Cordis Anima Physical Modeling and Simulation System Analysis

Alexandros Kontogeorgakopoulos, Claude Cadoz

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Kontogeorgakopoulos Alexandros*, Cadoz Claude †
* ACROE-ICA laboratory INPG, Grenoble, France, Alexandros.Kontogeorgakopoulos@imag.fr
† ACROE-ICA laboratory INPG, Grenoble, France, Claude.Cadoz@imag.fr

Abstract — **The CORDIS-ANIMA physical modeling system is one of the oldest techniques for digital sound synthesis via physical modeling. This formalism which is based on the mass-interaction paradigm has been designed and developed by ACROE in several stages since 1978. The aim of this article is to enlighten some special and particulars features of this approach by exploiting it mathematically. Linear CORDIS-ANIMA (CA) models are studied and presented using several useful system representations like system function input/output external descriptions, state space internal descriptions, finite difference model, modal decomposition, electrical analogous circuits, CA networks and digital signal processing block diagrams.**

I. INTRODUCTION

Physical modelling is the most active field in digital sound synthesis nowadays. Mechanical and acoustical systems governed by physical laws are modelled using several mathematical formalisms and simulated with the use of numerical techniques and digital computers.

In the domain of computer music various methodologies mainly oriented to musical applications than to musical acoustics have been introduced and proposed in the last thirty years:

- simulation techniques based on numerical analysis i.e. the finite differences scheme [1]
- the mass-interaction modular scheme adopted by CA [2]
- other lumped models used to approximate parts of physical systems like a singer's vocal folds or a brass player's lips
- the wide popular wave guide scheme oriented in the wave equation [3]
- the modal approach where the vibrating structure is represented through a series of elementary oscillators [4][5]
- algorithms based on the coupling of simple dynamic non-linear systems with numerical filters [6].
- State Space modular methodologies [7]
- the functional transformation method which provides a multidimensional transfer function by the application of a suitable functional transformation [8]

Every physical modelling scheme reflects various aims and may be considered "optimal" according the preliminary philosophy that has been conceived. In 2003 Castagne and Cadoz proposed 10 general criteria for evaluating physical modelling techniques oriented to music creation [9].

PM1 (*): How Efficient is the Algorithm?
PM2: How Faithful are the Synthesized Sounds?
PM3: How Diverse are the Categories of Instruments that can be Modeled?
PM4: Is the Scheme Exclusively Dedicated to Sound Synthesis or more General?
PM5: How Robust is Sound 'Plausibility'?
PM6: How Modular is the technique?
PM7: How Intuitive and Effective is the Associated Mental Model?
PM8: How Deep is the Modeling Process Enabled By the Scheme?
PM9 (*): Do Generation Algorithms Exist?
PM10: Is There a Friendly Musician-Oriented Environment for Using the Scheme?

Fig. 1. 10 Criteria for evaluating physical modeling techniques.

Vibrating structures like all kind of elastic bodies, strings, membranes, bars, plates can be considered as linear deformable objects. Each physical modelling scheme represents those physical objects differently in a discrete-time and discrete-space form. Those various structures may often be mathematically equivalent even if they represent and realize the physical object using different formalisms. However their realizations may not at all be equivalent: different realizations leads to system configurations with different complexity, different memory requirements and different finite-precision "after-effects".

It is appealing in some cases to pass from one formalism to another and represent a certain model with other mathematical schemes. For example in recent years there has been an interest to combine the waveguide scheme with finite difference methods and lumped elements to enhance the modelling possibilities of digital wave guides [10]. Also, the digital waveguide structure has been used by other models as the functional transformation method mostly for his algorithmic efficiency [11].

A further essential motivation to use several formalisms is the analysis. This corresponds to the 9th criterion of Castagne and Cadoz criteria for evaluating a physical modelling scheme as illustrated to figure 1 or to the 10th criterion of Jaffe criteria for evaluating synthesis technique [12]. It is evident that as every formalism offers a different type of system description, it is useful to choose the appropriate one for the desired analysis purposes. These purposes may be strictly scientific that can help the study and the development of the physical modelling scheme or more artistic to offer modelling techniques based on the paradigm of synthesis by analysis. These reasons stimulated us to study how the CA formalism is transformed to other representations.

A crucial question at this point might be:

"Why we bother changing formalisms for the analysis and synthesis while it is possible to stick with the most convenient formalism and start the simulation directly?"

Apart from the fact that Even if the model has an equivalent mathematical description the different configuration will produce slightly different *simulacrum* - as we have already mentioned that previously-, there is a much more vital and essential reason. Every formalism permits and allows a different way of manipulation and control due to its structure and to the mental image that conveys to the user. Consequently one user can use other representations for the analysis and even the concept of its model and then pass to a preferable physical modelling scheme for further manipulation and musical creation.

In this article, after taking a brief look in the CA physical modelling and simulation system and the GENESIS graphical environment based on this system, we will transfer it and represent it by other system formalisms used widely in the signal processing, in the control and in the physical modelling domain: CA network representation, block diagram representation, finite difference representation, state space representation, electrical analogous circuits representation, modal representation and input/output transfer function representation.

II. CORDIS ANIMA AND GENESIS

CORDIS-ANIMA is a real-time mass-interaction physical modeling and simulation system [2]. This lumped parameter formalism starts from the quantization of physical matter and time. It allows designing and simulating virtual objects that can be seen, heard and handled. Moreover it offers a complete internal description of the system in all the levels of the simulation, as it provides all the essential information for its internal physical structure. All the parts of the physical objects are modeled without the use of "black boxes" The simulated objects are composed only from two types of elements, called modules:

- <MAT> modules represent punctual material elements. The most used is the MAS module, which simulates an ideal inertia. <MAT> modules are elementary subsystems and can be characterized in terms of their input/output relationships.

- <LIA> modules represent physical interactions between pairs of <MAT> modules. Available interactions are based on linear or nonlinear elasticity and friction. <LIA> modules are elementary subsystems and can be characterized in terms of their input/output relationships.

Thus, CA models are combinations of several <MAT> and <LIA> modules based on some simple construction rules.

Position and force are the two fundamental variables upon which CA modules operate. At each sample a <LIA> computes two opposite forces according to the relative distance and/or velocity of the two <MAT> it links while a <MAT> computes its position according to the forces it receives from the <LIA> modules it is linked with. The algorithms can be found on [13].

The CA system is designed and intends to simulate the "Physical Instrumental Interaction". In this interaction, the "ergotic function" [14][15][16] which is what allows in a direct way to act on the physical instrument and to feel it by the haptic sense, plays an essential role. This is what permits to perform the gesture in an expressive way and then to produce and even transform expressively sounds. In the digital sound synthesis or transformation, the "ergotic function" can be supported by specific force-feedback gestural transducers [17][18].

GENESIS [19] is a graphical environment for musical creation based on CA. The user builds CA models at an elementary level, since models are created by direct graphical manipulation and connection of individual modules on a virtual workbench. A number of higher-level tools are available for editing multiple parameters at the same time, generating large structures, visualizing models during simulation, etc. GENESIS implements ten types of modules. While CA does not specify the dimensionality of the modules, GENESIS' simulation space is one-dimensional. <MAT> modules can only move in a perpendicular direction to the workbench, and distances and velocities are computed along this axis. For convenience, graphical manipulations take place in the 2D-space of the workbench, but the position of the modules on this plane have absolutely no consequence on the simulation: the workbench representation is only topological.

The normal set of GENESIS' building blocks is composed of:

- Linear modules: ideal mass (MAS), fixed point (SOL), second-order damped oscillator (CEL), elasticity (RES), friction (FRO), elasticity and friction combined (REF);
- Nonlinear interactions: the BUT and the LNL;
- Output modules: the SOX and the SOF, which respectively record a position and a force signal.
- Input modlules: the ENX and the ENF, which read an input file and respectively translate its data into a time-changing position (ENX) or force (ENF).

III. CORDIS ANIMA NETWORKS, BLOCK DIAGRAMS AND ELECTRICAL ANALOGOUS CIRCUITS

A. CORDIS-ANIMA network

In CORDIS-ANIMA formalism a physical object is modelled as a modular assembly of elementary mechanical components. Hence it is straightforward to represent the model as a plane topological network whose nodes are the punctual matter elements <MAT> and links are the physical interaction elements <LIA> (figure 2). The simulation space used for sound and musical applications is limited to one dimension. Forces and displacements are projected on a single axis, perpendicular to the network plane. Consequently the geometrical distance between two <MAT> elements is reduced to their relative distance on the vibration axis [20].

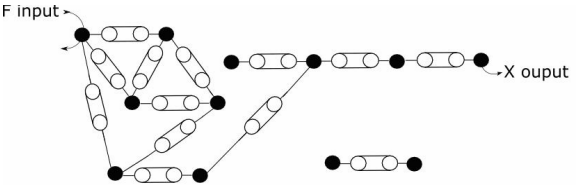

Fig. 2. A CORDIS-ANIMA network.

By using linear <LIA> elements such as springs and dumpers whose parameters do not change with time, we obtain a system which is linear and time-invariant. In table I we illustrate the algorithms for the <MAT> and the linear <LIA> modules. A model is fully described by its topology-network, its values of inertia $M=m$, elasticity $K=k/F_s^2$ and viscosity $Z=z/F_s$ where F_s is the sampling rate and its initial conditions $x0$ and $v0$. In GENESIS all CA models are designed graphically directly on the workbench as networks using a simplified representation of CA networks enriched with colours.

This highly modular representation gives the possibility to design a model based on intuition. As the basic building elements have actually a strong physical counterpart they remain pertinent to human senses and create a very realistic mental model. Therefore the design phase allows a purely physical approach carried out by "Physical Thinking". Castagne points out [19] "…Models are more easily internalized as representations of real objects than with more mathematical or signal processing physical modelling techniques…" Furthermore it is very often possible to guess and predict the general behaviour of a model by examining its network without the use of mathematical analysis tools.

CA networks offer directly another type of control based on the "Physical Instrumental Interaction". In this control scheme we don't affect the parameters of the model -even though it is possible and previewed within the CA system- but we apply forces to the <MAT> elements of the model using <LIA> elements like in reality. It is straightforward that this type of control is totally physical and energetic coherent. Since physical models enable an intuitive representation of the action we perform with real objects we can imagine several physical gestures to manipulate and control our model: dumping, pulling, pushing, e.t.c. This is still feasible for non real-time simulations and without the use of force feedback gestural interfaces but by designing models that simulate the physical gesture. The deferred-time simulation permits to design accurate and valid models of the control gesture with a precision that is not possible in the real-time situations

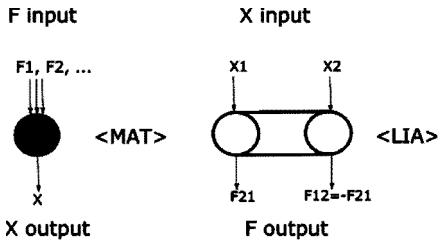

Fig. 3. <MAT> and the linear <LIA> modules.

TABLE I.
ALGORITHMS FOR THE <MAT> AND THE LINEAR <LIA> MODULES

$< MAT > \quad m \neq \infty$ $x_i(n) = 2x_i(n-1) - x_i(n-2) + \frac{1}{m_i F_s^2} \sum F^{j->i}(n-1)$
$< MAT > \quad m = \infty$ $x_i(n) = c$
$< LIA > \quad linear$ $F_K^{i->j}(n) = k_{ij}[x_i(n) - x_j(n)]$ $F_R^{i->j}(n) = F_s z_{ij}[x_i(n) - x_i(n-1) - x_j(n) + x_j(n-1)]$

One important drawback of CA networks is that they do not give information about their functional structure. The algorithms and realization structures beyond the model do not appear in this representation. Consequently it is not possible to implement directly the model using only the information furnished from these diagrams. On the other hand we may pass over this problem if the few basic algorithms of each module accompany those networks. Even in this case the precise implementation of the model is hidden. CA as a simulation language has been designed and to offer an optimal implementation correspondence to its modularity. It is clear immediately the reason we identify CA as a modelling and simulation language.

Another more important weakness of the network representation is that it does not offer a direct mathematical analysis. The tools of linear algebra and calculus are not directly applicable. Hence it is inconvenient to set off right away a mathematical study of these networks even if it is possible.

B. Block diagrams

The block diagrams offer a convenient structural representation of the computational algorithm of a system. This kind of representation using interconnected basic building blocks as adders, multipliers and unit delays, is the first step in the software or hardware implementation of a digital signal processing system [21]. The interconnections may be in cascade, in parallel or in feedback. Block diagrams contain all the information for the modelling and the simulation of a physical system.

It is interesting to notice that as digital signal processing block diagrams offer a decomposition of the system on interconnected subsystems performing *elementary mathematical operations*, CA networks suggest a similar decomposition on subsystems performing "*elementary physical operations*". Each subsystem in both cases is characterized by its input/output relationships. The *mathematical blocks* are near to the computing machine and allow a more "*Signal Thinking*", while the *physical blocks* are near our mental image for the physical world and allow a more *"Physical Thinking"*. The first approach is more symbolic/mathematical and the second one more material/physical. This type of CA modularity where each element preserves an "experimentable" physical nature and quality was one of the basics demands on the design of this formalism.

From the input/output relationships of the CA modules given in figure 3 we can construct the block diagrams.

There are various ways to realize or simulate these algorithms and consequently represent them by these elementary functional elements. In figure 4 we represent the CA linear modules using ordinary signal processing block diagrams. As we can see the subsystems are always interconnected using feedback links. This is a direct derivation from the Newtonian Mechanics.

When we represent a CA model with block diagrams, its physical constitution is no more easily perceptible or detectible. A straight consequence of this entirely functional point of view is the lost of the

"Physical Instrumental Interaction". In this case the control problem normally is taking the form of a mapping between the control signals and the available input parameters of the system. Nearly every physical modeling approach follows this non-physical control paradigm.

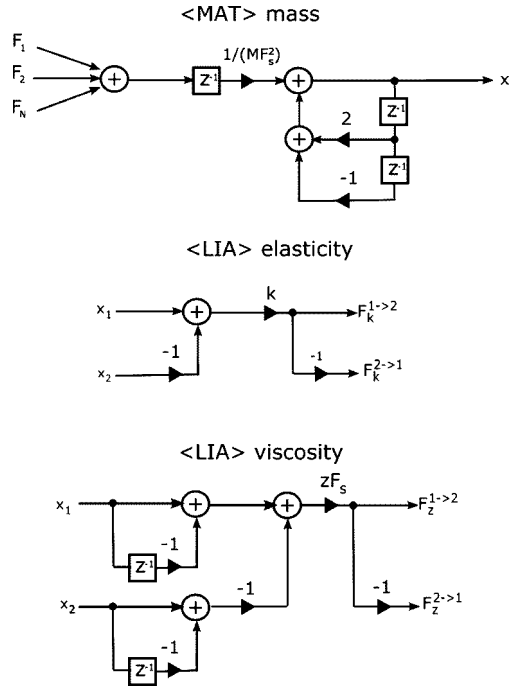

Fig. 4. Block diagrams of CA linear modules.

There are certainly many advantages in representing the physical model in a block diagram form. Mitra summarizes them as follows [21]: i) ease in the derivation of the computational algorithm by inspection ii) ease in the determination of input/output relation iii) ease in the manipulation of diagram to derive an "equivalent" with different computational algorithm iv) ease in the determination of the hardware requirements and the complexity v) ease in the developing of different block diagram representations from transfer functions. We may add to this list the ease to determine the complexity of the computational algorithm and the ease to detect delay-free loops i.e feedback branches without delay elements.

We must note that the one important reason for the double discretization scheme –centered for the acceleration and backward for the velocity- adopted by the CA formalism was the delay-free loop difficulty. Within the framework of physical modeling, this problem leaded to the notions of T-simulable and *-simulable objects [2].

C. *Electrical analogous circuits*

An electrical analogous circuit of a mechanical system is an electrical circuit in which currents/voltages are analogous to velocities/forces in the mechanical system [22]. If voltage is the analog of force and current is analog of velocity the circuit is called impedance analogous. In a similar way if voltage is the analog of velocity and current is the analog of force the circuit is called mobility analogous. In Electroacoustics, mechanical and acoustical systems are modelled with electrical circuits and simulated using digital computers by special software packages as SPICE®.

CA models are lumped systems and consequently can be represented easily by electrical circuits. In that case the basic linear building modules <MAT> and <LIA> correspond to one-ports. The immittance functions of these one-ports in the z domain are calculated in chapter five.

Sometimes confusingly the CA formalism is thought that it has been conceived as a way of discretizing analog circuits containing resistors, capacitors and inductors or its mechanical counterparts. We hope that this article will make clear that the concept and the design of this physical modelling and simulation language goes further from this limiting point of view. It would be interesting though to compare it with the Wave Digital Filters as developed by Fettweis [23], which were used principally for the discretization of analog filters. This comparison could offer an interesting interface and link with the digital waveguide physical modelling scheme, which share a similar formalism with them.

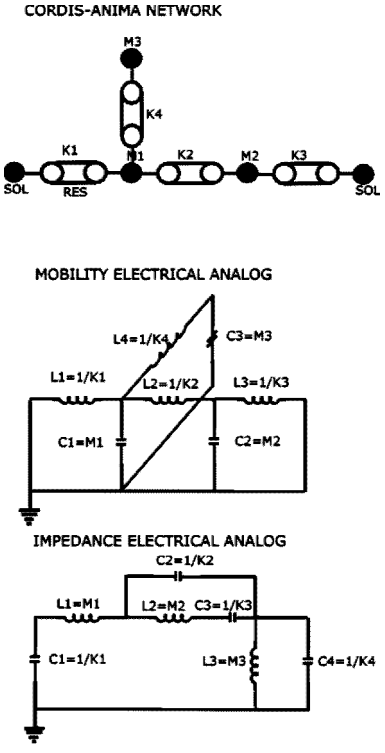

Fig. 5. Electrical analogous circuits of a CA model.

It is far more convenient to represent a CA model by its mobility electrical analog where the voltages are analogs of individual velocities. In the other case the currents are analogs of velocity differences, which pose

difficulties since in CA all positions are measured compared to a predefined point. The necessary steps for forming analogous electrical circuits are described clearly in [22]. In figure 5 we illustrate the mobility analogous and the impedance analogous of a CA model. The mobility analog circuit can be designed directly from a CA network simple by using as reference this example. A simple methodology is described below [24]:

- If we have n mobile masses we will have n+1 nodes
- One of the n+1 nodes is the ground. The other nodes are connected with that node with the correspondents masses dipoles.
- When two masses (or one mass and the ground) are connected with a interconnection-link, we will use the correspondents dipoles for that interconnection.

The impedance analogs circuits of structures further complicated than strings unfortunately do not offer intuitive networks and their design demands a certain but simple manipulation of the mobility analog network. There are three rules that we have to respect for the representation of a CA network by a Kirchhoff network: i) in CA we define the velocity by a stable point: the ground. So in the equivalent Kirchhoff network all the mass dipoles are connected with the same node: the ground (=infinity mass-no movement) ii) in CA it is not possible to connect two interactions-links in serial way without a mass in the middle. Therefore in the equivalent Kirchhoff network every node has to be connected at one side with a mass dipole. iii) in CA two masses are connected with one interaction-link of elastic and/or friction type. Therefore in the equivalent Kirchhoff network the correspondent nodes have to be connected with the dipoles that serve this interconnection. These dipoles can only be interconnected in parallel mode.

The representation of CA models by electrical circuits is helpful. Most of the techniques and theories conceived in the field of electrical networks are directly applicable: Kirchhoff laws, Thevenin and Norton theorems, Impedance analysis, calculation of two-ports and scattering parameters e.t.c. A number of methods developed for filter design and synthesis may be adopted as well. Additionally, many acoustical and mechanical systems modelled in elecroacoustics may be simulated with CA networks.

IV. FINITE DIFFERENCE MODEL AND STATE SPACE MODEL

A standard approach to study linear vibrating systems involving more than one degree of freedom is the finite derivative representation using matrices and vectors [25][26]. The equations of motion in the general case are given by:

$$[M]\ddot{X}_t + [R]\dot{X}_t + [K]X_t = F_t \tag{1}$$

where the matrices [*M*], [*R*], and [*K*] respectively represents the inertia, the viscosity and the elasticity of the system, the vector *X* the position of the masses and the vector *F* the external forces. If the system has the form of linked masses by viscoelastic forces moving in one-dimensional space as in CA then these matrices are symmetric [30]. The proportional viscosity networks where the matrices [*K*] and [*R*] are related by the expression

$$[R] = a[M] + b[K] \tag{2}$$

are diagnosable by the same transformation. These networks cover a sufficient number of interesting situations.

Using the double discretization scheme employed by CA we obtain a similar expression:

$$[M](\frac{X_{n+1} - 2X_n + X_{n-1}}{T_s^2}) + [K]X_n + [R](\frac{X_n - X_{n-1}}{T_s}) = F_n \Rightarrow$$
$$[M]X_n + (\frac{[K]}{F_s^2} + \frac{[R]}{F_s} - 2[M])X_{n-1} + ([M] - \frac{[R]}{F_s})X_{n-2} = \frac{F_{n-1}}{F_s} \tag{3}$$

The last one is a finite difference problem. In this equation F_s denotes the sampling rate in Hertz and T_s the sampling period in seconds. Apparently equations (1) and (3) are diagonalizable by the same transformation under the condition of proportional viscosity.

The finite difference models offer a direct way of implementation by iterations. Furthermore they can be handled and treated by numerous software dedicated to scientific computation as matlab®. Optimization problems, the inverse problem and many others are convenient to be approached by this representation.

An interesting remark is that similarity transformations produce "equivalent" CA networks with different internal structure. Hence we are able to modify the physical structure of the system and conserve the initial transfer function (chapter 5). It would be interesting to develop similarity transformations to pass from one classical topology to another i.e. transform a membrane to a sting.

The equations (1) and (3) can be written in first-order form [26]:

$$\dot{Y}_t = [A_c]Y_t + [B_c]F_t, \quad where \quad Y_t = \begin{bmatrix} X_t \\ \dot{X}_t \end{bmatrix}$$
$$[A_c] = \begin{bmatrix} 0 & [I] \\ -[M]^{-1}[K] & -[M]^{-1}[R] \end{bmatrix}, \quad [B_c] = \begin{bmatrix} [0] \\ [I] \end{bmatrix} \tag{4}$$

$$Y_{n+1} = [A_d]Y_n + [B_d]\frac{F_{n-1}}{F_s}, where \quad Y_n = \begin{bmatrix} X_n \\ X_{n-1} \end{bmatrix}, \quad [B_d] = \begin{bmatrix} [I] \\ [0] \end{bmatrix}$$
$$[A_d] = \begin{bmatrix} -\frac{[M]^{-1}[K]}{F_s^2} - \frac{[M]^{-1}[R]}{F_s} + 2[I] & -[I] + \frac{[M]^{-1}[R]}{F_s} \\ [I] & 0 \end{bmatrix} \tag{5}$$

This formulation is called state space and is widely used in the control system theory. It offers internal description of the system since not only a relationship between the input and the output signals is involved, but it also provides information about the state of the system by the state variables. The equations (4) and (5) are the state equations. The output equation is computed from the state vector Y_t or Y_n and the input vector F_t *or* F_n*:*

$$Z_t = [C_c]Y_t + [D_c]F_t \tag{6}$$

$$Z_n = [C_d]Y_n + [D_d]F_n \tag{7}$$

We reached the state equation from the finite difference representation of CA. We could find out other state equations by choosing different state variables. The state of a system at a certain moment is the set of variables, which are sufficient to determine the future behaviour of the system. Accordingly by choosing another set we derive to another expression. The analogous electrical circuit description of CA for example imposes as to work easier with forces and positions via a systematic procedure described in [27] and not with velocities and positions as in equation (5).

State space models are remarkably interesting because they offer a link between most of the physical modelling formalisms -as these intend to provide internal description of the systems- and the signal processing world. Depalle et all [7] provide a methodology for modular construction of musical instruments within this formalism. Further more these models propose a direct realization scheme [26].

V. System functions and modal representation

The system function for a discrete time system may be defined in the most general case as the ratio of the Z transform of the output response to the Z transform of the input excitation. All initial conditions are set to zero. For a CA model the excitation or the response may be either a force or a position.

A system function can have the form of an immittance function or a transfer function [29]. An immittance denotes both impedances and immittances. The immittance function is referred as driving-point immittance when it relates the force and the position at the same <MAT> or <LIA> element (*Zd* for driving-point impedance, *Yd* for driving-point admittance) and as transfer immittance when it relates them at different elements (*Zt* for transfer impedance, *Yt* for transfer admittance). The transfer function is referred as position transfer function *Hx* when both the excitation and the response are positions and as force transfer function *Hf* when they are both forces. The above cases are illustrated in figure 6.

Additionally the immittances of the linear CA modules are given in table II. This approach is better understood in the electrical representation of CA where the basic modules correspond to one-ports. The parameters of mass, stiffness and damping are not given the International System but in a system more convenient for GENESIS: $M=m$, $K=k/F_s^2$, $R=r/F_s$ where F_s is the sampling rate. We use R instead of Z for the dumping parameter to avoid confusion with the Impedance and the z variable of the z-Transform.

An easy method to compute the system function of a CA model is to use its electrical analog and perform all the mathematical operations in the z-domain where they are expressed algebraically. The rules concerning the immittance combinations in series and in parallel appear very helpful for this task.

One other method to compute the transfer admittance is to use the state space model. The transfer function matrix is given by the expression:

$$Yt(z) = [C_d](z[I] - [A_d])^{-1}[B_d] + [D_d] \Rightarrow$$
$$Yt(z) = \frac{1}{\det(z[I] - [A_d])}[C_d]adj(z[I] - [A_d])[B_d] + [D_d] \quad (8)$$

$Yt_{ij}(z)$ is the admittance function relating the output Z_{ni} to the input F_{nj} (equation (5)-(6)). The zeros of the polynomial $det(z[I]-[A_d])$ are the characteristic roots of the system.

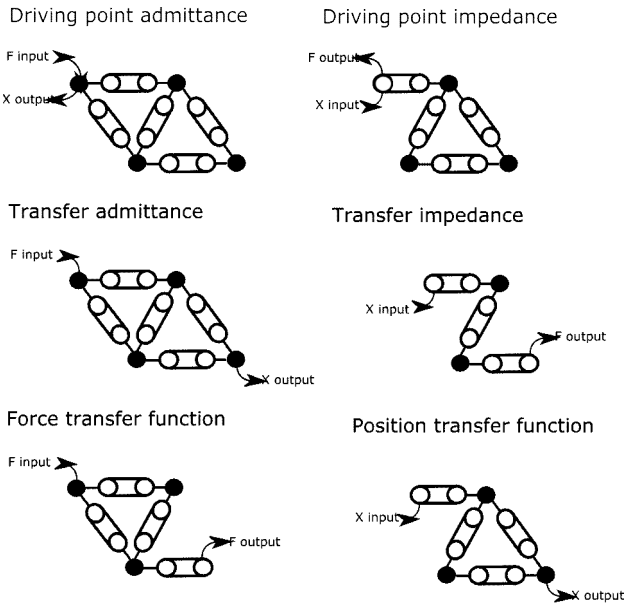

Fig. 6. System functions in CA

TABLE II.
IMMITTANCES OF THE LINEAR CA MODULES

<MAT> Inertia	$Zd_M(z) = \frac{X(z)}{F(z)} = \frac{Mz^{-1}}{1-2z^{-1}+z^{-2}}$ $Yd_M = \frac{1}{Zd_M(z)}$
<LIA> Stiffness	$Zd_K(z) = \frac{X(z)}{F(z)} = K \quad Yd(z) = \frac{1}{K}$
<LIA> Dumping	$Zd_R = \frac{X(z)}{F(z)} = Rz^{-1} \quad Yd_R = \frac{1}{Rz^{-1}}$

It is preferable to compute the characteristic polynomial by making the matrix $[A_d]$ diagonal. In this case we obtain the modal representation of the system. The modes are decoupled and we may write each mode independently. Consequently we obtain a 2N parallel first order systems where N is the number of masses of our CA system. It is clear that the diagonalized state space form is equivalent to a partial-fraction expansion of a transfer function [30].

If our system forms a viscosity compatible network, we can combine the conjugate poles to obtain a system of N parallel classical two-pole filters. Then our model can physically be represented as a set of independent elementary oscillators. It is more convenient to reach the modal description of a CA system from its finite

difference (equation (2)) or its finite derivative model (equation (1)). The modal representation of a network ([*M*], [*K*], [*R*]) is characterized by the transformation matrix [*Q*], where each column represents a mode shape and the diagonal stiffness and viscosity matrices, [K_m] and [Z_m]. All these are given by the equations [5]:

$$[Q]^t[M][Q]=[I],\ [K_m]=[Q]^t[K][Q],\quad [R_m]=[Q]^t[R][Q] \quad (9)$$

The matrix [*Q*] is calculated in GENESIS by the Jacobi transformation algorithm [31].

Modal models have many advantages. As Djoharian points out "…modal modelling bridges the gap between the structural representation (geometric an dynamic) of the vibrating system objects and their perceptual properties". They preserve in a certain way the physicality of the modelled object and additionally they furnish directly its perceptual characteristics. Moreover they are very simple in terms of their computer simulation. The modal data (frequencies, damping coefficients and mode shapes) can be obtained mathematically or by physical measurements. Therefore it is easy to design accurate linear models like an instrument resonators by using these data.

The modal representation is really useful when we need to pass from Laplace s-domain to z-domain. The double discretization scheme adopted by CA unfortunately does not permit the use of a direct transformation method from the s-domain to the z-domain. Nevertheless this can be done for some case (viscosity compatible networks) if we use the modal expressions and compare the form of the elementary oscillator in the continuous time domain and in the CA formalism.

VI. FINITE DIFFERENCE MODEL AND STATE SPACE MODEL

In this article, CORDIS-ANIMA physical modeling and simulation system has been represented and analyzed by the use of other useful mathematical formalisms. The aim for this study was to enlarge the analysis tools of this mass-interaction approach and to add new modeling strategies based on a more mathematical framework; further more, was to make clear the particularities of this classical physical modeling technique.

The choice of the representation language is extremely important especially for artistic situations like the musical creation. The CA was originally designed to fulfill the necessary requirements as an artistic tool within the domain of physical modeling. Its simple but efficient structure offers a different perspective in the modeling process, giving an experimental and intuitive character and hopefully a much more artistic nature.

We demonstrated that this formalism can be easily analyzed and combined with other mathematical representations. This capability strength furthers more its "modeling potential" and verifies its strong theoretical background.

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