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# Numerical experimentations on the coupling between PEEC and volume integral method

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**Abstract** — In order to model ferromagnetic material in quasi-static PEEC method, a coupling with a volume integral equation is proposed. This coupling enables to take advantage of the strong points of each method. The modeling of complex conductor geometry is achieved thanks to PEEC method and magnetic materials are taken into account thanks to a volume integral equation. The coupling is carried out by introducing an impedance matrix (computed with PEEC method in the vacuum) within an integral magnetostatic equation solved by moment method..

## I. INTRODUCTION

The PEEC method (Partial Element Equivalent Circuit) is mainly used for the modeling of complex interconnections and can be applied to a large range of devices where the air region is dominant [1] (printed circuits, bus bars...). However, the classical PEEC method does not enable the 3D modeling of ferromagnetic materials widely present in devices (ferromagnetic shielding, disruptive magnetic masses, and cores of inductance). The magnetostatic moments method is well-known and derives from a volume integral equation solved by a point matching approach. It is particularly well adapted to model lightly very simple magnetic materials volume. Like the PEEC method, it does not require the meshing of the air region; on the other hand, it is limited to modeling of magnetostatic effects. Both methods are complementary. A strong coupling of them can be accomplished by modeling non conductive magnetic regions with a method of moments, while PEEC method allows the modeling the contributions of the inductors fed with alternative currents.

## II. COUPLING PRINCIPLE

### A. Magnetostatic moment method

Let us consider a non conductive magnetic material placed in an inductor field  $\mathbf{H}_0$ . The total magnetic field  $\mathbf{H}$  is the sum of  $\mathbf{H}_0$  and  $\mathbf{H}_{\text{red}}$ , the reaction of the material. A well-known integral volume equation links the local field to the magnetization of the whole material volume  $V_{\text{mat}}$ :

$$\mathbf{H} = \mathbf{H}_0 + \frac{1}{4\pi} \int_{V_{\text{mat}}} \left( 3 \frac{(\mathbf{M}, \mathbf{r})}{r^5} \mathbf{r} - \frac{\mathbf{M}}{r^3} \right) dV \quad (1)$$

where  $\mathbf{r}$  is the vector linking the integration point to the point where the field is expressed,  $V_{\text{mat}}$  the ferromagnetic volume and  $\mathbf{M}$  its magnetization. Usually, this integral is solved with a numerical technique. The easier way is to mesh the volume into  $n$  elements and to consider that the magnetization is uniform on each of them. Then, thanks to a point matching approach of the linear magnetic material's law at the center of

each element, a linear matrix system is obtained. It remains to solve it, to obtain the magnetization of the whole volume  $V_{\text{mat}}$ . This method is usually called magnetostatic moment method. This formulation is known to suffer of some inaccuracies in some specific configurations [2]. However, the purpose of this paper is the coupling and very similar methodology can be applied to more sophisticated volume integral formulations.

Let's now assume that the inductor field is created by  $m$  unknown alternative currents  $I$  flowing in  $m$  conductors. In a very similar way to the previous one, we can get a linear system of equations. For instance, the equation associated to magnetic element  $k$  is:

$$\frac{\mathbf{M}_k}{\mu_r - 1} = \frac{1}{4\pi} \sum_{i=1}^n \int_{V_{\text{mat}_i}} \left( \frac{(\mathbf{M}_i, \mathbf{r})}{r^5} \mathbf{r} - \frac{\mathbf{M}_i}{r^3} \right) dV_i + \frac{1}{4\pi} \sum_{j=1}^m \int_{V_{\text{cond}_j}} \frac{d\mathbf{l} \times \mathbf{r}}{r^3} I_j dV_j \quad (2)$$

where  $\mu_r$  is the relative permeability of the material. The second integral term correspond to the Biot and Savart's law integrated on each conductor volume  $V_{\text{cond}_j}$ . The global matrix system obtained has  $6n$  equations (a vector complex equation per element) and  $6n+2m$  unknowns (a vector complex magnetization per element plus  $m$  complex currents).

### B. Inductive PEEC method

Let us consider  $m$  volume conductors fed with alternative sources placed in a surrounding air region without any magnetic materials. The well-known PEEC method is particularly reliable to solve this kind of problem. It is based on the determination of partial voltage generated on each conductor by electromagnetic sources. To compute these voltages volume integration on the conductor of the magnetic vector potential created by all the others conductor is provided. For instance, for the conductor  $k$ , the expression is [1]:

$$V_{k\_cond} = \frac{\mu_0}{4\pi} \frac{j\omega}{S_k} \int_{V_k} \left( \sum_{i=1}^m \frac{I_i}{S_i} \int_{V_i} \frac{1}{r} dV_i \right) dV_k \quad (3)$$

where  $\omega$  is the angular frequency and  $S_i$  is the section of the  $i$ -th conductor. This equation links partial voltages of conductors to currents flowing in them. If we write this equation for all conductors, we get a matrix system known as impedance matrix system. By combing these electromagnetic equations with the circuit ones representative of the conductors wiring and adding resistance source effect, it is possible to get a simplified system representative of the device which can be easily coupled with a standard circuit simulator. This inductive PEEC method has already shown its efficiency for the modeling of complex conductor geometries in comparison with FEM.

Let us now consider that linear ferromagnetic materials are present in the surrounding air region. Equation (3) has to be

modified by taking into account the influence of the field created by the material. In fact, like in the first approach, we have to integrate the magnetic vector potential on the volume conductor  $k$ . A new voltage has to be added to the previous one, not generated by the current but by the magnetization [3].

$$V_{k\_mat} = \frac{\mu_0}{4\pi} \frac{j\omega}{s_k} \int_{V_k} \left( \sum_{i=1}^n \int_{V_i} \frac{\mathbf{M}_i \times \mathbf{r}}{r^3} dV_i \right) \cdot \mathbf{u}_k dV_k \quad (4)$$

Finally, equations (3), (4) and (5) are brought together in a global square  $(6n+2m) \times (6n+2m)$  matrix system:

$$\begin{bmatrix} \mathbf{MoM} & \mathbf{BS} \\ \mathbf{L}_M & \mathbf{L}_{standard} \end{bmatrix} \times \begin{bmatrix} \mathbf{M} \\ \mathbf{I} \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ \mathbf{V} \end{bmatrix} \quad (5)$$

where MoM is the standard moments matrix (first two rearranged term of equation (2)), BS is the Biot and Savart integral term (last term of (2)),  $L_M$  is the influence of magnetization on conductors in PEEC approach (equation (4)) and  $L_{standard}$  the classical inductive and resistive PEEC matrix. By taking into account circuit equations, it is possible to reduce (5) to get a lighter system and to solve it to get magnetizations and currents in conductors.

### III. NUMERICAL EXAMPLES

We consider two numerical examples (Fig.1.). For both, three numerical methods are compared. The first one is the scalar magnetic potential FEM coupled with circuit equations. In this modeling, a special care is given to the mesh around both conductors to ensure accurate results. The second one is a coupling between PEEC to model inductances and FEM to take into account the ferromagnetic material. This approach has already shown its good accuracy with a reduced number of elements in comparison with standard FEM [4]. The last one is the considered coupling.

In the first example, two conductors are considered with a ferromagnetic bar ( $\mu_r = 1000$ , linear) placed between them (Fig.1, left). The first conductor is fed by a voltage source (1V, 1 kHz). The second one is in short circuit mode. To compare different approaches, we focus on the computed current in the second conductor with different meshes for the three approaches (see table I).

TABLE I. Currents values obtained (A)

MEF			
Nb of elements	230.000	600.000	1.000.000
Current values	-36.25+6,00i	-32.47+7,94i	-32.61+7.87i
MEF/PEEC			
Nb of elements	60.000	150.000	
Current values	-32.60+8,38i	-32.50+8.34	
MoM/PEEC			
Nb of elements	32	126	392
Current values	35.97+8,95i	-32.94+8.34i	-32.70+8.32i

Whereas the convergence is quickly reached with the MEF/PEEC method, the problem needs a very fine mesh to be accurately solved with FEM. We can see a small difference between both computed values. It can be explained by a small difference in the modelling method used to represent conductors. Results provided by our coupling are very encouraging, the convergence being reached with a very few

number of elements (around 200). Of course, the obtained matrix is fully dense, but the computation time is divided per one hundred in comparison with FEM and ten with PEEC/FEM. Moreover, no specific mesh refinement has been needed.

The second test case is an inductance of microphone converter. It is composed of a complex-shaped conductor and of two ferrite parts (Fig.1., right). In our study, the conductor is fed by a voltage source (1V, 10 kHz) and the ferrite is considered as linear with a permeability of 1000. Values of currents are still compared.

TABLE II. Currents values obtained (A)

FEM 100.000 elements	FEM/PEEC 30.000 elements	MoM/PEEC 2000 elements
14,36 – 1,97i	14,39 – 1,85i	14,43 – 1,68i

Results seem to be not so good for the coupling MoM / PEEC especially if we have a look to the imaginary part of the current. This inaccuracy is mainly due to the small distance between conductors and the magnetic material, leading to important variation of magnetization in the neighborhood of currents. This configuration benefits FEM in comparison with our coupling. A good improvement would certainly be to couple PEEC with a more sophisticated implementation of the volume integral equation.

### IV. CONCLUSION

In this paper, we have presented a coupling between PEEC and an integral volume equation. Our approach can be very fast and accurate and enables the introduction of linear magnetic material in PEEC methodology. It is particularly capable for the modeling of complex shapes conductors and relatively simple magnetic material geometry.

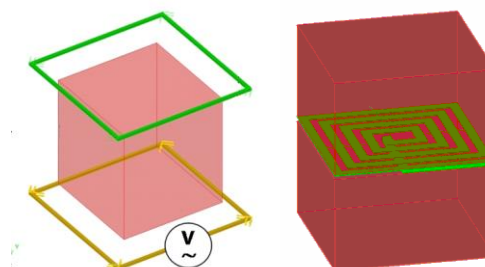


Fig.1. Geometry of both tested numerical cases.

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