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# Ranking of the nonlinearities of electrodynamic loudspeakers

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1 R. Ravaud, G. Lemarquand<sup>1</sup>, T. Roussel and V. Lemarquand

2 **Abstract**

3 The aim of this paper is to present a way of ranking the nonlin-  
4 earities of electrodynamic loudspeakers. For this purpose, we have  
5 constructed a nonlinear analytic model which takes into account the  
6 variations of the small signal parameters. The determination of these  
7 variations is based on a very precise measurement of the electrical  
8 impedance of the electrodynamic loudspeaker. First, we present the  
9 experimental method to identify the variations of these parameters,  
10 then we propose to study theoretically the importance of these nonlin-  
11 earities according to the input level or the input frequency. We show  
12 that the parameter which creates the most distortions is not always  
13 the same and depends mainly on both the input level and the input  
14 frequency. Such results can be very useful for optimizing electrody-  
15 namic loudspeakers.

16 **Keywords:** electrodynamic loudspeaker, nonlinearities, small signal pa-  
17 rameters, nonlinear modeling

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# 18 **1 INTRODUCTION**

19 The reference model describing the electrodynamic loudspeaker was designed  
20 by Thiele and Small [1]. Their model was adapted to describe an electro-  
21 dynamic loudspeaker as a linear system. This model is very useful because  
22 it is very simple to use and its parameters can be conveniently presented in  
23 terms of an electric analog circuit [2]. However, a loudspeaker shows non-  
24 linearities that produce distortions. These nonlinearities have three major  
25 sources [3][4]: the suspensions, the diaphragm [5]-[7] and the motor [8]-[10].  
26 They have been largely studied and there are many attempts to model them  
27 [11]-[16].

28 However, most of the papers dealing with the nonlinearities of electrody-  
29 namic loudspeakers do not take into account the variations of the mechanical  
30 damping  $R_{ms}$  or the equivalent mass  $M_{ms}$ . But these parameters are also  
31 nonlinear and their variations must be taken into account in order to pre-  
32 cisely characterize the distortions created by electrodynamic loudspeakers.  
33 It can be noted that alternative loudspeakers structure without iron have  
34 been proposed and studied [17]-[19] which use ferrofluid suspensions in order  
35 to delete the classical distortions generated by electrodynamic loudspeakers  
36 [20]-[22].

37 This paper has two objectives. In the first part, we present an experi-  
38 mental method which allows us to determine precisely the variations of the  
39 small signal parameters according to the input current. Our experimental  
40 method is based on a very precise measurement of the electrical impedance of

41 the electrodynamic loudspeaker. Indeed, the measurement accuracy is about  
42  $10^{-4}$  Ohm for both the real and imaginary parts of the electrical impedance.

43 However, all our approach does not use the classical definition of an elec-  
44 trical impedance. An electrical impedance, commonly defined by its ratio  $\frac{U}{I}$   
45 (in the frequency domain) does not depend on input current. Indeed, this  
46 electrical impedance is only defined for linear systems.

47 However, our experimental measurement shows that this ratio  $\frac{U}{I}$  depends  
48 on  $I$ . This can be easily verified experimentally by measuring the experimen-  
49 tal impedance for different input currents. Such a result is in fact known for  
50 scientifics involved in the modeling of nonlinear systems: a nonlinear system  
51 depends generally on the input level. In the case of an electrodynamic loud-  
52 speaker, the electrical impedance, that is to say, the ratio  $\frac{U}{I}$  depends on the  
53 input level.

54 We also show that it exists a bijective relation between the input current  
55  $I$  and the position of the voice coil  $X$ . This result is of great importance  
56 because it shows that the non linear parameters describing an electrodynamic  
57 loudspeaker ( $Bl(x)$ ,  $R_{ms}(x)$  and  $L_e(x)$ ) can also be described as parameters  
58 depending on input current (in the time-domain or in the frequency-domain).  
59 It can be noted that, strictly speaking, the force factor  $Bl$  which is commonly  
60 used in analog circuits represents the mean induction field times the length  
61  $l$  of the voice coil inside the air gap. Therefore, when the voice coil position  
62  $x(t)$  is sinusoidal, in complex notations, the force factor should be seen as a  
63 mean force factor  $\tilde{B}l$ .

64 In the second part, we discuss the behaviour of each nonlinearity accord-

65 ing to both the input level and the input frequency. It is noted that all this  
66 part is treated theoretically. Indeed, a very good agreement between our an-  
67 alytical model and experimental measurements has confirmed that our model  
68 can be used for modelling the distortions created by an electrodynamic loud-  
69 speaker. We show here that the lumped parameter whose relative variation  
70 according to input current is the most important is not always the same and  
71 depends greatly on both the input level and the input frequency.

72 Another drawback known in electrodynamic loudspeakers is that it is a  
73 time-varying system. In fact, the electrical resistance increases with time and  
74 the mechanical compliance of the outer rims depends also on time. These  
75 dependence have been studied with a similar approach in previous papers.  
76 In this paper, some care has been taken in the experimental measurements  
77 for omitting these temporal effects.

78 This paper proposes a way of ranking the nonlinearities of electrody-  
79 namic loudspeakers. To our knowledge, this way of characterizing these non-  
80 linearties has never been used for electrodynamic loudspeakers. However,  
81 many new phenomena can be seen by using such an experimental approach.  
82 This is why such results are very interesting for many manufacturers involved  
83 in the design of electrodynamic loudspeakers. We precise here that we use  
84 for the rest of this paper the small notations  $x(t)$  and  $i(t)$ , the displacement  
85 of the voice coil and the input current in the voice coil in the time-domain  
86 and the capital letters  $(X, I, U)$  in the frequency domain. We also precise  
87 that  $U$  is the input voltage.

## 88 **2 Classical description of a loudspeaker and** 89 **its limits**

### 90 **2.1 The small signal model using lumped parameters**

91 A common way of characterizing an electrodynamic loudspeaker is to measure  
92 its electrical impedance  $Z_e$ . Its theoretical expression is well known and is  
93 given by Eq. (1).

$$Z_e = R_e + \frac{jL_e w R_\mu}{jL_e w + R_\mu} + \frac{Bl^2}{R_{ms} + jM_{ms}w + \frac{1}{jC_{ms}w}} \quad (1)$$

94 where all the parameters are defined below.

95  $Bl$  = electrodynamic driving parameter [ $T.m$ ]

96  $R_{ms}$  = mechanical damping, drag force [ $N.s.m^{-1}$ ]

97  $C_{ms}$  = mechanical compliance of suspension( spider, outer rim)[ $N^{-1}.m$ ]

98  $L_e$  = inductance of voice coil [ $H$ ]

99  $M_{ms}$  = equivalent mass of moving voice coil, cone and air [ $kg$ ]

100  $R_e$  = electrical resistance of voice coil [ $\Omega$ ]

101  $R_\mu$  = eddy current resistance [ $\Omega$ ]

102  $w$  = angular frequency [ $rad.s^{-1}$ ]

103

104 These parameters can be represented in terms of an analog circuit in Fig.

105 1.

106 According to (1), the electrical impedance  $Z_e$  does not depend on  $I$ . How-  
107 ever, an experimental measurement shows the contrary. Indeed, the real part  
108 and the imaginary part of this electrical impedance are represented by using  
109 the Nyquist diagram in Fig 2 and we see that this electrical impedance de-

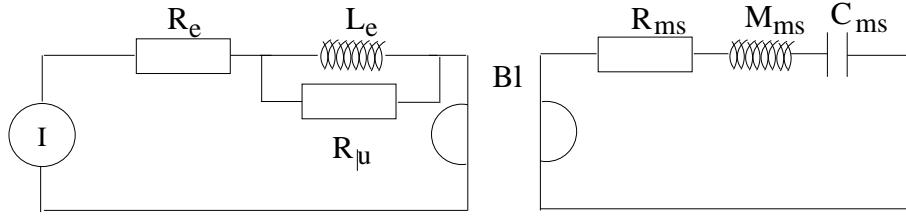


Figure 1: Representation of the lumped parameters in terms of an analog circuit

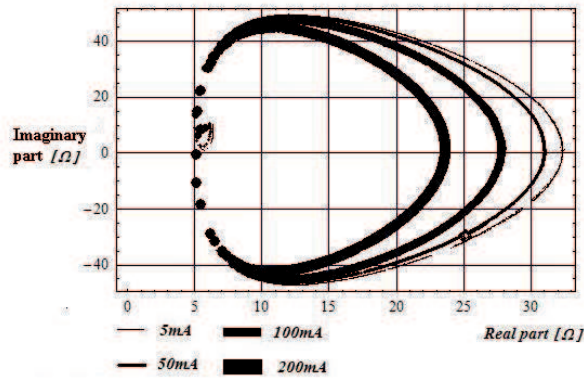


Figure 2: Experimental representation in the complex plane of the electrical impedance of the electrodynamic loudspeaker for four coil current levels (5mA, 50mA, 100mA, 200mA)

110 depends on the input current. Consequently, this electrical impedance can be  
 111 seen as a nonlinear electrical impedance. We denote it  $Z_e^{(NL)}(I)$ . In short,  
 112 we can say that the classical electrical impedance  $Z_e$  characterizing an elec-  
 113 trodynamic loudspeaker is not constant when the input current varies. Con-  
 114 sequently, we use the notation  $Z_e^{(NL)}(I)$  to describe the electrical impedance.  
 115 However, strictly speaking, the term -electrical impedance- should not be  
 116 used to describe a nonlinear system like an electrodynamic loudspeaker.

117 As a consequence, the lumped parameters defined in Eq. (1) depend on

118 the input current. The main problem is thus to know how to characterize  
119 their dependence according to the input current. According to the current  
120 state of art, cited in the introduction of this paper, the dominant nonlinear-  
121 ities in electrodynamic loudspeakers are  $Bl(x)$ ,  $C_{ms}(x)$  and  $L_e(x)$ . However,  
122 it exists a relation between the voice coil position  $x$  and the input current  $i$ .  
123 This relation is shown in next section. Consequently, it can be noted that  
124 these nonlinearities can depend on  $x$  or  $i$ : it is in fact equivalent. In this pa-  
125 per, we choose to work directly with the input current  $i$  because  $Z_e^{(NL)}(I)$  is  
126 easier to determine experimentally. Consequently, thanks to this very precise  
127 experimental measurement, the variations of the lumped parameters can be  
128 determined according to the input current  $i$ .

## 129 **2.2 Relation between the input current $I$ and the voice** 130 **coil position $X$ according to the Thiele and Small** 131 **model**

132 This section presents the relation between the input current  $I$  and the voice  
133 coil position  $X$  with the linear approximation of the Thiele and Small model.  
134 It is noted here that the aim of this section is merely to show that it exists a  
135 bijective relation between the input current  $I$  and the voice coil position  $X$ .  
136 To find this relation, we use one differential equation describing the electro-  
137 dynamic loudspeaker and we use complex notations. We find:

$$\left( M_{ms}(jw)^2 + R_{ms}(jw) + \frac{1}{C_{ms}} \right) X = BLI \quad (2)$$

138 So, we can write:



$$X = \frac{Bl}{\left(M_{ms}(jw)^2 + R_{ms}(jw) + \frac{1}{C_{ms}}\right)} I \quad (3)$$

139 Therefore, we see that Eq. (3) is the bijective relation between the input  
 140 current  $I$  and the voice coil position  $X$ , in complex notation. This relation  
 141 shows that we can work either with the input current  $I$  or with the voice coil  
 142 position  $X$  so as to describe the nonlinear variations of the Thiele and Small  
 143 parameters.

### 144 **2.3 Relation between the input current $I$ and the voice** 145 **coil position $X$ according to the nonlinear model** 146 **used in this paper**

147 Strictly speaking, the relation between the input current  $I$  and the voice coil  
 148 position is nonlinear. Indeed, all the small signal parameters depend on the  
 149 input current. It is emphasized here that the transfer function describing the  
 150 ratio  $\frac{U}{I}$  is considered as the ratio of the part of response with fundamental  
 151 frequency  $w$  to the excitation. We call it a nonlinear electric impedance in  
 152 this paper but it is useful to precise that nonlinear systems cannot be de-  
 153 scribed with complex notations.

154

155 We describe in the next section a way of determining the nonlinear laws  
 156 of all the small signal parameters. Therefore, we precise here for the rest of  
 157 this paper that (3) can be written in the frequency-domain as follows(4):

$$X = \frac{Bl(I)}{\left(M_{ms}(jw)(I)^2 + R_{ms}(I)(jw) + \frac{1}{C_{ms}(I)}\right)} I \quad (4)$$

158 It is noted that the determination of the small signal parameters can be  
159 determined in the frequency domain. We explain in next section how to find  
160 these variations.

## 161 **3 Determination of the nonlinear variations** 162 **of the lumped parameters**

### 163 **3.1 Principle of the measurement**

164 We have shown in the previous section that the nonlinear variations of the  
165 lumped parameters describing an electrodynamic loudspeaker depend on the  
166 input current. We explain now how to find these variations.

167 The electrodynamic loudspeaker used is a boomer (mark: Eminence,  
168 number: beta 15 ). We use the electrical impedance of the electrodynamic  
169 loudspeaker because its measurement is very precise. Our measurement de-  
170 vice is a Wayne Kerr bridge which has an excellent precision ( $10^{-4}\Omega$ ). This  
171 experimental devices is dedicated to the impedance measurement and func-  
172 tions as a classical impedance bridge.

173 In order to measure the electrical impedance of a loudspeaker, it is mounted  
174 in a normalized plane in an anechoic room [23]. Dalmont [24] has shown that  
175 the acoustical impedance in these conditions is the same as the one when the  
176 loudspeaker is mounted in an infinite baffle in an anechoic room. We mea-  
177 sure the electrical impedance by varying the frequency and the coil current.  
178 So, we build an experimental impedance layer by using a Runge Kutta al-  
179 gorithm to determine all the experimental measured points. In other words,  
180 this measurement algorithm chooses the best measurement point according

181 to the gradient method. Such a measurement method allows us to detect all  
182 the subtle effects due to the nonlinearities in electrodynamic loudspeakers.  
183 This algorithm has been improved in relation to the one used in a previ-  
184 ous paper. In the previous paper, the algorithm took some experimental  
185 points by using constant intensities. In other words, our measurement sys-  
186 tem used the algorithm of gradient only in two dimensions for one intensity  
187 and then it was repeated for another intensity. For example, an intensity  
188 was fixed (for example 50 mA), and a two-dimensional algorithm allowed us  
189 to determine the measurement points, that is to say, our algorithm auto-  
190 matically settled one intensity and took some experimental points by using  
191 a method based on the gradient method. In this paper, the algorithm uses  
192 a Runge Kutta algorithm in three dimensions ( $Z_e, I, f$ ) and we can rapidly  
193 obtain an impedance layer which is very precise. Consequently, the temporal  
194 effects can be neglected. A two-dimensional representation of the electrical  
195 impedance is shown in Fig. 2: the imaginary part is a function of the real  
196 part ( it is Nyquist diagram for different coil currents). We can say that the  
197 Wayne Kerr bridge cannot supply currents greater than 0.2A. Consequently,  
198 the parameter variations are determined in this current interval.

199 Figure 3 confirms that the electrical impedance is a function of the input  
200 current.

## 201 **3.2 Determination of the nonlinear electrical impedance**

202 The next step is thus to determine the dependence of the small signal param-  
203 eters with the coil current so as to construct the nonlinear transfer function.

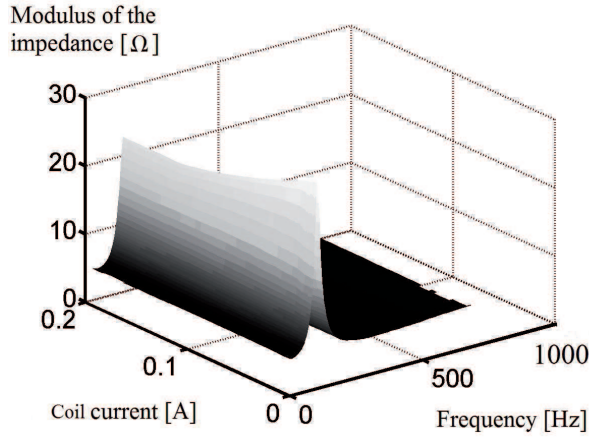


Figure 3: Experimental three-dimensional representation of the modulus of the electrical impedance of the electrodynamic loudspeaker ( x : coil current 0 A 0,2 A)(y : frequency 0 Hz 1000 Hz)(z: impedance 0Ω 30Ω)

204 For this purpose, we use the Nyquist diagram constructed previously. Five  
 205 parameters ( $Bl(I)$ ,  $R_{ms}(I)$ ,  $C_{ms}(I)$ ,  $M_{ms}(I)$ ,  $L_e(I)$ ) are assumed to vary with  
 206 the coil current. Indeed, the electrical resistance does not depend on input  
 207 current because our experimental measurements have been done with a sta-  
 208 bilized temperature (the electrodynamic loudspeaker has been run in during  
 209 24 hours before the experiment) and the eddy current resistance does not  
 210 seem to vary with the input current  $I$ . In a first approximation, we use a  
 211 polynomial writing to represent the dependence of the parameters with the  
 212 coil current. We write:

$$213 \quad Bl(I) = Bl(1 + \mu_{Bl}I + \mu_{Bl}^2 I^2 + \dots + \mu_{Bl}^n I^n) \quad (5)$$

$$214 \quad R_{ms}(I) = R_{ms}(1 + \mu_{R_{ms}}I + \mu_{R_{ms}}^2 I^2 + \dots + \mu_{R_{ms}}^n I^n) \quad (6)$$

$$215 \quad C_{ms}(I) = C_{ms}(1 + \mu_{C_{ms}}I + \mu_{C_{ms}}^2 I^2 + \dots + \mu_{C_{ms}}^n I^n) \quad (7)$$

$$M_{ms}(I) = M_{ms}(1 + \mu_{M_{ms}}I + \mu_{M_{ms}}^2 I^2 + \dots + \mu_{M_{ms}}^n I^n) \quad (8)$$

$$L_e(i) = L_e(1 + \mu_{L_e} I + \mu_{L_e}^2 I^2 + \dots + \mu_{L_e}^n I^n) \quad (9)$$

216 So the electrical impedance becomes :

$$Z_e(I) = R_e + \frac{\sum_{s=0}^n jL_e w R_\mu (\mu_{L_e} I)^s}{R_\mu + \sum_{s=0}^n jL_e w (\mu_{L_e} I)^s} + \frac{(\sum_{s=0}^n Bl(\mu_{Bl} I)^s)^2}{\sum_{s=0}^n jM_{ms} w (\mu_{M_{ms}} I)^s + \sum_{s=0}^n R_{ms} (\mu_{R_{ms}} I)^s + \sum_{s=0}^n \frac{1}{jwC_{ms} (\mu_{C_{ms}} I)^s}} \quad (10)$$

217 We use a least square method to identify all the parameters; this method is  
 218 based on Symplex algorithm. The Simplex method is a systematic procedure  
 219 which selects the variable that will produce the largest change towards the  
 220 minimum solution. This algorithm selects the best choice at each iteration  
 221 without needing information from previous and future iterations. In our case,  
 222 the principle of this algorithm is to minimize the difference D between the  
 223 experimental and the theoretical Nyquist diagrams. Consequently, the two  
 224 parameters which are minimized are the real part  $Real(I)$  and the imaginary  
 225 part  $Imag(I)$  of the electrical impedance defined by eq. (11).

$$Z_e^{(NL)}(I) = Real(I) + jImag(I) \quad (11)$$

226 When the algorithm converges, we obtain the values of the parameters of  
 227 the equations (5), (6), (7), (8) and (9).

## 228 4 Experimental results

229 When we take into account the variations of the small signal parameters  
 230 with the coil current, the mean difference between the experimental and the

Ranking	Parameter	Law of variation (100 Hz)	D [ $\Omega$ ](100 Hz)
1	$R_{ms}$	$1.1(1 + 4.09I - 8.36I^2)$	1.24
2	Bl	$5.5(1 + 0.33I - 1.02I^2)$	1.67
3	$M_{ms}$	$0.009(1 + 0.56I - 0.22I^2)$	1.74
4	$C_{ms}$	$0.00013(1 + 2.02I - 9.3I^2)$	1.86
5	$L_e$	$0.0017(1 - 1.68I + 7.58I^2)$	1.98

Table 1: Ranking of the parameters according to their sensitivity to the least square algorithm

231 theoretical values is  $0,4\Omega$  whereas the mean difference is  $2,0\Omega$  with Thiele and  
 232 Small model with constant parameters. We present in table 1 the laws of  
 233 variations of the five parameters that vary according to the coil current, and  
 234 we give for each parameter the sensitivity to the least square. We propose  
 235 a ranking of these parameters based on the criterion D. To get this ranking,  
 236 we proceed as follows : we write that one parameter ( $L_e$ ,  $R_{ms}$ ,  $M_{ms}$ ,  $Bl$   
 237 or  $C_{ms}$ ) is a function of the coil current. We input the polynomial in the  
 238 theoretical impedance and we use the least square algorithm to determine at  
 239 the value of the mean difference between the experimental impedance and  
 240 the theoretical impedance. We obtain in this case the ranking shown in table  
 241 1. Moreover, the value of the eddy current resistance is  $2,2\Omega$  and the value  
 242 of the electrical resistance is  $3,3\Omega$ . The table 2 shows the laws of variations  
 243 of Thiele and Small parameters. The representations of these variations are  
 244 given in Figs 4, 5, 6, 7 and 8.

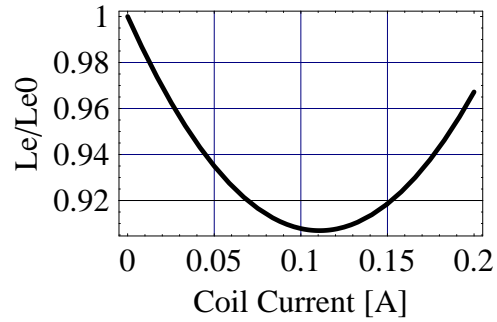


Figure 4: The ratio  $\frac{Le}{Le0}$  is a function of the coil current ( x : coil current 0A 0,2A) (f=100Hz);  $Le0 = 0.0017H$

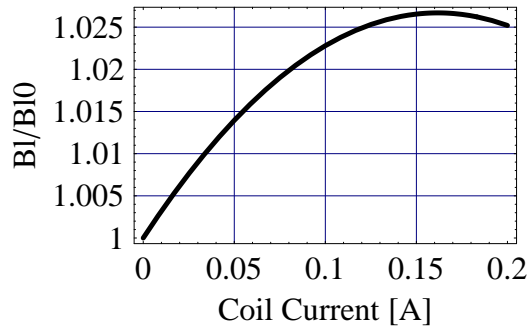


Figure 5: The ratio  $\frac{Bl}{Bl0}$  is a function of the coil current ( x : coil current 0A 0,2A)(f=100Hz);  $Bl0 = 5.5Tm$

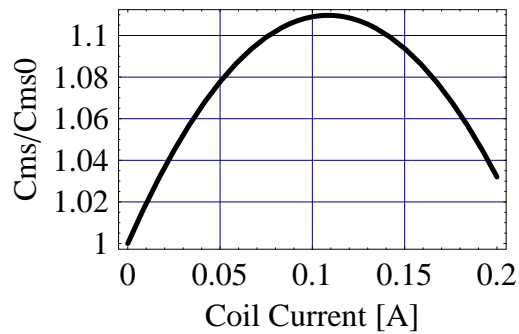


Figure 6: The ratio  $C_{ms}/C_{ms0}$  is a function of the coil current ( x : coil current 0A 0,2A)(f=100Hz);  $C_{ms0} = 0.00013m/N$

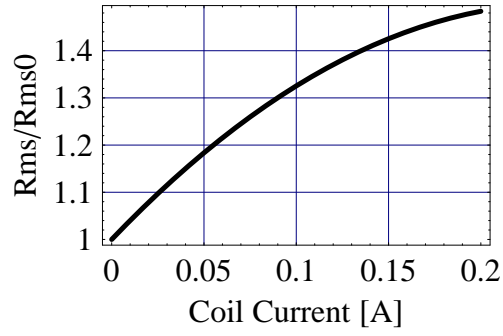


Figure 7: The ratio  $\frac{R_{ms}}{R_{ms0}}$  is a function of the coil current ( x : coil current 0A 0,2A)(f=100Hz);  $R_{ms0} = 1.1kg/s$

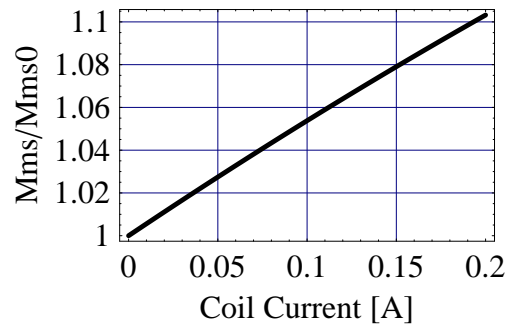


Figure 8: The ratio  $\frac{M_{ms}}{M_{ms0}}$  is a function of the coil current ( x : coil current 0A 0,2A)(f=100Hz);  $M_{ms0} = 0.009kg$



## 245 4.1 Discussion

246 The most sensitive parameter to the variations of the coil current is the  
247 equivalent damping parameter  $R_{ms}$ . The parameter which is the less sensitive  
248 to the variations of the coil current is the inductance  $L_e$  of the voice coil.  
249 However, though this inductance sensitiveness is rather weak, the relative  
250 variation of the inductance  $L_e$  is rather important. In other words, even if  
251 the inductance  $L_e$  is a parameter that contributes just a little to the variation  
252 of its electrical impedance, we see that its relative variation is important.  
253 Another interesting point can be adressed. With the values found in this  
254 paper, at  $200Hz$ , we see that the eddy current resistance  $R_\mu$  equals roughly  
255  $wL_e$ . We can conclude that the effects of the eddy currents become important  
256 from  $200Hz$  for this electrodynamic loudspeaker.

## 257 5 Study of the distortions created by the elec- 258 trodynamic loudspeaker

### 259 5.1 Obtaining the nonlinear differential equation

260 The nonlinear differential equation describing the electrodynamic loudspeaker  
261 is given by Eq. (12). We use the parameter  $k = \frac{1}{C_{ms}}$  to solve it.

$$a(i) \frac{d^3 x(t)}{dt^3} + b(i) \frac{d^2 x(t)}{dt^2} + c(i) \frac{dx(t)}{dt} + d(i)x(t) = u(t) \quad (12)$$

262 with

$$a(i) = \frac{(M_{ms}(1 + \mu_{M_{ms}}i + \mu_{M_{ms}}^2 i^2)) (Le(1 + \mu_{Le}i + \mu_{Le}^2 i^2))}{Bl(1 + \mu_{Bl}i + \mu_{Bl}^2 i^2)} \quad (13)$$

263

$$b(i) = \frac{((M_{ms}(1 + \mu_{M_{ms}}i + \mu_{M_{ms}}^2 i^2))Re)}{Bl(1 + \mu_{Bl}i + \mu_{Bl}^2 i^2)}$$

$$+ \frac{(R_{ms}(1 + \mu_{R_{ms}}i + \mu_{R_{ms}}^2i^2)R_e)}{Bl(1 + \mu_{Bl}i + \mu_{Bl}^2i^2)} \quad (14)$$

$$c(i) = \frac{Re(R_{ms}(1 + \mu_{R_{ms}}i + \mu_{R_{ms}}^2i^2)) + (Bl(1 + \mu_{Bl}i + \mu_{Bl}^2i^2))^2}{Bl(1 + \mu_{Bl}i + \mu_{Bl}^2i^2)} + \frac{(Le(1 + \mu_{Le}i + \mu_{Le}^2i^2))k(1 + \mu_ki + \mu_k^2i^2)}{Bl(1 + \mu_{Bl}i + \mu_{Bl}^2i^2)} \quad (15)$$

$$d(i) = \frac{Re(k(1 + \mu_ki + \mu_k^2i^2))}{Bl(1 + \mu_{Bl}i + \mu_{Bl}^2i^2)} \quad (16)$$

## 5.2 Solving the nonlinear differential equation

The relation between the coil current  $i$  and the position of the voice coil  $x$  is nonlinear (Eq. (4)). Each small signal parameter is a function of the voice coil position and we obtain, for example, for one parameter :

$$R_{ms}(x) = R_{ms}(1 + \tilde{\mu}_{R_{ms}}x + \tilde{\mu}_{R_{ms}}^2x^2) \quad (17)$$

with  $|\tilde{\mu}_{R_{ms}}^2| \ll |\tilde{\mu}_{R_{ms}}| \ll 1$ .

If we assume that the input voltage  $u(t)$  is sinusoidal, in this case, the solution of the nonlinear differential equation is a trigonometric expansion. The solution can be developed until the order 2 ( $\mu^2$ ).

$$x(t) = x_0(t) + \mu x_1(t) + \mu^2 x_2(t) + \dots \quad (18)$$

where  $x_0(t)$  is the solution of the nonlinear differential equation of the electrodynamic loudspeaker when we neglect the terms with orders higher than zero,  $x_1(t)$  is the solution of the nonlinear differential equation when we omit the terms with orders higher than one and smaller than one,  $x_2(t)$  is the solution of the nonlinear differential equation of the electrodynamic loudspeaker when we neglect the terms with orders smaller than two and higher than two.

280 It can be noted that some methods like Volterra Series are interesting but  
 281 do not show what parameters are really nonlinear. The way of solving the  
 282 nonlinear differential equation is very important but we think that it is not  
 283 the most important thing for characterizing electrodynamic loudspeakers.  
 284 The real problem is to know what parameters vary and how. Furthermore,  
 285 the ranking of these parameters according to the input current or frequency  
 286 is of great importance. Indeed, we can think that if the structure of an  
 287 electrodynamic loudspeaker must be improved, we must know all the defects  
 288 in the motor or the suspensions.

289 Let us consider now Eq. (12). The denominator in Eq. (12) contains  
 290 a nonlinear term. Consequently, it is very difficult to solve this nonlinear  
 291 differential equation with a nonlinear denominator. One possible solution is  
 292 to approximate this denominator as follows :

$$\frac{1}{Bl(x)} = \tilde{B}l_0 + \tilde{B}l_1x + \tilde{B}l_2x^2 + \dots \quad (19)$$

293 The previous relation is used as a simplification for solving numerically the  
 294 nonlinear differential equation. Moreover, we can use a classical trigono-  
 295 metric expansion to solve Eq. (12). In short, the solution of the nonlinear  
 296 differential equation of the electrodynamic loudspeaker is

$$x(t) = A \cos(\omega t) + B \sin(\omega t) + C \cos(2\omega t) + D \sin(2\omega t) + \dots \quad (20)$$

297 All the terms  $A, B, C, D, \dots$  are found numerically but an analytical solution  
 298 is possible if the force factor is approximated. Indeed, the terms A and B  
 299 can be found by inserting  $A \cos(\omega t) + B \sin(\omega t)$  in the equation (12) with an

300 excitation  $u(t)$  equals to  $P \sin(\omega t)$  where  $P$  is an amplitude. The terms  $C$   
301 and  $D$  can be found by taking the terms with orders higher than one into  
302 account, etc...

### 303 **5.3 Theoretical results and position of the small signal** 304 **parameters according to their created distortions**

305 We present the position of the small signal parameters according to their  
306 created distortions. For this purpose, we solve the nonlinear differential  
307 equation by using the serial expansion presented in the previous section but  
308 we take into account only one variation of a parameter at a time. Figure 9  
309 shows the created distortions by each Thiele and Small parameter. The level  
310 of the input voltage is 10V and the frequency of the excitation is 50Hz. We  
311 can see that for this electrodynamic loudspeaker, the nonlinear parameter  
312 which creates the most important second-harmonic is the inductance  $L_e$  of  
313 the voice coil, and the nonlinear parameter which creates the most important  
314 third-harmonic is the equivalent damping parameter  $R_{ms}$ . However, when  
315 the frequency of the excitation increases, the nonlinear parameters which  
316 create more distortions are not the same. Figure 10 presents the created  
317 distortions by each Thiele and Small parameter when the frequency of the  
318 excitation is 150 Hz. As shown in the previous figure, the level of the input  
319 voltage is 10V and the frequency of the excitation is 150 Hz. We see that  
320 the nonlinear parameter which creates the most important second-harmonic  
321 is the equivalent mass  $M_{ms}$  and the nonlinear parameter which creates the  
322 more important third-harmonic is the damping parameter  $R_{ms}$ . We sum up

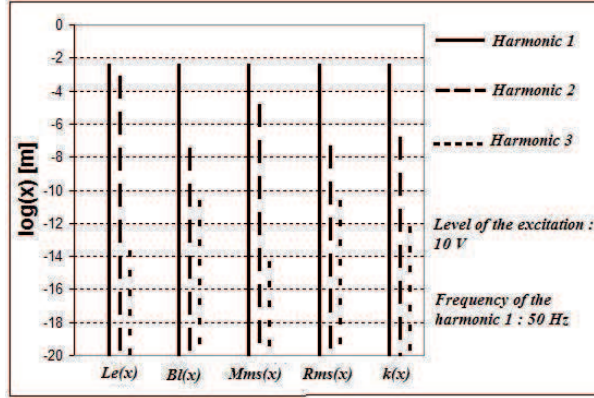


Figure 9: Theoretical spectrums of Thiele and Small parameters: the frequency of the excitation is 50Hz and the input voltage is 10V

Ranking ( 50 Hz)	Parameter(harmonic 2)	log[x]
1	$L_e$	-3.2
2	$M_{ms}$	-4.9
3	$R_{ms}$	-6.8
4	$C_{ms}$	-7.7
5	$Bl$	-7.8

Table 2: Ranking of the parameters according to their created distortions( the frequency of the excitation is 50 Hz)

323 all the results in tables 2, 3, 4 and 5.

## 324 5.4 Discussion

325 The previous section shows that the nonlinear parameters which create the  
326 most distortions are not those which are the most nonlinear according to the  
327 coil current. For example, figure 7 shows that the equivalent damping pa-  
328 rameter  $R_{ms}$  is the most nonlinear parameter (  $R_{ms}$  varies a lot with the coil  
329 current ) but it is not the parameter which creates the most distortions. On  
330 the contrary, the inductance  $L_e$  of the voice coil varies very little with the coil

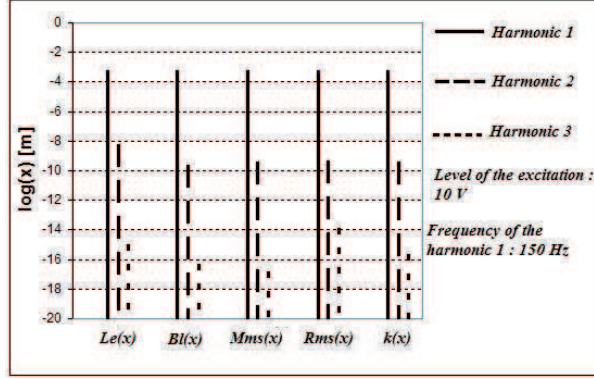


Figure 10: Theoretical spectrums of Thiele and Small parameters: the frequency of the excitation is 150Hz and the input voltage is 10V

Ranking ( 50 Hz)	Parameter(harmonic 3)	log[x]
1	$R_{ms}$	-11
2	$Bl$	-12.5
3	$C_{ms}$	-12.6
4	$L_e$	-12.9
5	$M_{ms}$	-14.6

Table 3: Ranking of the parameters according to their created distortions( the frequency of the excitation is 50 Hz)

Ranking ( 150 Hz)	Parameter(harmonic 2)	log[x]
1	$M_{ms}$	-5.3
2	$L_e$	-6.4
3	$R_{ms}$	-7.7
4	$C_{ms}$	-8.2
5	$Bl$	-8.4

Table 4: Ranking of the parameters according to their created distortions( the frequency of the excitation is 150 Hz)

Ranking ( 150 Hz)	Parameter(harmonic 3)	log[x]
1	$R_{ms}$	-11.7
2	$L_e$	-12.6
3	$C_{ms}$	-14.2
4	$Bl$	-15
5	$M_{ms}$	-15.1

Table 5: Ranking of the parameters according to their created distortions( the frequency of the excitation is 150 Hz)

331 current but at low frequency this parameter creates important distortions.  
332 Moreover, we see that the mechanical compliance  $C_{ms}$  is not an important  
333 nonlinear parameter since it does not create important distortions. In ad-  
334 dition, the ranking of these nonlinearities depend also on input frequency.  
335 Such results are interesting for the design of electrodynamic loudspeakers.

## 336 **6 BEHAVIOR OF THE CREATED DISTOR-** 337 **TIONS ACCORDING TO THE VARIA-** 338 **TIONS OF THE INPUT VOLTAGE**

339 The previous section presents the ranking of the nonlinear Thiele and Small  
340 parameter according to the variation of the frequency excitation. The aim of  
341 this section is to show that the distortions created by the nonlinear param-  
342 eters are more sensitive than the fundamental component according to the  
343 variation of the input voltage. To show this, we solve the nonlinear differen-  
344 tial equation of the electrodynamic loudspeaker (12). We solve this equation  
345 by taking four different amplitudes for the excitation ( 1V, 5V, 10V, 50V )  
346 and we represent the spectrum in Fig. 11. The frequency of the excitation

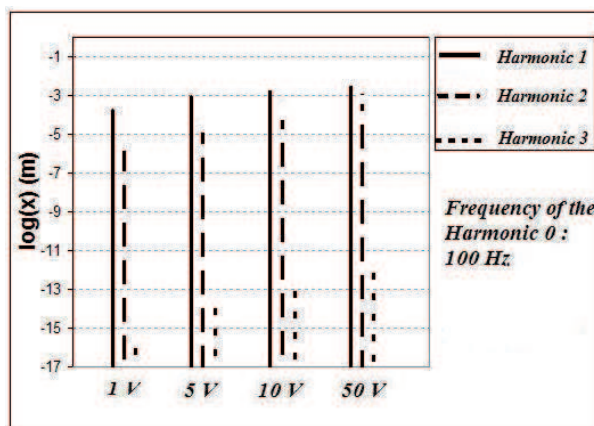


Figure 11: Theoretical spectrum : influence of the input voltage on the created harmonics

Harmonics	1V	5V	10V	50V
first-harmonic	-3.8	-3	-2.8	-2.7
second-harmonic	-5.9	-5.6	-4.2	-3.4
third-harmonic	-15.8	-13.5	-13	-11.4

Table 6: Ranking of the parameters according to their created distortions (the input frequency is  $100Hz$ )

347 is 100 Hz.

## 348 7 CONCLUSION

349 The aim of this paper was the study of the spectrum of the electrodynamic  
350 loudspeaker. The experimental method, based on the impedance measure-  
351 ment of an electrodynamic loudspeaker allows us to find all the variations  
352 of the Thiele and Small parameters. We can say that this experimental  
353 method can be used to characterize many transducers which are described  
354 with their electrical impedance. Indeed, an electrical impedance can be seen



355 as a nonlinear transfer function which varies with the input current or the  
356 input voltage [25]. In section 2, we have presented a method to derive the  
357 coefficients of the nonlinear parameters based on Symplex algorithm. It is  
358 noted that a simplifying method has been used in a previous paper [10] and  
359 has been improved in this paper.

360 The small signal coefficients, inserted in the differential equation of the  
361 electrodynamic loudspeaker enable us to find the generated harmonics. Many  
362 new results are discussed. The equivalent damping parameter is the param-  
363 eter which is the most nonlinear if we look at its relative variation according  
364 to the input current. However, it is not the parameter which creates the most  
365 important distortions. This result is important because it gives information  
366 about the way of manufacturing an electrodynamic loudspeaker.

367 More generally, when we take into account the variations of the small  
368 signal parameters with the coil current, the mean difference between the  
369 experimental and the theoretical values is  $0,4\Omega$ , whereas the mean difference  
370 is  $2,0\Omega$  with Thiele and Small model with constant parameters.

371 Another interesting result is the weak variation of the electrical induc-  
372 tance with the input current. It is noted that the electrodynamic loudspeaker  
373 characterized in this paper is a good one and is less nonlinear than a bad  
374 loudspeaker. However, we see that this weak variation creates important dis-  
375 tortions. This is why it can be very important to try to build electrodynamic  
376 loudspeakers with constant inductance.

377 Furthermore, we have seen that these generated harmonics become more  
378 and more important when the input voltage increases. This result is in fact

379 consistent with all the studies dealing with the modeling of nonlinear systems.

380 This paper is a first step to derive and class the defects of electrodynamic  
381 loudspeakers. The experimental approach taken is certainly the more precise  
382 for characterizing the variation of a nonlinear transfer function according to  
383 the input level. We can say that such defects are very important to determine  
384 because they lower the quality of loudspeakers.

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