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Error estimation for sub-structured problems

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Abstract

This study presents a strategy to verify computations in a substructured context: after a finite element solution has been obtained from a domain decomposition based iterative solver, discretization error is estimated through the recovery of statically admissible stresses. A first parallel recovery technique is presented and assessed.

1 INTRODUCTION

Virtual testing has become a short term aim of industrials which will to replace expensive experimental studies and validations by numerical simulations, even in order to certify large structures as planes and bridges. So, one keypoint of the new numerical methods to develop is the *verification* of computations which enables to warranty that the computed solution is sufficiently close to the original continuum mechanics model. This topic of numerical analysis has been the subject of many studies for the last decades.

Another key point of new numerical methods is their ability to quickly provide solutions to large (nonlinear) systems. The most classical answer to this issue is to use domain decomposition methods in order to take advantage of the parallel hardware architecture of recent clusters and grids. In engineering, non-overlapping domain decomposition methods are mostly employed.

Our aim is to provide a fully integrated adaptative computational strategy to compute large structural mechanical problems with certified quality. Our method is based on the error in constitutive relation [4] to measure the quality of our results, and on a generic vision of non-overlapping domain decomposition methods [3].

In this paper, we focus especially on the computation of elementary contribution to the global error in a fully parallel context. First, we give basic notions about non-overlapping domain decomposition and constitutive error for elastic mechanical problem. Then, we present a strategy to compute elementary contribution fitted to sub-structuration. At last, we give some results and conclusions.

2 FORMULATION AND BASIC NOTIONS

We study the static equilibrium of a linear elastic structure occupying the domain Ω under the small perturbation hypothesis. The problem is solved using finite element discretization.

2.1 Non-overlapping domain decomposition

Domain Ω is decomposed into a set of subdomains ($\Omega^{(s)}$), in the following superscript (s) stands for data associated to domain $\Omega^{(s)}$. Assuming conforming decomposition of the structure (one-to-one correspondance of nodes on the interface between subdomains), the equilibrium equation $Ku = f$ (K is the stiffness matrix, u and f the displacement and force nodal fields), can be written in a substructured-fashion:

$$\begin{aligned} K^{(s)}u^{(s)} &= f^{(s)} + t^{(s)T}\lambda_b^{(s)} \\ \sum_s \underline{\mathbb{A}}^{(s)}u_b^{(s)} &= 0 \\ \sum_s \mathbb{A}^{(s)}\lambda_b^{(s)} &= 0 \end{aligned} \tag{1}$$

First equation of (1) stands for equilibrium of substructure (s), $\lambda_b^{(s)}$ is the nodal reaction of neighbouring subdomains and $t^{(s)}$ is the trace operator (which extracts the boundary nodes from the subdomain nodes). Second equation of (1) sets the continuity of displacements between substructures ($u_b^{(s)} = t^{(s)}u^{(s)}$ is the boundary displacement, and $\underline{\mathbb{A}}^{(s)}$ is a signed boolean assembly operator like in FETI method [2]). Third equation of (1) sets the equilibrium of reactions (action-reaction principle) ($\mathbb{A}^{(s)}$ is a boolean assembly operator like in BDD method [5]).

Whatever the (most often iterative) resolution strategy to system (1), in the end one obtains boundary fields $u_b^{(s)}$ and $\lambda_b^{(s)}$ respectively satisfying interface continuity and equilibrium conditions.

2.2 Constitutive relation error in elasticity

A posteriori error estimation, which has been widely studied for the last decades, contains a large number of techniques. The associated estimators are usually

classified into three main categories: flux projection techniques, residual on equilibrium equation or error in constitutive relation [4].

In the present study, we have chosen to use an estimator based on the constitutive relation error whose principle is recalled below. The usual continuum formulation of any mechanical problem can be rewritten in term of kinematic admissibility (2, KA) , static admissibility (2, SA) through the principle of virtual work and constitutive relation (2, CR):

$$\begin{aligned}
u \text{ KA} : u|_{\partial_1\Omega} &= u_d \\
\sigma \text{ SA} : \int_{\Omega} \sigma : \varepsilon(u^*) d\Omega &= \int_{\Omega} f_d \cdot u^* d\Omega + \int_{\partial_2\Omega} F_d \cdot u^* dS, \quad \forall u^* \text{ KA}_0 \\
\text{CR} : \sigma &= \mathbf{K} : \varepsilon(u)
\end{aligned} \tag{2}$$

where σ is Cauchy stress tensor, $\varepsilon(u)$ is the symmetric part of the gradient of displacement field u , u_d is given displacement on the part $\partial_1\Omega$ of the boundary, F_d is given surface effort on the part $\partial_2\Omega$ of the boundary, f_d is the given volume force.

Usually, a finite-element solution pair (u_h, σ_h) satisfies (2, KA&CR) but not (2, SA). Then, the starting point of the measure of the constitutive relation error is to build a new displacement-stress pair $(\hat{u}_h, \hat{\sigma}_h)$ from (u_h, σ_h) , which satisfies (2, KA&SA). Thereafter, the constitutive relation error is computed through the residual on the constitutive relation (2, CR) with the use of following energetic norm (3):

$$e_{CR}(\hat{u}_h, \hat{\sigma}_h) = \|\hat{\sigma}_h - \mathbf{K} : \varepsilon(\hat{u}_h)\|_{\sigma, \Omega} \quad \text{with} \quad \|x\|_{\sigma, \Omega}^2 = \int_{\Omega} (x : \mathbf{K}^{-1} : x) d\Omega \tag{3}$$

An important feature is that whenever (u_h, σ_h) satisfies (2, KA&SA), the associated estimator overestimates the true error.

3 RECOVERING SA STRESS WITHIN SUB-STRUCTURED CONTEXT

Most classical finite element formulations provide displacement field u_h satisfying kinematic constraints so that \hat{u}_h is chosen equal to u_h . However, σ_h is seldom statically admissible so that $\hat{\sigma}_h$ satisfying (2, SA) has to be computed from σ_h . A systematic two steps process (illustrated fig. 1(a)) consists in: first, building equilibrated force densities \hat{F} set on the edges of each element of the mesh, using “star-patches” of elements and previous field σ_h ; second, recovering equilibrated stress field $\hat{\sigma}_h$ from an element-by-element procedure with \hat{F} applied as Neumann boundary condition, using when possible analytical solution or richer approximation basis ([1] recommends $p + 3$ interpolation).

Since the second step is fully local, it can be easily driven in a sub-structured context. Unfortunately, the first step involves, for each node, systematic computations on a patch of elements centered on it (cf fig. 1(b)). In a substructured

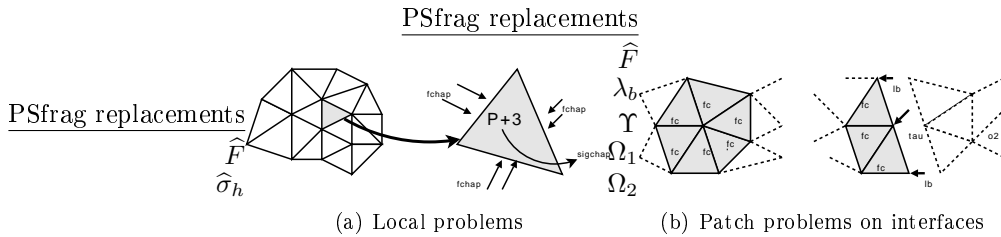


Figure 1: Recovering of SA stress

context, difficulties occur for interface nodes, for which associated patches are split between distinct substructures. Thus classical reconstruction of SA stress field would require inter-subdomains communications.

Though such an exchange of data would not be excessively expensive, we study fully-parallel alternative construction techniques. As stated earlier, in the end of the DD-based resolution continuous displacement field $u_b^{(s)}$ and equilibrated reaction field $\lambda_b^{(s)}$ are available on each interface node. We propose to build an approximated interface force density function $\tilde{\lambda}_b$ from nodal reaction λ_b in order to decouple the construction of equilibrated stress density \hat{F} on edges. As a first approximation, we choose to represent $\tilde{\lambda}_b$ on the basis constituted by the trace of the finite element shape functions. Then, $\tilde{\lambda}_b$ is computed thanks to :

$$\tilde{\lambda}_b = \sum_i \tilde{\lambda}_{bi} \varphi_i \quad \text{with } \tilde{\lambda}_b \text{ verifying } \tilde{\lambda}_{bi} = \int_{\Gamma} \tilde{\lambda}_b d\tau \quad (4)$$

4 RESULTS AND CONCLUSION

Tests have been conducted on a Γ -shape structure. For a fixed mesh, global error has been computed for the sequential initial problem and for different decompositions obtained with automatic splitting, see Tab. 1.

# sub-domains	1	2	3	5	8	9
e _{CR} (%)	9.40	9.31	13.59	9.23	9.32	8.77

Table 1: Global error for different substructurations

These first results enable to conclude that in most cases our strategy provides satisfying SA fields since the estimated error is smaller when the problem is substructured. The 3-subdomain substructuration is an exception, in this case, an interface between subdomain passes through the stress concentration zone making the linear interpolation of reaction density irrelevant.

Future work will try to overcome this problem, so that no a priori knowledge of stress concentrations will be necessary to provide decompositions leading to correct fully-parallel estimation of constitutive relation error.

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