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► **To cite this version:**

Pierre Devillard, Adeline Crépieux. Noise in superconductor-quantum dot-normal metal structures in the Kondo regime. 2009. hal-00440732

HAL Id: hal-00440732

<https://hal.science/hal-00440732>

Preprint submitted on 11 Dec 2009

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Noise in superconductor-quantum dot-normal metal structures in the Kondo regime

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We consider a N-dot-S junction in the Kondo regime in the limit where the superconducting gap is much smaller than the Kondo temperature. A generalization of the floating of the Kondo resonance is proposed and many body corrections to the average subgap current are calculated. The zero frequency noise is computed and the Fano factor sticks to the value 10/3 for all voltages below the gap. Implications for finite frequency noise are briefly discussed.

PACS numbers: 71.10.Ay, 71.27.+a, 72.15.Qm, 73.63.Fg

Possible realizations of quantum dots have revived the interest in Kondo physics^{1,2}. For normal electrodes, the dot spin is screened and the system behaves in first approximation as a resonant level at the Fermi energy. However, this picture is not sufficient; the system is a Fermi liquid with interactions, moreover, the ratio of elastic and inelastic backscattering obeys some universality³, which can be obtained from the concept of floating of the Kondo resonance. Current noise is a useful tool to obtain informations on interactions, which are not present in the average current $\langle I \rangle$. Recently, the zero frequency noise S and the Fano factor $F = S/2e\langle I \rangle$ in the SU(2) case⁴ and the SU(4) case^{5,6} were calculated and confirmed experimentally⁷ for SU(2).

What happens to this effect in the superconducting case? This poses the problem of interplay between Kondo physics and superconductivity, already present in heavy fermion compounds² and underdoped high- T_c materials⁸. Devices with a dot between two superconducting electrodes have been extensively studied both theoretically^{9,10,11,12,13,14,15} and experimentally^{16,17} with emphasis on the Josephson current, and on how increasing the gap Δ destroys the Kondo effect. Here, we study the noise of the subgap Andreev current in a normal metal-dot-superconductor structure for $\Delta \ll T_K$. While in this regime, there is still no destruction of the Kondo resonance by the presence of the gap Δ ^{1,11}, the interplay between one and many-particle scattering, Andreev and normal reflexion, is far from trivial. The central result of this paper is a generalization of the floating of the Kondo resonance in the case where one electrode is superconducting. The most noticeable consequence is a constant Fano factor, equal to 10/3 for all voltages bias below the gap. This could be tested experimentally on carbon nanotubes.

Model: A quantum dot with effectively one level of energy ϵ_0 is placed between two electrodes, the left one being normal and the right one being a usual BCS superconductor, see Fig. 1. The on site repulsion U on the dot is supposed to be the largest energy of the problem. Electrons can hop from the lead to the dot with amplitude τ , implying a broadening of the level $\Gamma = 2\pi\rho(\epsilon_F)|\tau|^2$, with $\rho(\epsilon_F)$ the density of states at the Fermi energy in the

normal metal. We abide by the particle-hole symmetric case, for which $\epsilon_0 = -U/2$. The Hamiltonian reads

$$H = \sum_{k,\sigma,p} (\epsilon_k - \mu_p) c_{k,\sigma,p}^\dagger c_{k,\sigma,p} + \tau c_{k,\sigma,p} d_\sigma^\dagger + h.c. + \sum_k \Delta c_{k,\uparrow,R}^\dagger c_{-k,\downarrow,R}^\dagger + h.c. + \epsilon_0(n_\uparrow + n_\downarrow) + U n_\uparrow n_\downarrow, \quad (1)$$

where Δ is the superconducting order parameter; $p = L$ for the normal lead on the left and $p = R$ for the superconducting one; the operators $c_{k,p,\sigma}^\dagger$ and d_σ^\dagger create an electron on the p lead and on the dot, respectively and $n_\sigma = d_\sigma^\dagger d_\sigma$. The chemical potentials are $\mu_L = eV$ and $\mu_R = 0$, with V the voltage bias. For $\Delta \ll T_K$, and even when both electrodes are superconducting, it was shown by various methods¹, both analytical and numerical^{11,12} that the Kondo resonance subsists despite the gap. In the non-Kondo case, the average current has been calculated in Ref. 10. Also, in the low temperature regime, in the Kondo case, the average current and the noise were estimated by using slave-boson methods but this method is an effective one-body approximation to the full Kondo Hamiltonian and thus, is not sufficient to capture the complexity of the Kondo Hamiltonian, already when both electrodes are normal⁴.

Method and one-body setting: Here, we want to generalize the calculation of the Kondo noise when two electrodes are normal to the case where one electrode is superconducting. In order to include the many-body effects, we consider a slightly different model for the Kondo dot, which has been used for modelling an imperfect NS junction¹⁸, see Fig. 2. The dot is a scatterer placed in front of the superconductor and produces dephasing, whereas in reality, the dot acquires off-diagonal order due to hopping on and from the superconductor. An incident electron of energy ϵ is scattered by the dot and then Andreev reflected as a hole by the NS interface, supposed to be perfect, with a modulus one and a phase $-i \operatorname{atan}(\epsilon/\Delta)$, and then, this hole gets scattered by the dot, (see Fig. 2). The dephasing of the electron by the scatterer is $\delta_\epsilon = \delta_\epsilon^{(0)} + \delta_\epsilon^{(1)} + \dots$, where $\delta_\epsilon^{(0)} = -\pi/2$, $\delta_\epsilon^{(1)}$ is first order in ϵ/T_K and the dots represent higher orders in ϵ/T_K . The energy transmission of electrons through the scatterer is $T(\epsilon) = \sin^2(\delta_\epsilon)$. The same thing happens

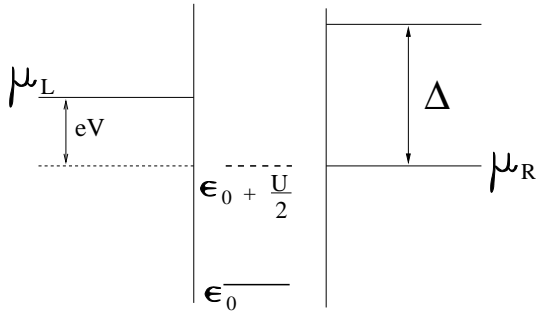


FIG. 1: Quantum dot sandwiched between a normal electrode on the left with chemical potential μ_L and a superconducting one on the right with gap Δ and chemical potential $\mu_R = 0$. Only one level of negative energy ϵ_0 intervenes. Hopping amplitudes on and from the dot to both electrodes are assumed to be equal. The effective energy level taking into account the Hartree correction is $\tilde{\epsilon}_0 = \epsilon_0 + U/2$, in heavy dashed line.

for the holes with dephasing δ_h . Using the Bogolubov-de Gennes (BdG) equations enables to express the s -matrix in Nambu space for the whole structure, which is 4×4 but, below the gap, reduces to a 2×2 reflexion matrix, because no transmission of one-particle excitation occurs in the superconductor. The energy dependent normal reflection coefficient r_N and the Andreev amplitude reflection coefficient r_A can be extracted in terms of δ_e and

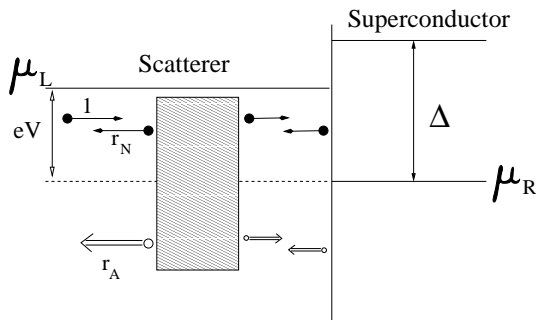


FIG. 2: Simplified model for the N -dot- S system. A scatterer, having no off-diagonal Green's function, is placed in front of the superconductor accounts for the imperfect transmission properties of the dot. The interface with the superconducting electrode is taken to be perfect so that only Andreev reflexion occurs at the interface and only normal reflexion or transmission occurs across the scatterer. Proximity effects are neglected. An incident electron is partially transmitted through the scatterer and then totally Andreev reflected as a hole. This hole is in turn partially transmitted as a hole but also partially reflected by the scatterer, etc... All processes need to be resummed to infinite order, the amplitudes r_N and r_A are the result of this (coherent) resummation. However, this is equivalent to solving the BdG equations.

δ_h . We want to make contact with the original model of Eq (1). If T_K were infinite, then, the one-body picture of a resonant model centered at the Fermi level, with width T_K would be valid. We are close to the unitary limit and we need the dot Green's function in this limit. In

the case where both electrodes are normal, the retarded Green's function is of the form $z^{-1}(\epsilon - \tilde{\epsilon}_0 - 2i\tilde{\Gamma}sgn(\epsilon))^{-1}$ with $z = 1 - (\partial\Sigma^{ret}(\epsilon)/\partial\epsilon)|_{\epsilon=\epsilon_F}$ and $\tilde{\Gamma} = z^{-1}\Gamma \simeq T_K/2$; $\Sigma^{ret}(\epsilon)$ being the retarded self-energy of the dot and $\tilde{\epsilon}_0 = \epsilon_0 + U/2$. This has been used justified theoretically by Ref. 2. When both electrodes are superconducting, for $\Delta \ll T_K$, a similar procedure exists and has been used in Ref. 13 to study the dynamics of Andreev states. We thus use these Green's functions, adapted to our case, and following the same steps as in Ref. 10, the conductance g_A reads

$$g_A = 4\frac{e^2}{h}(eV)^{-1} \int_0^{eV} \left[\left(\frac{\epsilon}{\tilde{\Gamma}/2}\right)^2 \left(\frac{\epsilon}{\Delta}\right)^2 \right] d\epsilon. \quad (2)$$

On the other hand, the model of Ref. 18 leads to a conductance

$$g_B = 4\frac{e^2}{h}(eV)^{-1} \int_0^{eV} \left(\frac{T(\epsilon)}{2 - T(\epsilon)} \right)^2 d\epsilon. \quad (3)$$

Thus, for g_A and g_B to have the same expression, we adjust the dephasing of the scattering center to be $\delta_e^{(1)} = (\epsilon/\Delta)(\epsilon/\tilde{\Gamma}) \equiv (\epsilon/T_K)\alpha_1(\epsilon)$, where α_1 is a function of ϵ . The same occurs for $\delta_h^{(1)}$. This form does not correspond to a resonant level at $\epsilon = 0$ and width $\tilde{\Gamma}/2$. In the original model, the dot acquires some off-diagonal matrix element in Nambu space. Very qualitatively, this favors direct Andreev reflexion of a wave incoming on the dot. Since, for the whole structure, $|r_N|^2 + |r_A|^2 = 1$, there will be less normal scattering. The amount of normal scattering increases as $(\epsilon - \epsilon_F)^2$ instead of $(\epsilon - \epsilon_F)$ for the normal case. Let us denote by $G_{dot1,1}^{ret}$ the upper Nambu component of the retarded dot Green's function, calculated without the many-body corrections, using the renormalized parameter $\tilde{\Gamma}$ and putting aside the usual multiplicative renormalization factor z . For energies higher than the gap, the form of the normal part of the spectral density of the dot $\rho_{1,1}(\epsilon) = -\pi^{-1}ImG_{dot1,1}^{ret}$ is very close to the usual shape for the normal case. Surprisingly, this persists down to energies barely larger than Δ . Of course, this is not the exact $\rho_{1,1}(\epsilon)$ but this feature seems to be shared by more elaborate numerical solutions for the S-dot-S case, such as numerical renormalization group (NRG)¹¹ or functional renormalization group (fRG)^{14,17}.

Many-body calculation: Now, we want to put the many-body corrections since T_K is not infinite. The electrons build the Kondo resonance from hopping to the normal lead and also from hopping to the superconductor. Unlike the normal case, the argument of doping the normal side to establish the floating of the Kondo resonance^{3,6} does not work directly, because doping the metal and changing the chemical potential for the electrons will also change it for the holes. However, for $\Delta \ll \epsilon \ll T_K$, $\delta_e^{(1)}$ is linear in ϵ , (α_1 becomes energy independent) and the argument can be used. Thus we have to cancel the linear contribution in $1/T_K$ of δ_e by a many-body interaction between electrons (and holes),

as in usual Fermi-liquid theory. The fixed point Hamiltonian is

$$H_{FP} = H_0 + H_{int}, \quad (4)$$

with H_0 being the one-body Hamiltonian, involving scattering states (first order in $1/T_K$) and H_{int} is the interaction between quasiparticles. The simplest form maintaining particle-hole symmetry is

$$H_{int} = \frac{1}{T_K} \left[\sum_{k_1, k_2, k_3, k_4} \beta_1(\epsilon_{k_1} + \epsilon_{k_2} + \epsilon_{k_3} + \epsilon_{k_4}) \times \left(b_{k_1, \uparrow}^\dagger b_{k_2, \downarrow}^\dagger b_{k_3, \downarrow} b_{k_4, \uparrow} + B_{k_1, \uparrow}^\dagger B_{k_2, \downarrow}^\dagger B_{k_3, \downarrow} B_{k_4, \uparrow} \right) \right], \quad (5)$$

with $b_{k, \sigma} = (c_{k, \sigma, L} + c_{k, \sigma, R})/\sqrt{2}$ and $B_{k, \sigma} = (c_{-k, \bar{\sigma}, L}^\dagger + c_{-k, \bar{\sigma}, R}^\dagger)/\sqrt{2}$ with $\bar{\sigma} = -\sigma$ and $\beta_1(\epsilon)$ is a function of energy. Here, a right-mover state with energy ϵ will have some normal reflected part. No elastic scattering Hamiltonian is necessary⁶.

Because of Hartree-Fock corrections to order one in β_1 , the dephasing $\delta_e^{(1)}$ will be changed to $\delta_e^{(1)} - (\beta_1/T_K)\delta n_{\bar{\sigma}}$, so that $\beta_1 = \alpha_1$ to cancel the $1/T_K$ overall contribution.

The main assumption of this paper is that this can be continued all the way down to $\epsilon = 0$. Then, $\beta_1(\epsilon)$ will have to cancel $\alpha_1(\epsilon)$.

Now, we calculate the average current in the Keldysh formalism, perturbatively to order $1/T_K^2$, following the method of Ref. 5. The right mover scattering states are thus of the form, in Nambu space, far on the left of the scattering center

$$\psi_{\mathcal{R}}(\epsilon_k, x) = \frac{1}{\sqrt{k}} \left[\left(e^{ikx} + r_N e^{-ikx} \right) \begin{pmatrix} 1 \\ 0 \end{pmatrix} + r_A e^{-ikx} \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right], \quad (6)$$

The left-mover scattering state consists in sending a hole from the left, which is partially normally reflected and also Andreev reflected as a left propagating electron. At the one-body level, the current operator has two components and is then expressed in terms of the BdG wave function, and falls into three parts, I_R , involving only right-moving scattering states, I_L with solely left-movers and I_{OD} which involves both right and left-movers. Each of these three terms gives rise to two contributions. One contribution, denoted by subscript 1 comes from the electron part of the two component Nambu wave function and the other one from the hole part (subscript 2). For example $I_{R,1} = \sum_{k, \sigma} (1 - |r_N|^2) \psi_{k, \sigma, R, 1}^\dagger \psi_{k, \sigma, R, 1}$, with $\psi_{k, \sigma, R}$ is the operator creating a two-component Nambu right mover, and $\psi_{k, \sigma, R, 1}$ denotes its upper component. r_N and $(1 - |r_A|)$ are order 1 in $1/T_K$. We could work with these scattering states but we prefer to use the zeroth order scattering states, (i.e. $r_N = 0$ and $|r_A| = 1$). This is at the price of having to introduce an elastic scattering Hamiltonian. The fixed point Hamiltonian is $H'_{FP} = H'_0 + H'_\alpha + H'_{int}$ where H'_0 is the Hamiltonian for

a perfect scatterer (zeroth order in $1/T_K$) and

$$H'_\alpha = \frac{1}{\pi\nu T_K} \sum_{i=1}^2 \sum_{k, k', \sigma} \left[\frac{\epsilon_k + \epsilon_{k'}}{2} \right] \times \alpha_2(\epsilon_k + \epsilon_{k'}) \mathcal{B}_{k, \sigma, i}^\dagger \mathcal{B}_{k', \sigma, i}, \quad (7)$$

with $\mathcal{B}_{k, \sigma, i}^\dagger = (\Phi_{k, \sigma, R, i}^\dagger + \Phi_{k, \sigma, L, i}^\dagger)/\sqrt{2}$, where $\Phi_{k, \sigma, R, i}^\dagger$ are the Nambu components of operators creating a scattering state to zeroth order in $1/T_K$. α_2 is adjusted so that H'_α gives the same dephasing δ_e to order $1/T_K$ as H_0 (Eq. (4)). α_2 is proportionnal to α_1 ; we obtain $\alpha_2(\epsilon) = A\epsilon/\Delta$, with $A = 2$. The two-body Hamiltonian is written in the form

$$H'_{int} = \frac{1}{\pi\nu T_K} \sum_{i=1}^2 \sum_{k_1, k_2, k_3, k_4} \beta_2(\epsilon_{k_1} + \epsilon_{k_2} + \epsilon_{k_3} + \epsilon_{k_4}) \times \mathcal{B}_{k_1, \uparrow, i}^\dagger \mathcal{B}_{k_2, \downarrow, i}^\dagger \mathcal{B}_{k_3, \downarrow, i} \mathcal{B}_{k_4, \uparrow, i}, \quad (8)$$

β_2 depends on energy ϵ . For the same reason that β_1 and α_1 were not independent, β_2 and α_2 are also linked. We find $\beta_2(\epsilon) = B\epsilon/\Delta$ with $B = 2$. For calculating the inelastic part, it suffices to expand to second order in β_2 the quantity $(1/2) \sum_{\eta=\pm} \langle T_{\mathcal{C}} I(t^\eta) \exp(-i \int_{\mathcal{C}} H'_{int}(t^{\eta_1}) dt^{\eta_1}) \rangle$, where η and η_1 are Keldysh indices, \mathcal{C} denotes the Keldysh contour and $T_{\mathcal{C}}$ the corresponding time ordering. Lumping these with the elastic backscattering contribution results in the following expression for the averaged current

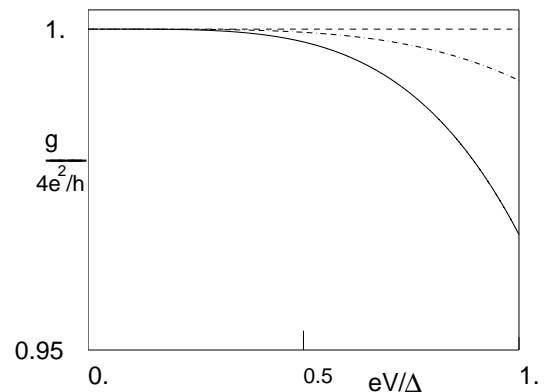


FIG. 3: Conductance in units of $4e^2/h$ vs. eV/Δ for $\Delta/T_K = 0.2$, taking into account the many-body terms (solid line), in the resonant level approximation (dash-dotted line) and for infinite T_K (BTK result), (dashed line).

$$\langle I \rangle = 4 \frac{e^2}{h} V \left[1 - \left(\frac{A^2 + 3B^2}{20} \right) \left(\frac{eV}{\Delta} \right)^2 \left(\frac{eV}{T_K} \right)^2 \right]. \quad (9)$$

As usual, this can be retrieved in a simple way by using Fermi golden rule for the elastic process and the three inelastic processes, $(\mathcal{R}, \mathcal{R}) \rightarrow (\mathcal{R}, \mathcal{L})$, $(\mathcal{R}, \mathcal{R}) \rightarrow (\mathcal{L}, \mathcal{L})$ and $(\mathcal{R}, \mathcal{L}) \rightarrow (\mathcal{L}, \mathcal{L})$, where \mathcal{R} is a right-mover and \mathcal{L} a left-mover. For example, for the second process, two

right-movers are backscattered; one right-mover carries a charge e for the incoming electron and also e from the Andreev reflected hole, so $2e$ in total. The current is of the form $(1/2) d(N_{\mathcal{L}} - N_{\mathcal{R}})/dt$, where $N_{\mathcal{L}}(\mathcal{R})$ is the operator number for left (right) movers and the change in the current is $4e$. The diagram multiplicities are the same as for the normal case but phase space integrals are different because of the ϵ dependence of α_2 and β_2 . Collecting the four contributions gives the average current

$$\langle I \rangle = 2 \int \left[\alpha_2^2(\epsilon) (2e) \Gamma_\alpha(\epsilon) m_\alpha + \sum_{j=1}^3 \beta_2^2(\epsilon) e_j^* \Gamma_{\beta,j}(\epsilon) m_j \right] d\epsilon, \quad (10)$$

where the factor 2 comes from the spin; $\Gamma_\alpha(\epsilon)$ and the $\Gamma_{\beta,j}$ are $2\pi/\hbar$ times the right mover self-energies due to the above mentioned processes and m is the multiplicity of the diagram. For instance, for the second process, $m_2 = 2$ and $e_2^* = 4e$, so that the contribution is $2B^2(\pi^2\nu^2T_K^2)^{-1}m_2e^*\nu^2J$, with $J = \int_{-eV}^{eV} (\epsilon/\Delta)^2 (2\pi/\hbar) (1/4) \int_{-eV}^\epsilon dx \int_0^{\epsilon-x} dy d\epsilon$. For the first and third processes, $e_1^* = e_3^* = 2e$ and $m_1 = m_2 = 4$, but the self-energy triple integrals obtained by integrating $\Gamma_{\beta,j}(\epsilon)$ on energy ϵ give a contribution 1/16 smaller than for $(\mathcal{R}, \mathcal{R}) \rightarrow (\mathcal{L}, \mathcal{L})$.

These results are summarized in Fig. 3, showing the conductance $g = \langle I \rangle / V$ versus eV/Δ , in units of $4e^2/h$, for $\Delta/T_K = 0.2$. Many-body terms make g decrease. In the case of $\Delta/T_K = 0$, the BTK result¹⁹ is retrieved. For comparison, the result given by a resonant level formulation, without the many-body terms is shown.

Noise: We now turn to the zero frequency shot-noise calculation. As, to zero-th order in α_2 and β_2 , there is no partition noise (unlike SU(4) case^{5,6}), applying the

Schottky formula for each process is sufficient⁴. As in the normal case, direct inspection of the four processes gives

$$S = 4 \int \left[\alpha_2^2(\epsilon) (2e)^2 \Gamma_\alpha(\epsilon) m_\alpha + \sum_{j=1}^3 \beta_2^2(\epsilon) (e_j^*)^2 \Gamma_{\beta,j}(\epsilon) m_j \right] d\epsilon. \quad (11)$$

The resulting Fano factor with $A = B = 2$ is thus 10/3, to second order in eV/T_K for any $eV \leq \Delta$.

The finite frequency noise is now discussed. For a normal NS junction, at zero temperature, the absorption finite frequency noise $S(\omega)$ goes to zero for $\omega > 2eV$. Here, within perturbation theory in Keldysh to second order in $1/T_K^2$, this is again the case. However, inspection of the diagrams suggests that inelastic terms bring more noise than in the case of an imperfect NS junction having the same Andreev conductance.

In conclusion, we have proposed a generalization of the floating of the Kondo resonance to the case where one electrode is superconducting, in the regime of small Δ/T_K , close to the unitary limit. This enables a calculation of the many-body correction to the subgap Andreev current and zero frequency noise. Remarkably, the Fano factor sticks to the value 10/3 as long as $eV \leq \Delta$. In the region $eV \geq \Delta$, not studied here, it is expected to decrease and to reach eventually the normal state limit 5/3 for $eV \gg \Delta$.

Acknowledgments

M. Lavagna, J. L. Pichard and P. Simon are gratefully acknowledged for discussions and comments.

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