Fast and robust dominant point detection on digital curves

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ABSTRACT

A new and fast method for dominant point detection and polygonal representation of a discrete curve is proposed. Starting from results of discrete geometry [1, 2], the notion of maximal blurred segment of width ν has been proposed, well adapted to possibly noisy and/or not connected curves [3]. For a given width, the dominant points of a curve C are deduced from the sequence of maximal blurred segments of C in $O(n \log^2 n)$ time. Comparisons with other methods of the literature prove the efficacity of our approach.

Index Terms— corner detection, dominant point, critical point

1. INTRODUCTION

The work on the detection of dominant points started from the research of Attneave [4] who said that the local maximum curvature points on a curve have a rich information content and are sufficient to characterize this curve. Therefore, these points play a critical role in curve approximation, image matching and in other domains of machine vision. Many works have been realised about the dominant point detection and an interesting survey is presented in [5]. Several problems have been identified in the different approaches: time computation, number of parameters, selection of start point, bad results with noisy curves, ...

In this paper, we present a new fast and sequential method issued from theoretical results of discrete geometry, it only requires to fixe one parameter, it is invariant to the choice of the start point and it works naturally with general curves : possibly being noisy or disconnected. It relies on the geometrical structure of the studied curve obtained by considering the decomposition of the curve into maximal blurred segments for a given width [3].

In section 2, we recall theoretical results of discrete geometry used in this paper to analyse a curve. The section 3 describes our method for dominant point detection. Finally, the section 4 presents experimental results and comparisons with other methods.

2. DECOMPOSITION OF A CURVE INTO MAXIMAL BLURRED SEGMENTS

The notion of blurred segment [2] relies on the notion of arithmetic discrete line [6]. An ***arithmetic discrete line***, noted $D(a, b, \mu, \omega)$, is a set of points (x, y) that verifies this double inequation: $\mu \leq ax - by < \mu + \omega$ with a, b, μ, ω integer parameters. A ***width*** ν ***blurred segment*** is a set of integer points which belong to a discrete line $D(a, b, \mu, \omega)$ verifying $\frac{\omega-1}{max(|a|,|b|)} \leq \nu$. The notion of ***width*** ν ***maximal blurred segment***, used in this paper, was proposed in [3] (deduced from [1,2]). We consider a discrete curve $C = \{C_i\}_{i=1..n}$ of n points, let us recall that the predicate "$S_{i,j}$, a set of points indexing from i to j in C, is a blurred segment of width ν" is noted by $BS(i, j, \nu)$.

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Definition 1
*$S_{i,j}$ is called a **width** ν **maximal blurred segment** and noted $MBS(i, j, \nu)$ iff $BS(i, j, \nu)$ and $\neg BS(i, j + 1, \nu)$ and $\neg BS(i - 1, j, \nu)$.*

An algorithm is proposed in [3] to determine the sequence of maximal blurred segments of width ν of a discrete curve C of n points. The complexity of this algorithm is $O(n \log^2 n)$. For a given width ν, the sequence of the maximal blurred segments of a curve C entirely determines the structure of C.
Let $C = \{C_i\}_{i=1..n}$ be a discrete curve and $MBS_\nu(C)$ the sequence of all maximal blurred segments of C, in which the i^{th} maximal blurred segment $MBS(B_i, E_i, \nu)$, is a set of point indexing from B_i to E_i. We recall below two important properties [3].

Property 1
Let $MBS_\nu(C)$ the sequence of width ν maximal blurred segments of the curve C. Then, $MBS_\nu(C) = \{MBS(B_1, E_1, \nu), MBS(B_2, E_2, \nu), ..., MBS(B_m, E_m, \nu)\}$ and satisfies $B_1 < B_2 < ... < B_m$. So we have: $E_1 < E_2 < ... < E_m$.

Property 2
Let $L(k)$, $R(k)$ be the functions which respectively return the indices of the left and right extremities of the maximal blurred segments on the left and right sides of the point C_k. So:

- $\forall k$ *such that* $E_{i-1} < k \leq E_i$, *then* $L(k) = B_i$
- $\forall k$ *such that* $B_i \leq k < B_{i+1}$, *then* $R(k) = E_i$

Thanks to Property 2 and the sequence $MBS_\nu(C)$, it is easy to obtain the left and right extremities of the width ν blurred segments starting from each point of the studied curve C. For each point M of C, the width ν blurred segment between M and left (resp. right) extremity is called ***width*** ν ***maximal left (resp. right) blurred segment*** of this point (see Fig. 1).

3. DOMINANT POINT DETECTION

We present here a new method for dominant point detection based on theoretical results of discrete geometry (recalled in section 2) : the sequence of maximal blurred segments of a curve permits to obtain important informations about the geometrical structure of the studied

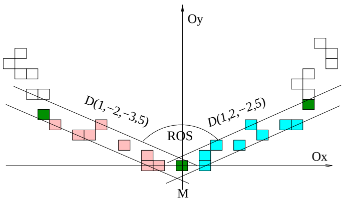

Fig. 1. Region of support based on maximal left (pink points) and right (blue points) blurred segments of M

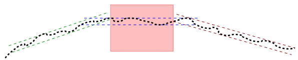

Fig. 2. Pink zone is not a common zone of successive maximal blurred segments

curve. The width of the maximal blurred segments permits to work in different scales and permits to consider the noise present in the studied curve. The width is the only parameter uses in our method.

3.1. Dominant point and region of support (ROS)

Deducing from [7], we propose in this section the notion of ROS that is compatible with the blurred segment notion.

Definition 2
Width ν maximal left and right blurred segments of a point constitute its region of support (ROS) (see figure 1). The angle between them is called the ROS angle of this point.

Remark 1
The smaller the ROS angle of a point is, the higher the dominant character of this point is.

This remark is deduced from the work [3], where ***curvature at a point*** C_k is estimated as inverse of the radius of the circumcircle passing through C_k and the extremities of its left and right width ν maximal blurred segments. Therefore, we have a corollary of this remark: if the ROS angle of a point is nearly $180°$, this point cannot be a dominant point.

3.2. Relation between dominant points and maximal blurred segments

In this paragraph, we study the relation between position of dominant points and maximal blurred segments.

Proposition 1
A dominant point of the curve must be in a common zone of successive maximal blurred segments.

Proof 1
Let us consider the points on the pink zone (see Fig. 2) which are not in a common zone of successive maximal blurred segments but which belong to one blurred segment. By applying the Property 2, the left and right end points of the blurred segments of these points are also in the same blurred segment. The ROS angles of these points are nearly $180°$. Therefore these points are not candidates as dominant points.

Let us now consider the common zone of more than 2 successive maximal blurred segments.

Proposition 2
The smallest common zone of successive width ν maximal blurred segments whose slopes are increasing or decreasing contains a candidate as dominant point.

Proof 2
Let us consider k successive width ν maximal blurred segments which share the smallest common zone. Without loss of generality, we assume that these k maximal blurred segments do not intersect any other smallest zone. Suppose that there are k first maximal blurred segments with the extremities below: (B_1, E_1), (B_2, E_2), ...,(B_k, E_k). Their slopes satisfy $slope_1 < slope_2 < ... < slope_k$ (similarity to decreasing case). Due to Property 1, we must have: $B_1 < B_2 < ... < B_k$; $E_1 < E_2 < ... < E_k$. Because these maximal blurred segments share the smallest common zone, we must have $B_k < E_1$. So, the smallest common zone is $[B_k, E_1]$. By applying Property 2, the left and right extremities of the points of the k partial common zones $[B_1, B_2[$, $[B_2, B_3[$, ...$[B_k, E_1[$ respectively are (B_1, E_1), (B_1, E_2),...(B_1, E_k). The slopes of the left blurred segments of the points of these partial common zones are always equal to $slope_1$. On the contrary, the slopes of the right blurred segments of the points of these partial common zones respectively are $slope_1$, $slope_2$, ... ,$slope_k$. By a similar way, we deduce that on the partial common zones $]E_1, E_2]$,...,$]E_{k-1}, E_k]$, the slopes of the right blurred segments of the points of these partial common zones are equal to $slope_k$ and the slopes of the left blurred segments respectively are equal to $slope_2$, ... ,$slope_k$. The ROS angle of the points in the zone $[B_k, E_1)$ is equal to the angle $(slope_1, slope_k)$ and this value is minimal for all the points indexed from B_1 to E_k, due to the hypothesis of the increasing slopes of maximal blurred segments. Therefore, this zone contains a candidate as dominant point.

To eliminate the weak dominant point candidates, we use the following natural property of a maximal blurred segment, due to the shape of straight line of a maximal blurred segment and also due to the property of corner of a dominant point.

Property 3
A maximal blurred segment contains at most 2 dominant points.

3.3. Proposed algorithm

3.3.1. Algorithm

We propose below a heuristic strategy for localizing the position of each dominant point candidate. **Heuristic strategy:** *In each smallest common zone of successive maximal blurred segments whose slopes are increasing or decreasing, a candidate as dominant point is detected as middle point of this zone.*

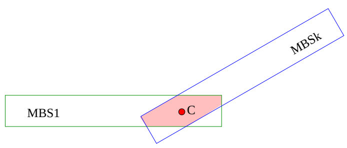

Fig. 3. A dominant point is expected as center of smallest common zone.

Let us consider a smallest zone that satisfies this condition. This zone contains a candidate as dominant point (cf. proposition 2). By using the property 1, this zone must be the intersection of the first and the last maximal blurred segments in the set of successive maximal blurred segments that share this zone (see Fig. 3). We recall that each point in this zone has the same region of support. We then propose to locate the candidate as dominant point that has geometric properties close to the expected corner point. So, the candidate as dominant point is detected as ***middle point*** of the partial curve corresponding to this zone.

Based on the above theoretical framework and using the heuristic strategy above, we present hereafter our proposed algorithm for dominant point detection. It is decomposed into two parts :

- the scan of the interesting common zones of maximal blurred segments according to the Proposition 2 and the Property 3,
- the detection of dominant points in common zones of successive maximal blurred segments whose slopes are increasing or decreasing.

Algorithm 1: Dominant point detection

Data: C discrete curve of n points, ν width of the segmentation
Result: D set of extracted dominant points
begin
  Build $MBS_\nu = \{MBS(B_i, E_i, \nu)\}_{i=1}^m$, $\{slope_i\}_{i=1}^m$;
  $m = |MBS_\nu|$; $p = 1$; $q = 1$; $D = \emptyset$;
  while $p \leq m$ **do**
    while $E_q > B_p$ **do** $p++$;
    Add $(q, p-1)$ to stack;
    q=p-1;
  while $stack \neq \emptyset$ **do**
    Take (q, p) from stack;
    Decompose $\{slope_q, slope_{q+1}, ..., slope_p\}$ into monotone sequences;
    Determine the last monotone sequence $\{slope_r, ..., slope_p\}$;
    $D = \{D \cup C_{\lfloor \frac{r+p}{2} \rfloor}\}$;
end

3.3.2. Complexity

The complexity of our method depends on the decomposition of a curve into maximal blurred segments that can be done in $O(n \log^2 n)$ time [3]. The slope estimation of maximal blurred segments is done in linear time. On the other hand, each maximal blurred segment is considered at most twice while the curve is decomposed into common zone of maximal blurred segments whose slopes are monotone sequence. So, in this phase, the dominant points are detected in linear time. Therefore the complexity of this method is $O(n \log^2 n)$.

4. EXPERIMENTAL RESULTS

The figures 4 and 5 show our obtained results and compare them with other methods (see table I) on some classical criteria: number of dominant points (nDP), compression ratio (CR), ISE error, max error, and figure of merit (FOM). CR is the ratio between number of curve points and number of detected dominant points, ISE is the sum of squared perpendicular distance of the curve points from approximated polygon. As a low error of approximation leads to a low ratio of compression, Sarkar [8] propose a FOM criterion to combine these measures: $FOM = CR/ISE$.

Because FOM criterion is not suitable for comparation with different dominant point number, Rosin [9] proposed other criteria to evaluate obtained result by comparing with the optimal result.

Fig. 4. Detected dominant points with default parameter (width=0.9). From left to right: leaf, chromosome, semicircle curves.

He proposed 2 measures, fidelity for error measurement and efficiency for compression ratio. $Fidelity = (E_{opt}/E_{appr}) * 100$, $Efficiency = (N_{opt}/N_{appr}) * 100$ where E_{appr} and N_{appr} are respectively error and DP number of tested algorithm, E_{opt} is error of optimal algorithm with the same approximated DP number, N_{opt} is DP number of optimal algorithm with the same approximated error. The merit measure is based on these measures. $Merit = \sqrt{Fidelity * Efficiency}$. Our method is not as efficient as the top 3 of the Rosin's list [9], that contains 30 different methods (see table II), but it is better than all others.

Moreover, our method also gives good results on noisy curves as it can be seen on figure 6 and in the table hereafter. The noisy image is created by using Kanungo model [10]:

Without noise, nPoint=338				With noise, nPoint=370			
width	ISE	CR	nDP	width	ISE	CR	nDP
1	62.22	7.51	45	2	262.68	11.21	33
2	203.25	12.52	27	3	506.62	16.09	23
3	424.65	17.79	19	4	878.72	20.56	18

5. CONCLUSION

We have presented a new method for dominant point detection. This method utilizes recent results in discrete geometry to work naturally with noisy curves. The width parameter permits to take into account the noise present in the curve. For the future work, we will compare our method with other methods that can also work with noisy curves.

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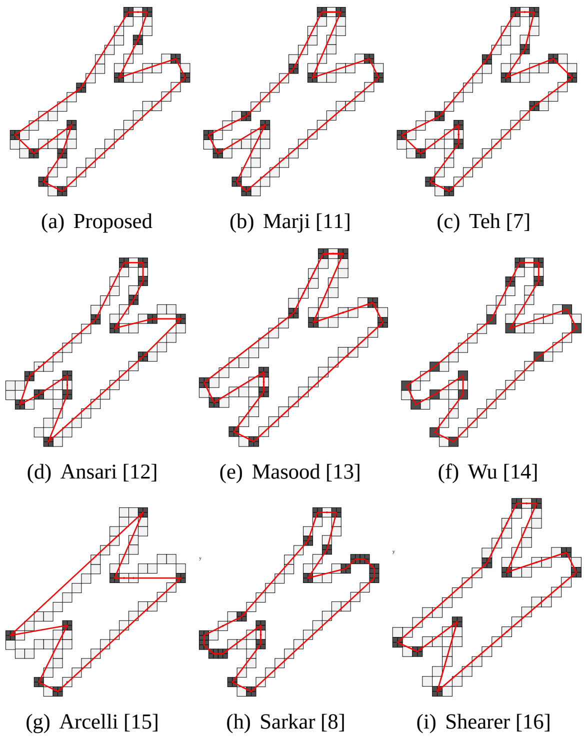

Fig. 5. Dominant points of the chromosome shape

$Shape$	$Algo.$	N_d	CR	$MaxE$	ISE	FOM
Chro.	Proposed	13	4.61	0.742	5.853	0.798
	Masood [13]	12	5	0.88	7.76	0.65
	Shearer [16]	10	6		6.086	0.744
	Wu [14]	17	3.53	0.64	5.01	0.704
	Marji [11]	11	5.45	0.895	9.96	0.548
	Teh [7]	15	4.00	0.74	7.2	0.556
	Ansari [12]	16	3.75	2	20.25	0.185
	Arcelli [15]	7	8.57			
	Sharkar [8]	19	3.16	0.55	3.857	0.819
Leaf	Proposed	23	5.217	0.74	11.52	0.4527
	Masood [13]	23	5.217	0.74	10.61	0.49
	Shearer [16]	22	5.46		13.06	0.418
	Wu [14]	23	5.22	1	20.34	0.256
	Marji [11]	22	5.45	0.78	13.21	0.413
	Teh [7]	29	4.14	0.99	14.96	0.277
	Ansari [12]	30	4.00	2.13	25.57	0.156
	Arcelli [15]	16	7.50			
	Sarkar [8]	23	5.22	0.784	13.17	0.396
Semi.	Proposed	21	4.857	0.84	10.41	0.466
	Masood [13]	19	5.37	1.00	23.9	0.23
	Shearer [16]	13	7.85		22.97	0.342
	Wu [14]	27	3.78	0.88	9.19	0.411
	Marji [11]	18	5.67	1	24.2	0.234
	Teh [7]	22	4.64	1	20.61	0.225
	Ansari [12]	28	3.64	1.26	17.83	0.24
	Arcelli [15]	10	10.20			
	Sarkar [8]	19	5.37	1.474	17.37	0.309

Table 1. Comparisons using Sarkar's criteria

Method	nDP	ISE	Fidel.	Effi.	Merit	Rosin's rank
Massod [13]	21	9.82	81.7	95.8	88.5	-
Proposed	21	10.41	77.07	90.47	83.5	-
Sarkar II [8]	20	13.65	66	78.9	72.2	4
Sarkar I [8]	19	17.38	57.8	73.7	65.3	6
Arcelli [15]	10	75.10	51.8	80.3	64.5	7
Teh [7]	22	20.61	34.0	59.2	42.4	17
Ansari [12]	28	17.83	18.8	49.1	28.8	26
Wu [14]	27	9.01	41.09	74.07	55.17	-

Table 2. Comparisons using Rosin's criteria on semicircle curve

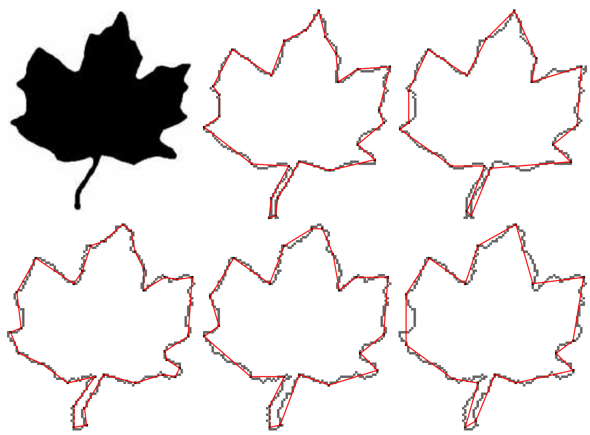

Fig. 6. From left to rigth : First line -Leaf image, segmentations for width=2 and 3- Second line - segmentations of noisy curve [10] for width = 2,3 and 4

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