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Analytical Calculation of the Interactions between two Cylinder-shaped Magnets

Hicham ALLAG, Jean-Paul YONNET and Benoit DELINCHANT

Laboratoire de Génie Electrique de Grenoble, G2E Lab (UMR 5269 CNRS/INPG-UJF),
 Institut Polytechnique de Grenoble, ENSIEG, BP46, 38402 St Martin d'Hères cedex, France
 Tel : 33 (0) 476 82 62 97 – Fax : 33 (0) 476 82 63 00
 E-mail : <Jean-Paul.Yonnet@g2elab.grenoble-inp.fr>

Abstract — Analytical calculation of interaction forces between two cylindrical magnets is a difficult problem. Solutions can be obtained by elliptic integrals or by numerical computation. Thanks to the development of the analytical calculation for parallelepiped magnets (interaction energy, force and torque), the calculation of the force component between two cylinders can be obtained in analytical form by decomposition in a finite number of elements.

This type of calculation allows the calculation of many systems that uses the forces of interaction between cylinder magnets. For example it can be used to directly calculate the force exerted in permanent magnet bearings in non-centred position.

I. INTRODUCTION

Recent works on the analytical calculation of forces between permanent magnets allow calculating the interactions between magnets of parallelepiped shapes. The forces, interaction energy and couples can be expressed as analytical expressions, parameterized by the geometrical dimensions and magnetization of the magnets [1 to 4].

However, the problem of rod-shaped magnets has never been analytically solved. The simpler problem of the 3D calculation of the magnetic field is made with elliptic integrals. This field calculation needs two integrations. For the calculation of interaction energy and forces, four successive integrations are necessary.

We will present an innovative analytical method, to calculate the interaction forces between two magnets of cylindrical shapes. The final analytical expression is a series with a limited term number. The accuracy depends on the number of elements.

This method allows calculating the interaction forces in three directions, as well as energy interaction, through a relatively simple analytical expression.

II. CALCULATION PRINCIPLE

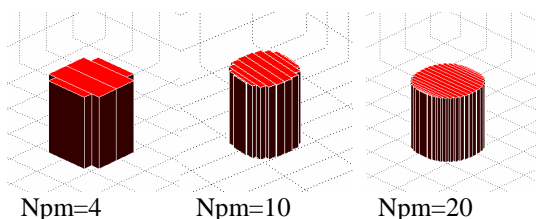


Figure 1: Principle of cutting of the cylindrical magnet in parallelepiped elements.

Figure 1 shows the principle of the cutting of the cylinder in parallelepiped elements. The first case corresponds to a cut in 4 parts, the second 10 and third 20. The number of elements is given by the parameter Npm (Number of elementary permanent magnets).

III. INTERACTION ENERGY

We give in the page 2 of this summary only the expression of the energy of interaction. The forces are then obtained by derivation of this energy.

IV. EXAMPLE OF CALCULATION

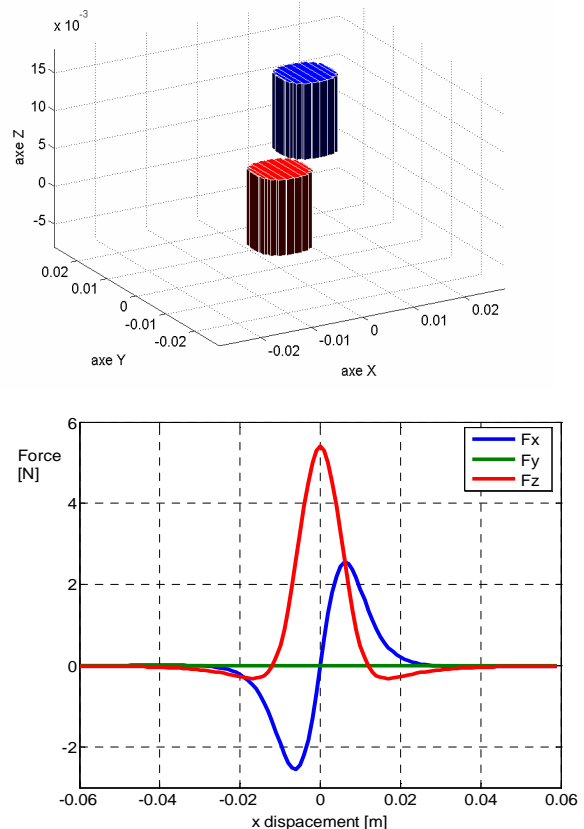


Figure 2: Calculation of the forces components for a magnet moving above a fixed one.

Figure 2 shows a direct example of use of these analytical results. It is the calculation of forces between two cylinder-shaped magnets. The two magnets have a diameter of 10 mm and a height of 10 mm. The second moves horizontally above the first and the distance between the magnets is 5 mm. Polarization J is

Analytical expression of the interaction energy :

$$E = \frac{J \cdot J'}{4\pi\mu_0} \sum_{n=2}^N \sum_{n1=1}^N \sum_{i=0}^{n1-1} \sum_{j=0}^{n1-1} \sum_{k=0}^{n1-1} \sum_{l=0}^{n1-1} \sum_{p=0}^{n1-1} \sum_{q=0}^{n1-1} (-1)^{i+j+k+l+p+q} \cdot \psi(U_{ij}, V_{kl}, W_{pq}, r)$$

with:

$$\psi(U, V, W, r) = \frac{U(V^2 - W^2)}{2} \ln(r - U) + \frac{V(U^2 - W^2)}{2} \ln(r - V) + UVW \cdot \operatorname{tg}^{-1}\left(\frac{UV}{rW}\right) + \frac{r}{6}(U^2 + V^2 - 2W^2)$$

The parameters are given by:

$$U_{ij} = (\alpha - \alpha 1(n1) + \alpha 2(n2)) + (-1)^j A(n2) - (-1)^i a(n1)$$

$$V_{kl} = \beta + (-1)^l B - (-1)^k b$$

$$W_{pq} = \gamma + (-1)^q C - (-1)^p c$$

$$r = \sqrt{U_i^2 + V_j^2 + W^2}$$

With:

$$\phi 1(n) = n \cdot \pi i / N p m 1$$

$$x s 1(n) = \sqrt{K c} \cdot \sqrt{R^2 / (1 + \operatorname{tg}^2(\phi 1(n)))}$$

$$\alpha 1(n) = \frac{x s 1(n) + x s 1(n+1)}{2}$$

$$a 1(n) = \frac{|x s 1(n) - x s 1(n+1)|}{2}$$

$$b 1(n) = |\operatorname{tg}(\phi 1(n)) \cdot x s 1(n)|$$

$$K c = \frac{\pi \cdot R^2}{4 \sum_{n=1}^{N p m} A(n) \cdot B(n)}$$

normalized to 1 tesla. The results show the values of the three force components, Fx, Fy and Fz.

The accuracy of analytical results is directly related to the number of elementary parallelepiped components used for each of the magnets. The complete study of the accuracy shows that one can easily obtain a relative error less than 1/1000, when the number of elements is sufficient. For this example of two cylindrical magnets, the precision of 1/1000 is achieved when the number of elements is about 20, even for a gap down to 2 mm.

V. APPLICATION TO MAGNETIC BEARING CALCULATION

For example, the calculation of the force exerted in a radial magnetic bearing is a relatively complicated. The permanent magnet magnetic bearings are composed of two rings in magnetic repulsion. Each magnet may be likened to two nested cylinders, representing the external volume, the other representing the internal volume. The problem of calculating the interaction forces between the two rings can be treated as one of four cylinders in interactions. The radial force can be expressed as an analytical expression involving a limited set of sums

. SUMMARY AND CONCLUSION

The calculation of interaction forces between two cylindrical magnets had never been resolved so far as analytical. Following the development of the analytical calculation of parallelepiped magnets, the calculation of these forces between two cylinders can be obtained in analytical form by decomposition to a finite number of elements.

This type of calculation allows the calculation of many systems that uses the forces of interaction between the cylinder magnets. For example it can be used to directly calculate permanent magnet magnetic bearings.

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