the wind on the extreme wave event is to shift the focusing point downstream, and to increase its amplitude and duration. It is also interesting to emphasize that the amplitude and lifetime of the steep wave increases with wind velocity.

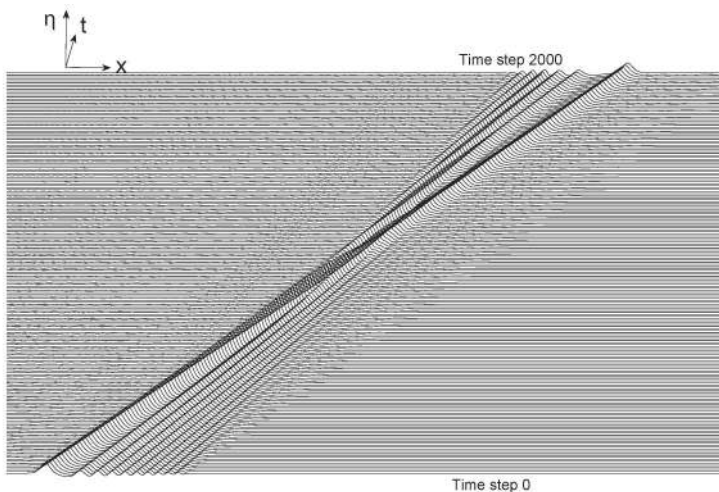


FIGURE 6. Spatio-temporal evolution of the focusing and defocusing wave group in the absence of wind (from bottom to top)

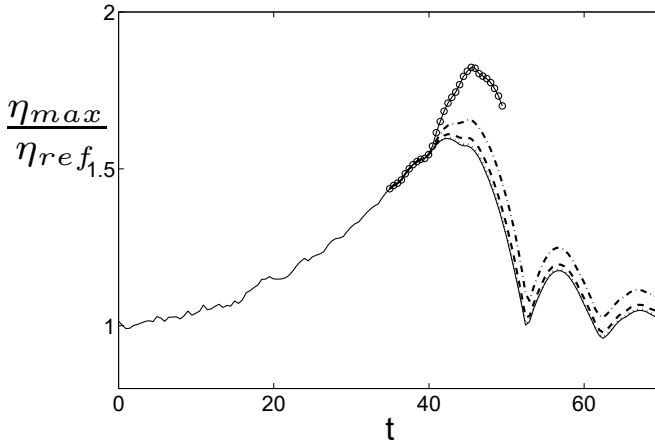


FIGURE 7. Evolution of the amplification factor $A(t, U) = \eta_{\max}/\eta_{\text{ref}}$ as a function of time, for several values of the wind speed. $U = 0 \text{ m}\cdot\text{s}^{-1}$ (solid line), $U = 8 \text{ m}\cdot\text{s}^{-1}$ (dotted line), $U = 16 \text{ m}\cdot\text{s}^{-1}$ (dashed line), $U = 20 \text{ m}\cdot\text{s}^{-1}$ (dot-dashed line), $U = 30 \text{ m}\cdot\text{s}^{-1}$ (open circle)

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