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► **To cite this version:**

Mohamad Sayed Hassan, Karine Amis Cavalec. Turbo product code decoding for concatenated space-time error correcting codes. 20th Symposium on Personal, Indoor and Mobile Radio Communications (PIMRC 2009), Sep 2009, Tokyo, Japan. 10.1109/PIMRC.2009.5450262 . hal-00437649

HAL Id: hal-00437649

<https://hal.science/hal-00437649>

Submitted on 21 Jun 2022

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Turbo Product Code Decoding for Concatenated Space-Time Error Correcting Codes

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Abstract—This paper presents iterative decoding structures for concatenated space-time error correcting codes (STECCs), based on the turbo product code decoding algorithm. Thanks to linear combinations of forward error correcting (FEC) codewords, established to create a space-time redundancy, a product code is reconstructed from concatenated STECC ensuring at the receiver an iterative turbo product code decoding scheme that outperforms the existing turbo-like receiver with an interference canceller optimized according to the minimum mean square error (MMSE) criterion.

I. INTRODUCTION

Multiple Input Multiple Output (MIMO) technology has attracted much attention in wireless communication systems, since it increases the data throughput without additional bandwidth. Several families of space-time codes have been proposed to combat the fading or to increase the throughput of the transmission. The first family maximizes data rate by transmitting independent data streams from each of the multiple transmit antennas without exploiting all the available transmit diversity (this technique is referred as the spatial multiplexing). Alternatively, orthogonal space-time block (OSTB) codes [1], [2] are a second family of space-time codes that looks for improving the overall quality transmission, through maximizing the diversity gain, taking into account a low complexity detection at the receiver. The third family is composed of STB codes trying to achieve a tradeoff between diversity and multiplexing gains [3]. This family contains full-rate and full-diversity STB codes such as the threaded algebraic space-time (TAST) codes [4], the perfect STB codes [5] and the cyclotomic ones [6].

However, the space-time encoder in all these structures deals with the error correcting code as an independent entity in the transmission scheme. An interesting method to systematically construct a family of full transmit diversity STB codes from linear error correcting codes, has been proposed in [13], [14]. This family is based on the binary rank criterion [13]. It differs from known algebraic STB codes by an error protection at the center of its design, and is referred in this paper as space-time error correcting codes (STECCs) in order to stress on its ability to correct errors due to the transmission. We refer also to the serial concatenation of both STECC and a linear FEC code as concatenated STECC. Independently and in the same context, an explicit construction of concatenated STECCs for

2 transmit antennas based on explicit linear combinations of FEC codewords has been presented in [7] and improved in [8] from a space-time design point of view. Thanks to linear combinations of FEC, it was shown in [7] that this family seems more suitable than other ones to be concatenated with linear error correcting codes.

At the receiver side, the decoding structure must have a reasonable complexity. But unfortunately the optimal decoder, which can exploit all the channel diversity, is based on an exhaustive search to find the maximum likelihood sequence. Thus, the decoder complexity increases exponentially with the number of bits constituting the transmitted sequence. Instead of a prohibitive complexity optimal joint detection and decoding, a turbo-like iterative receiver exhibiting good performance in terms of bit error rate (BER) with a reasonable complexity has been proposed and used in some literature among which [9]-[12]. For retrieving the information in an iterative way, this receiver structure is based on the cooperation between an interference suppression detector, optimized according to the MMSE criterion in order to reduce the detection complexity, and a soft input soft output (SISO) channel decoder. Thanks to its low detection complexity, this receiver structure, which is also referred to as MMSE turbo equalizer, can be adopted for any STB codes applied to FEC encoded data and in particular to concatenated STECCs.

It clearly appears that a concatenated STECC is constituted of 2 linear error correcting codes: the first one is used to construct the STB codes (inner code) and the second one is the FEC code (outer code). Thus, the ingenuity of this work is to benefit from these 2 linear codes to apply iterative receiver structures based on the turbo product code decoding algorithm, which as we will show in this paper are more suitable to this family of space-time codes concatenated with FEC codes than other iterative receiver structures. By rearranging the soft information delivered by a soft maximum *a posteriori* (MAP) detector, we can reconstruct a product code and then, an iterative turbo product code decoder can be applied [15].

The remainder of this paper is organized as follows. In section II, we introduce the concatenated STECCs and we show how a turbo product code can be reconstructed from these codes. In section III, we present the decoding of con-

This work was supported in part by Orange Labs.

catenated STECCs using a MMSE turbo equalizer. In section IV, we apply the turbo product code decoding algorithm to these codes. In section V, simulations carried out over a non frequency selective block fading channel, using the improved structure proposed in [8] for 2 transmit antennas, show that the proposed receiver structures, based on the turbo product code decoding algorithm, outperform the MMSE turbo equalizer receiver structure. Section VI presents our conclusions.

II. SYSTEM MODEL AND NOTATIONS

We consider an $n_t \times T$ STECC to transmit data over T modulation symbol durations, for a $n_t \times n_r$ MIMO system with n_t transmit and n_r receive antennas. As the unified construction proposed by Lu and Kumar [14] can encompass the explicit construction presented in [7], [8], we adopt the approach used in [14] to represent an $n_t \times T$ STECC. For the sake of simplicity, we present the construction of an $n_t \times T$ STECC using an $M = 2^{m_b}$ -order quadrature amplitude modulation (M -QAM), where m_b is the modulation efficiency, although the unified construction can be used for many types of modulation (PAM, PSK, ...). Let \mathcal{C}_T be a linear binary ($n_t \times T$ matrix) code, where the elements of each codeword are arranged in the form of an $n_t \times T$ matrix. We consider a linear binary matrix code which satisfies the binary rank criterion [13] (every binary matrix $\mathbf{C} \in \mathcal{C}_T$ is of full rank over the binary field $\mathbb{F} = \{0, 1\}$). Each entry of the space-time error correcting codeword \mathbf{S} can be written as

$$[\mathbf{S}]_{m,n} = \mu \sum_{u=0}^{U-1} 2^u \theta^{[\mathbf{C}_1^u + 2\mathbf{C}_2^u]_{m,n}} \quad 1 \leq m \leq n_t; 1 \leq n \leq T \quad (1)$$

where $\mathbf{C}_1^u, \mathbf{C}_2^u \in \mathcal{C}_T, i = \sqrt{-1}, \theta = i, \mu = 1+i$ and $m_b = 2U$ (we suppose that the modulation efficiency is even). $[\mathbf{X}]_{m,n}$ is the (m,n) th component of the matrix \mathbf{X} . Therefore, this STECC \mathbf{S} achieves full transmit diversity and it realizes the optimal rate-diversity tradeoff [14], thus in each space-time codeword \mathbf{S} there are T information symbols and the others are associated to redundancy bits. Let $\mathbf{c}_j, 1 \leq j \leq n_t \times T$ be the binary m_b -tuples that represents the label of an entry of \mathbf{S} such that the first sequences $\mathbf{c}_n, 1 \leq n \leq T$ correspond to the information symbols. Thus for $r = T+1, \dots, n_t \times T$ the redundancy sequences can be written as

$$\mathbf{c}_r = (p_{1,r} \mathbf{c}_1) \oplus (p_{2,r} \mathbf{c}_2) \oplus \dots \oplus (p_{T,r} \mathbf{c}_T) \quad (2)$$

where $p_{n,r} \in \mathbb{F}$, and \oplus stands for mod-2 addition. $\{p_{n,r}\}$ defines the linear inner code.

Let $\mathcal{C}_O(N, L)$ be a linear FEC code (outer code), where N denotes the code length and L its dimension. To construct the concatenated STECC, we need T information codewords $\{\mathbf{c}_n\}_{1 \leq n \leq T}$. The redundancy codewords are linear combinations of the information codewords calculated using Eq. (2). After applying the unified construction using Eq. (1), we obtain $P = \frac{N}{m_b}$ space-time error correcting codewords $\mathbf{S}_{n_t \times T}(k), 1 \leq k \leq P$ to be transmitted. These $n_t \times T$ FEC

codewords form a product code represented as

$$\mathbf{T}_{n_t T \times N} = \begin{bmatrix} & & & \mathbf{c}_1 & & & \\ & & & \vdots & & & \\ & & & \mathbf{c}_T & & & \\ \mathbf{c}_{T+1} = (p_{1,T+1} \mathbf{c}_1) \oplus \dots \oplus (p_{T,T+1} \mathbf{c}_T) & & & & & & \\ & & & \vdots & & & \\ \mathbf{c}_{n_t T} = (p_{1,n_t T} \mathbf{c}_1) \oplus \dots \oplus (p_{T,n_t T} \mathbf{c}_T) & & & & & & \end{bmatrix} \quad (3)$$

where the column code is a simple linear block code with an information rate equal to $1/n_t$ referred to as inner code, while the row code is a linear FEC code without any constraint on its rate neither on its length. We note that the information rate of the inner code can be increased by reducing the diversity of the STECC according to the optimal rate-diversity tradeoff [14]. This ability to represent a concatenated STECC as a turbo product code improves performance from a BER point of view by using appropriate receiver structures which will be described in details in section IV.

To clarify this idea, we present explicitly the concatenated STECC proposed in [7] and improved in [8] for $n_t = 2$ transmit antennas and $T = 3$ modulation symbol durations.

$$\mathbf{S}(k) = \begin{bmatrix} s_1(k) & s_{1 \oplus 2}(k) & s_3(k) \\ s_{1 \oplus 3}(k) & s_2(k) & s_{2 \oplus 3}(k) \end{bmatrix} \quad (4)$$

where $\{s_j(k)\}$ and $\{s_{l \oplus j}(k)\}, 1 \leq j \leq 3, 1 \leq l \leq 3, j \neq l, 1 \leq k \leq P$ represent QAM symbols which correspond to the linear FEC codewords $\{\mathbf{c}_j\}$ and $\{\mathbf{c}_{l \oplus j}\}$ respectively. The information rate of the inner code is equal to $1/2$.

In this paper, we consider a coherent system, where the receiver knows the channel state-information (CSI), over a non frequency-selective $n_r \times n_t$ block fading channel. The received signal matrix can then be expressed as :

$$\mathbf{Y}_{n_r \times T}(k) = \mathbf{H}_{n_r \times n_t}(k) \mathbf{S}_{n_t \times T}(k) + \mathbf{N}_{n_r \times T}(k) \quad (5)$$

where $\mathbf{H}(k)$ is the channel matrix with independent and identically distributed (i.i.d) zero-mean complex Gaussian entries and $\mathbf{N}(k)$ is assumed to be the i.i.d zero-mean complex Gaussian noise matrix at the receiver. Subscripts indicate the dimensions of the matrices.

III. TURBO-LIKE RECEIVER WITH AN INTERFERENCE CANCELLER OPTIMIZED ACCORDING TO THE MMSE CRITERION

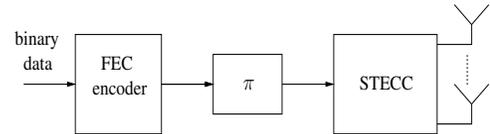


Fig. 1. Concatenated STECC transmitter scheme.

Regarding the prohibitive exponential complexity that makes the optimal receiver practically unrealistic, we present in this section and in the next one reasonable complexity iterative receivers. The transmitter scheme is represented in

Figure 1. An interleaver is used to protect the information against burst errors, and to benefit from the time diversity.

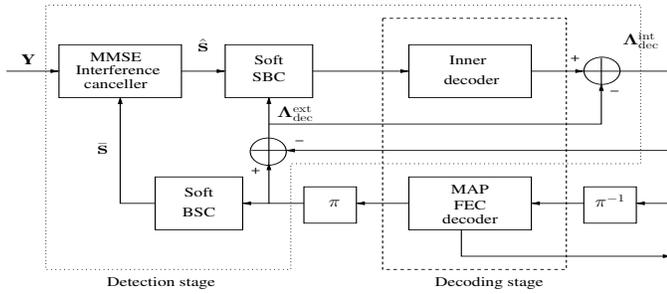


Fig. 2. MMSE turbo equalizer with interference canceller scheme.

This section deals with the iterative MMSE turbo equalizer for concatenated STECCs. This receiver as it was already mentioned can be adopted for any STB code associated to FEC codes. The turbo equalization principle is based on the cooperation of two entities: an equalizer detector with an interference canceller, optimized according to the MMSE criterion, and a SISO decoder. According to the turbo principle, the detector and the SISO decoder can exchange reliability information in order to improve their own process of extraction of the transmitted information. At each iteration, the equalizer computes an estimated symbol \hat{s} by using the received signal \mathbf{Y} and the soft symbols \bar{s} , produced from the previous decoding iteration, to suppress intersymbol interference. Then, from this estimated symbol, we can calculate the log-likelihood ratio (LLR) for each bit of the FEC codewords using the *extrinsic* information $\Lambda_{\text{dec}}^{\text{ext}}$ generated by the decoder at previous iteration. Moreover, this symbol to bit conversion (SBC) has a low complexity that increases linearly with the number of transmit antennas. As it is shown in Figure 2, the SISO decoder is constituted from a SISO FEC decoder preceded by an inner decoder that exploits the linear combinations between FEC codewords. The binary to symbol converter (BSC), in turn, uses the SISO decoder output to compute the soft symbols \bar{s} , which will be used in the next iteration as an input of the MMSE interference canceller. We note that the choice of the inner decoder depends only on the linear inner block code. Moreover, we can use an optimal inner decoder like the one described in the combining procedure in [11].

We mention that the complexity of the MMSE turbo equalizer presented in this paper has been reduced with respect to the MMSE turbo equalizer one used in [11] with an improved performance for low signal to noise ratios. This reduction of complexity, which is induced by using only one inner decoder instead of two, is offered thanks to a proper arrangement of the *extrinsic* information exchange in this receiver structure. The reader can refer to [11] to see in more details equation derivation of such a receiver.

IV. TURBO PRODUCT CODE DECODING FOR CONCATENATED STECC

As seen in section II, the concatenated STECC can be reformulated as a product code. Thus, the turbo-like iterative

decoding developed in [15] can be used. It requires as input soft values (*intrinsic* information) which can be provided by a preceding detector. As it is represented in Figure 3, the turbo

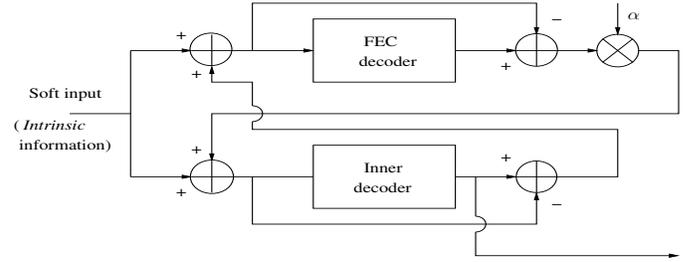


Fig. 3. SISO Turbo product code decoding structure.

product code decoding is based on the reliability information exchange between two elementary SISO decoders: the inner one and the MAP FEC decoder. Each elementary decoder benefits from two inputs: the soft values (*intrinsic* information) delivered by a detector, and the previous elementary decoder *extrinsic* information (*a priori* information). So as to ensure the convergence, weighting coefficients α ($\alpha < 1$) are applied to the *a priori* information generated by the FEC decoder.

A. MAP STECC detection

It clearly appears that the product code decoder soft input reliability has an important influence on the performance achieved by such a receiver structure. Therefore, in order to fully exploit the diversity of the $n_t \times T$ STECC, we apply a MAP detection on this space-time code (over T symbol durations). This MAP STECC detection benefits from linear binary combinations introduced among symbols in order to provide reliable input values to the product decoder (Figure 4). To compute these values (*intrinsic* information) the MAP STECC detector considers the $n_t \times T$ STECC set \mathcal{S} with cardinality equals to $2^{T n_t m_b}$, and the received signal matrix over each T symbol duration interval.

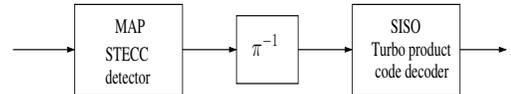


Fig. 4. MAP STECC detection with SISO turbo product code decoding structure.

Compared to the MMSE interference cancellation structure described in section III, this receiver structure does not require any matrix inversion and only one deinterleaving is required after the MAP detection. Moreover, as a turbo product code decoding is used, a stop criterion can be applied in order to reduce the iteration number and thus spare power consumption. This criterion consists in stopping the iterative decoding process when all the columns of the turbo product code at the output of the FEC decoder verify the parity check equations of the inner block code.

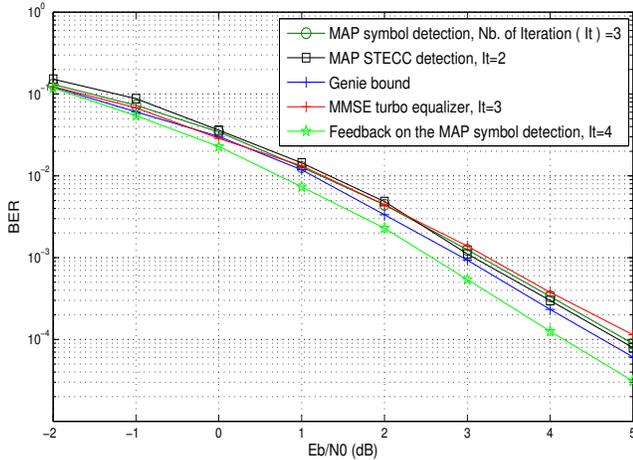


Fig. 9. Performance comparison between the MMSE turbo equalizer and the proposed receiver structures based on the turbo product code decoding algorithm. $L=510$. $n_r = 2$. $CC(7, 5)_{\text{Oct}}$. $\tau = 384$. 4-QAM (Gray).

9 show that the proposed receiver structures based on turbo product code decoding algorithm outperform the MMSE turbo equalizer, which converges within three iterations (the number of iteration in this paper is determined by simulation). For $\tau = 3$, one can see that the MAP STECC detection enables the receiver to achieve the so-called genie bound within two iterations only, whereas the MAP symbol detection performs differently from roughly 2 dB and progressively tends to the MMSE turbo equalizer performance curve within three iterations. For $\tau = 384$ the two previous receiver structures achieve the same performance and outperform the MMSE turbo equalizer. In addition, for $\tau = 3$ and $\tau = 384$ simulation results show that the turbo product code decoding scheme with feedback on the MAP symbol detection is interesting since it outperforms, especially from medium signal to noise ratios, all the other structures. We note that the genie bound is a BER lower bound for the MMSE turbo equalizer and it is not necessary to be the BER lower bound for other iterative receiver structures. For example at a $\text{BER}=10^{-4}$, a gain equals to 1 dB is offered by this receiver structure with respect to the other one using the turbo equalization principle. It requires four iterations to guarantee its convergence in severe channel conditions, but as it was already mentioned we can spare power consumption in most cases by applying a stop criterion in the turbo product code decoding algorithm.

VI. CONCLUSIONS

The STECCs are space-time codes with the ability to correct errors and are well suitable to be concatenated with linear FEC codes. In this paper, we have proved that a concatenated STECC can be associated to a product code and thus iterative receiver structures based on the turbo product code decoding algorithm have been proposed. These receiver structures differ by the receiver STECC detection employed at the input of the turbo product code decoding scheme.

Simulation results carried over a block fading channel show the performance of such a decoding scheme compared to the MMSE turbo equalizer. Using a MAP STECC detection, the resulting receiver structure can reach the genie bound and thus outperforms the MMSE turbo equalizer. In order to reduce the complexity of this detection, a MAP symbol detection has been proposed. But a loss in diversity gain with respect to the MAP STECC detection has been observed for block fading channel with fast variation rate. Motivated to improve the BER, a feedback on the MAP symbol detection is applied in the iterative procedure, resulting in a significant improvement, as this receiver outperforms all the other ones.

As a result, taking into account that concatenated STECCs can be handled as product codes, the turbo product code decoding should be preferred to other strategies as it better takes benefit from the available diversity and enables stopping criterion application to spare power consumption.

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