Calculation Method of Permanent Magnet Pickups for Electric Guitars

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Abstract

This paper first presents the structures of permanent magnet pickups for electric guitar and the considered device: string, magnet, coil. It then describes a method to calculate the induced electromotive force (EMF) in the pickup coil when the string moves. The method of calculation links the EMF in the pickup coil with the flux cut by the string when it moves. The EMF is of course nonlinear. The harmonics of the EMF can be calculated. This is a first step towards the final aim that will consist in studying the frequency spectrum of the electrical signal given by the pickup, which is quite complicated, in order to link the magnetic structure and the string movement with the musical effect. Analytical calculations using the coulombian model of magnets are used to evaluate the magnetic field created by the magnet and the electromotive force in the pickup coil.

Index Terms

Permanent magnets, pickups for electric guitar, analytical magnetic field calculation

I. Introduction

WHO has never heard some music, played with electric guitars? These musical instruments are nowadays well-known and of current use in pop groups. Their story backs up in the 1930s, when Rickenbacker fitted out a guitar with a magnet and coils, thus designing the first magnetic circuit for electric guitar pickups. The way these pickups are designed and tuned has an empirical basis and is often related to the know-how and the feeling [1]. Patents have been taken out by pickups designers, especially in the U.S.A. by well-known makers like Rickenbaker, Fender, Gibson, Seymour Duncan, Di Marzio [2][3], but few papers are to be found in scientific journals [4]. We want to describe these pickups from a scientific point of view and to give quantitative guidelines for their design. The description can be done from two points of view that have to be joined: the magnetic one and the acoustic one. We have first to look at the types of magnetic circuit for the guitar pickups. We consider in this paper the most usual one, which is a cylindrical piece of permanent magnet axially magnetized. There is one magnet for each string. A whole pickup can be described as the juxtaposition of six magnets, one for each string, with the coil around them. We suppose that the magnet is rigid and that its magnetic relative permeability is equal to one, which is the case for hard ferrite as well as for rare earth magnets. This study doesn't include Alnico magnets, that have been used for a long time - and are still used for vintage pickups - but do not have these properties. We neglect the eddy currents, either in the magnet or in the string.

The string and its behavior have then to be considered. We consider that the guitar neck and the string define the x axis.

The movement of the string is constituted by two elementary transversal movements. Some authors [5] [6] consider that they are not coupled for small movements, within the linear approximation, and that they become coupled for higher amplitudes. The transversal modes have close but nevertheless separate frequencies. This comes from the limit conditions that are slightly different for each mode at the bridge and at the nut [7] . We study each transversal movement separately. We first consider that the string movement in front of the pickup is sinusoidal and occurs in a xy plane above and parallel to the pickup (Fig.1). We then consider that the string movement in front of the pickup is sinusoidal and occurs in a xz plane above the pickup. In fact, a good player is able to exert a force on the string in the y direction only, while a less experienced player will also exert a force in the z direction. The string always has an elliptical movement, but the amplitude along the z axis is smaller than along the y axis and

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we may consider that the latter is between 10% and 20% of the former. We assume that the string moves freely, and we neglect the attraction force of the magnetic field on the string. The string movement possibly contains mechanical harmonics [8], but we only consider the fundamental vibration of the string by now. The amplitude of the movement is of course of great importance, as the non linear behavior depends on it.

The string dimensions depend on the corresponding note: the smaller the diameter, the sharper the note. A guitar has six strings corresponding to the notes E $(83Hz)$, A $(110Hz)$, D $(147Hz)$, G $(196Hz)$, B $(247Hz)$, E $(330Hz)$ in the standard tuning. In fact the string diameter is chosen with regard to the note but also to the string tension, so that the tensions of the six strings are nearly identical, in order not to twist the guitar neck. The length of the string depends on the choice of the sounding length of the guitar string made by the guitar maker. The height of the string above the pickup is also an important parameter, and it is not always the same for all of the six strings.

The strings are metallic and the usually used materials are pure nickel, nickelplated steel or stainless steel, all ferromagnetic, but that give different types of sound, because of their different mechanical properties.

A coil of a highly thin copper wire (diameter $0.07mm$) and very numerous windings (around 6500), is wound around the magnets. When the string moves in front of the pickup, the flux in the magnetic circuit changes and so does the electromotive force in the coil. This voltage, its amplitude and its frequency spectrum, constitutes the signal of the pickup. The magic of the sound depends on it and so, on the dimensioning of the pickup [9].

We will consider only one magnet to explain the method and calculations. To dimension a pickup for an electrical guitar, one has to consider that it is a three dimensional device, including a cylindrical magnet and a string. Average dimensions are a diameter between 4 and $5mm$ and a height of $10mm$ for the magnet, between 0.2 and $1.3mm$ for the strings' diameter and a length of around $5mm$ for the distance between the magnet and the string. This means that the string is rather small in comparison with the magnet and also rather distant. As a consequence, it is not possible to do the usual approximate calculations as for a two dimensional device. Finite elements calculations still take too many computing resources to take correctly into account the differences of scale in the dimensions. The difficulty is to obtain a good mesh for devices in which one dimension is small compared with the others. Moreover, we want to calculate the effects of the variation of the flux in the magnetic circuit and we need a precision compatible with the spectral analysis of the signal [10]. Such a precision on the flux value is not reachable with finite elements calculations programs.

We are confronted with the same problem as numerous authors which study permanent magnet electrical machines and try to find alternative methods to finite element analysis [11][12][13]. The considered system has the same problematics as permanent magnet motors for which the EMF has to be calculated [14][15]. We describe a method to calculate the induced electromotive force (EMF) in the pickup coil when the string moves. The method of calculation links the EMF in the pickup coil with the flux cut by the string when it moves: we show that the EMF in the pickup is proportional to the magnetic flux cut by the string in its movement. Analytical calculations using the coulombian model of magnets are used to evaluate the magnetic field created by the magnet and the EMF in the pickup coil. The EMF is of course nonlinear. The harmonics of the EMF can be calculated. This is a first step towards the final aim that will consist in studying the frequency spectrum of the electrical signal given by the pickup, which is quite complicated, in order to link the magnetic structure and the string movement with the musical effect.

II. Description of the method

We consider that the elementary pickup is constituted by a cylindrical modern permanent magnet and a coil around it. The diameter, d_m, of the magnet is $4.5mm$, its height, h_m, is $10mm$. The altitude, d, of the string above the pickup is $5mm$ at rest. The initial position of the string is centered above the upper face of the magnet. It has to be noticed that the magnet, the string and the coil constitute a magnetic circuit and we will work on the equivalent magnetic circuit[16]. In fact, the system works as a polarized reluctant system [17] [18], with a fixed source of magnetic field, the magnet, and a ferromagnetic moving part, the string.

All the hypotheses we make lead to the following expression of Ampere's theorem (1):

$$\mathcal{E} = \mathcal{R}\varphi = \ Ni \ + \frac{Jh_m}{\mu_0} \tag{1}$$

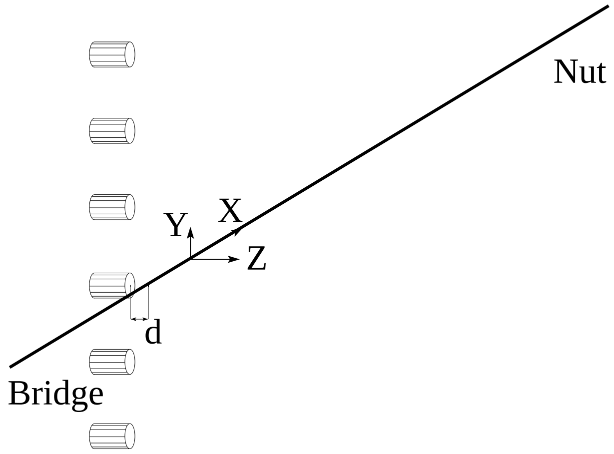

Fig. 1. Pickup geometry: the six magnets and one string. The coil is wound around the six magnets and isn't represented here

where $\mathcal{E}$ is the magnetomotive force in the circuit, $\mathcal{R}$, the reluctance of the circuit - string, magnet, air-, J, the magnetic polarization of the magnet, in Tesla, h_m, the height of the magnet, N the number of windings of the coil and i the current in the coil. As the interesting signal is the electromotive force in the coil, the measurement electronics have a high input impedance and the current in the coil is zero. Equation (1) shows then that the magnetomotive force in the circuit is a constant.

The magnetic circuit that has to be considered is constituted by a finite juxtaposition of flux tubes. All the tubes have the same section. Most of the tubes contain air and magnet. Some tubes go through the string, and contain iron, magnet and air. We index with j the tubes that contain the string at the time t and that the string is going to leave at the time $t + \Delta t$. We index with k the tubes that don't contain the string at the time t but that are going to receive it at the time $t + \Delta t$. We write φ_j the magnetic flux in the j tubes where the string is at the time t. At the time $t + \Delta t$, when the string has moved and has left them, we write $\varphi\prime_j$ the new flux it these tubes. Identically, we write φ_k the magnetic flux in the k tubes at the time t, and $\varphi\prime_k$ the new flux it these tubes at the time $t + \Delta t$, when the string has reached them. We want to calculate the flux variation in the coil, $\Delta\varphi$, between the time t and the time $t + \Delta t$. The tubes that contribute to the flux variation are the tubes j and k and we don't need to consider the other tubes. The part of the flux that contributes to the variation and that is seen by the coil at the time t is the sum $\varphi_j + \varphi_k$ for all the tubes j and k. This flux becomes the sum $\varphi\prime_j + \varphi\prime_k$ at the time $t + \Delta t$. The total variation is then the following (2):

$$\Delta\varphi = \sum_{all\ tubes\ j\ ,\ k} (\varphi\prime_j + \varphi\prime_k) - (\varphi_j + \varphi_k) \tag{2}$$

The reluctance of a j tube going through the string at the time t can be considered as the sum of the reluctance of the portion of tube going through the string $\mathcal{R}_s$, and the reluctance of the remaining length of the tube, $\mathcal{R}_j$. At the time $t + \Delta t$, the reluctance $\mathcal{R}_s$ becomes $\mathcal{R}_a$, because this portion of tube no longer contains iron, but contains air instead. The same kind of definitions are given for the tubes k. $\mathcal{R}_s$ appears as the "passing reluctance" of the string. Table I sums up all these notations. There is an assumption on the reluctances: we consider that the string passing reluctance, $\mathcal{R}_s$, is approximately the same at the time t and at the time $t + \Delta t$.

We assume that the flux density in a tube is uniform and corresponds to the value in the middle of the tube.

As the tubes are parallel connected, we also have the following relations (3) and (4):

$$\mathcal{E} = (\mathcal{R}_j + \mathcal{R}_a)\ \varphi_j = (\mathcal{R}_j + \mathcal{R}_s)\ \varphi\prime_j \tag{3}$$

$$= (\mathcal{R}_k + \mathcal{R}_s)\ \varphi_k = (\mathcal{R}_k + \mathcal{R}_a)\ \varphi\prime_k \tag{4}$$

From (2) and (3) we deduce (5):

	j Tube		k Tube	
	Flux	Reluctance	Flux	Reluctance
t	φ_j	$\mathcal{R}_j + \mathcal{R}_s$	φ_k	$\mathcal{R}_k + \mathcal{R}_a$
$t + \Delta t$	$\varphi\prime_j$	$\mathcal{R}_j + \mathcal{R}_a$	$\varphi\prime_k$	$\mathcal{R}_k + \mathcal{R}_s$

TABLE I
NOTATIONS FOR THE FLUXES AND RELUCTANCES

Where $\mathcal{R}_s$ is the string passing reluctance of a tube, $\mathcal{R}_a$, is the reluctance of the preceeding portion of tube with air instead of iron, $\mathcal{R}_j$ and $\mathcal{R}_k$, the remaining reluctances of the considered tubes.

$$\Delta\varphi = \sum_{all\ tubes\ j\ ,\ k} \frac{\mathcal{R}_s - \mathcal{R}_a}{\mathcal{R}_k + \mathcal{R}_a}\, \varphi_k - \frac{\mathcal{R}_s - \mathcal{R}_a}{\mathcal{R}_j + \mathcal{R}_a}\, \varphi\prime_j \tag{5}$$

And then we deduce (6) and (7).

$$\Delta\varphi = \sum_{all\ tubes\ j\ ,\ k} \frac{\mathcal{R}_s - \mathcal{R}_a}{\mathcal{E}}\, (\varphi_k \varphi\prime_k - \varphi_j \varphi\prime_j) \tag{6}$$

$$\simeq \sum_{all\ tubes\ j\ ,\ k} \frac{\mathcal{R}_a - \mathcal{R}_s}{\mathcal{E}}\, (\varphi_k - \varphi_j)(\varphi_k + \varphi_j) \tag{7}$$

We make the additional hypothesis that the sum of the fluxes varies very slightly and can be considered as a constant.

Finally, the flux variation in the coil can be expressed as (8):

$$\Delta\varphi \simeq \sum_{all\ tubes\ j\ ,\ k} K(\varphi_k - \varphi_j) \tag{8}$$

where K is a constant corresponding to (9):

$$\begin{aligned} K &= \sum_{all\ tubes\ j\ ,\ k} \frac{1}{\mathcal{E}}(\mathcal{R}_a - \mathcal{R}_s)(\varphi\prime_k + \varphi\prime_j) \\ &= \sum_{all\ tubes\ j\ ,\ k} (\mathcal{R}_a - \mathcal{R}_s) \left\{ \frac{1}{\mathcal{R}_j + \mathcal{R}_s} + \frac{1}{\mathcal{R}_k + \mathcal{R}_a} \right\} \\ &\simeq \sum_{all\ tubes\ j\ ,\ k} (\mathcal{R}_a - \mathcal{R}_s) \left\{ \frac{1}{\mathcal{R}_j} + \frac{1}{\mathcal{R}_k} \right\} \end{aligned} \tag{9}$$

Equation (8) shows that the variation of the flux in the coil is proportional to the difference of the fluxes in the tubes left by the string and the tubes reached by the string, so the variation of the flux between two positions of the string, which can also be qualified as the flux, φ_c, cut by the string when it moves.

Equation (9) shows that the variation of the flux in the coil depends on the reluctance variation divided by the total reluctance of the magnetic circuit. Shortly, it depends on the relative reluctance variation, which is normal. Equation (9) also shows that $\Delta\varphi$ will be large if the difference $(\mathcal{R}_a - \mathcal{R}_s)$ is great, so, if the passing reluctance of the string is small. This reluctance can be expressed by (10):

$$\mathcal{R}_s = \frac{l_s}{\pi \mu_0 \mu_s \delta_s} \tag{10}$$

where δ_s is the diameter of the tube occupied by the string, l_s the length of the tube occupied by the string and μ_s the relative magnetic permeability of its material. We thus see the importance of having a ferromagnetic string, and the influence of the string diameter -thick strings will give a greater variation.

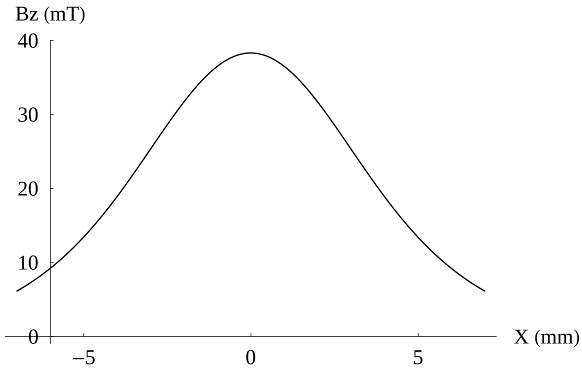

Fig. 2. z component of the magnetic flux density for a circular section of $4.5mm$ diameter along the x axis at a distance $d = 5mm$ along z above the magnet.

The electromotive force (EMF), e, which is also the signal in the pickup, is given by the Faraday law and can be written as in (11):

$$\begin{aligned} e &= \sum_{relevant\ tubes} -N\frac{\Delta\varphi}{\Delta t} \\ &\simeq \sum_{relevant\ tubes} -NK\frac{\varphi_c}{\Delta t} \end{aligned} \tag{11}$$

where N is the number of turns of the pickup coil.

The link established by (8) between the variation of the flux in the coil and the flux cut by the string is very interesting from a calculation point of view, because the cut flux, φ_c, can be evaluated with (12):

$$\varphi_c = \iint_S B\ \ dS, \tag{12}$$

where B is the component of the magnetic flux density along the normal to the cut surface S.

We will now present how to calculate the magnetic field created by the magnet and then discuss how the movement of the string creates the signal.

A. Permanent Magnet Field Calculation

1) Principle: We calculate the magnetic flux density, B, created by the permanent magnet in the space around it when the magnet is alone. We assume that the magnet is axially and uniformly magnetized, with a magnetic polarization, J, of $1T$. We use a magnetic masses model for the magnet [19]. The cylindrical magnet can be replaced by two discs of same diameter, d_m, as the magnet, representing the top and bottom faces of the cylinder. They are separated by the magnet height, h_m. We consider that the material between and around the discs is air. Both discs are charged with a uniform magnetic masses surface density, σ^*:

$$\sigma^* = \mathbf{J}.\mathbf{n} \tag{13}$$

but with opposite signs. Let us call $M1$ a point on the upper disc (surface $S1$, north pole of the magnet), and $M2$ a point on the lower disc (surface $S2$, south pole of the magnet). The magnetic flux density, B, created by the magnet at a point M of the space is calculated with (14).

$$\mathbf{B}(M) = \frac{J}{4\pi}\iint_{S_1}\frac{\mathbf{MM_1}}{|\mathbf{MM_1}|^3}\,d\mathbf{S_1} - \frac{J}{4\pi}\iint_{S_2}\frac{\mathbf{MM_2}}{|\mathbf{MM_2}|^3}\,d\mathbf{S_2} \tag{14}$$

We calculate the values of the magnetic field (modulus and direction) in the space where the string will be located. Fig.2 shows values along the center of the string, as if the string had no thickness.

2) Circular section and square section: Previous calculations are achieved numerically, as we do not have an analytical expression for B when the surfaces of the magnet are circular. But when the magnet's surfaces are rectangular or square, the analytical expressions for the magnetic field B are available [19]. Let us compare the values of B for two cases. In the first case, B_c is calculated numerically for a magnet with a circular section. In the second case, B_s is calculated analytically for a magnet with a square section. The dimensions are chosen so that the surfaces have the same value. We evaluate the relative difference, e_r, for the three components of the magnetic flux density at a given distance, d, of the magnet. Of course, d can't be too small: the comparison is not valid on the surface of the magnet, but for the generally chosen string height above the magnet, the distance d is large enough to legitimate the comparison.

The relative difference for the component along the x axis is maximal when the string is located above the edge of the square magnet. The maximum difference is around 1.4%. For the component along the y and the z axis, the maximal difference occurs when the string is centered above the magnet. Its value is respectively around 1.5% and 0.6%. The magnet with a circular section will be replaced by a magnet with a square section in the further study, as it allows us to calculate the electromotive force analytically.

B. String motion

The real movement of the string is elliptical in the space, but can be described as the composition of two movements in two perpendicular planes. An approach is to study the movement of the string in two planes xy and xz [20] [21] [22].

When we consider that the string is moving in a plane parallel to the xy plane, which is parallel to the upper face of the magnet and at a distance d above the magnet, we only need to calculate the B_z component to be able to evaluate the cut flux. The shape of the string is assumed to be a sine and is described by the following equation (15):

$$y(t,x) = Y_I \; cos(2\pi f t) \; sin(\frac{\pi x}{L}) \tag{15}$$

where L is the length of the string, f is the frequency of the vibration and Y_I is the initial amplitude of the excitation of the string .

When we consider that the string is moving in a plane parallel to the xz plane, which is perpendicular to the upper face of the magnet, we only need to calculate the B_y component to be able to evaluate the cut flux. The string is initially at a distance d above the magnet -along the z axis. The position along the y axis can't be $y = 0$, because the B_y component would always be zero. The string has thus to be shifted along the y axis. The shape of the string is assumed to be a sine and is described by the same kind of equation as previously (16):

$$z(t,x) = Z_I \; cos(2\pi f t) \; sin(\frac{\pi x}{L}) + d \tag{16}$$

where Z_I is the initial amplitude of the excitation of the string. Of course, we consider that Z_I remains smaller than d, to avoid the contact between the string and the magnet.

III. Illustration

The numerical values we take for our calculations are the characteristics of a Fender Stratocaster guitar with Seymour Duncan single pickups and we consider the neck pickup. The length of the vibrating part of the string, L, is $65cm$. The neck pickup is situated at a quarter of the string length from the bridge: this means that the magnet is at a distance of $16.25cm$ from the bridge extremity of the string. The magnet is circular of diameter $4.5mm$, but as demonstrated we consider a square one of same section, so with a $4mm$ side.

A. Movement in the xy plane

1) Centred string: We consider here that the initial position of the string is $5mm$ above the center of the upper magnet face, parallel to the x edges of the magnet. The cut surface considered to calculate the cut flux is the surface delimited in the xy plane by the shape of the string at rest - a straight line- and the sine shape it has when it moves. For small values of the mechanical excitation, the cut flux seems to vary sinusoidally with the time, this means, like the string amplitude. This is true, as long as the string remains in an area where the magnetic flux density is

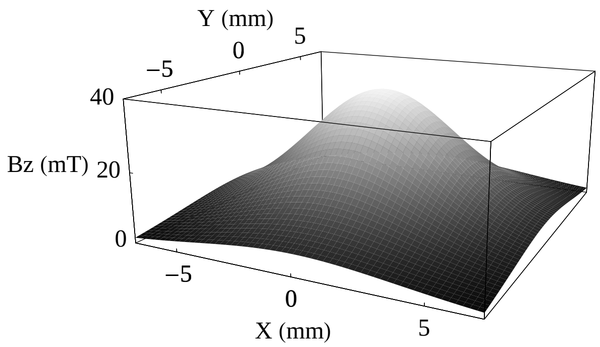

Fig. 3. B_z component of the flux density above the square section magnet. Altitude:$d = 5mm$

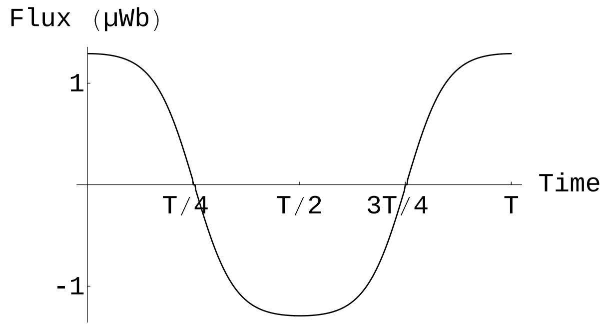

Fig. 4. Cut flux for an excitation of the string $Y_I = 10mm$ along y. String altitude:$d = 5mm$. Magnet position $L/4$: neck pickup.

almost constant (Fig.3). When the amplitude of the string movement increases, the cut flux should increase, but as the string comes into areas where the flux density amplitude decreases, the cut flux doesn't increase as rapidly as before and its shape is deformed, like for a saturation, as shown in Fig.4. The corresponding phenomenon is observed on the EMF, Fig. 5, as it is the derivative of the former. This implies that the obtained electrical signal is no longer sinusoidal but that a distortion appears.

A FFT decomposition of the EMF shows which harmonic frequencies appear when the excitation amplitude varies. We note E_n the amplitude of the n^{th} harmonic of the the EMF. We study their relative amplitudes, with regard to the fundamental, expressed in decibels (dB). This allows us to see how much a harmonic frequency is attenuated with regard to the fundamental and to conclude whether it is important or not. Table II shows the values

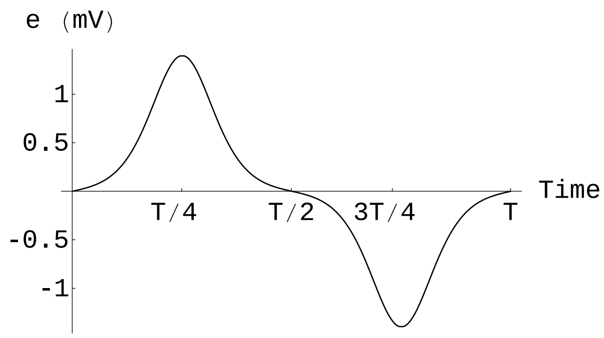

Fig. 5. Electromotive force for an excitation of the string $Y_I = 10mm$ along Oy. String altitude:$d = 5mm$. Magnet position $L/4$: neck pickup.

Harmonic rank	3	5	7
$\frac{E_n}{E_1}$ (dB)	-15	-34	-56

TABLE II
RELATIVE AMPLITUDE IN dB OF THE HARMONICS FOR AN EXCITATION $Y_I = 6mm$ ALONG y, STRING INITIALLY CENTRED.

Harmonic rank	2	3	4	5	6
$\frac{E_n}{E_1}$ (dB)	-7	-25	-25	-48	-50

TABLE III
RELATIVE AMPLITUDE IN dB OF THE HARMONICS FOR AN EXCITATION $Y_I = 6mm$ ALONG y, STRING INITIALLY SHIFTED OF $3mm$.

for the EMF. There are only odd rank harmonics. This is because of the symetry of the movement in the symetrical flux density. When the excitation increases, the level of the harmonics increases and so, the number of significant harmonics increases. This corresponds to the increasing distortion.

2) Shifted string: When the initial position of the string is not above the center of the face of the magnet, the flux density is no longer symetrical for the string movement. As a consequence, the EMF shows another type of distortion, that is dissymetrical (Fig.6). The FFT decomposition shows that even harmonics also appear.

It is interesting to notice that if the fundamental frequency is $110Hz$, which means that the note A has been played, the second harmonic frequency is $220Hz$ and corresponds to the note A, but an octave higher. The third harmonic frequency is $330Hz$ and the corresponding note is the E, which is the fifth of a A. This shows that added notes appear, that are different from the played one. We must remark here that a pickup is not considered as a linear movement sensor: we don't look for a linear response, but we want to generate harmonics. The question is rather which harmonics and with what amplitude. And although this analysis considers only a sinusoidal vibration of the string, in reality any note played on the guitar will excite many higher string modes, which will have the same harmonic frequencies (neglecting string stiffness) as the EMF distortions treatd here. Hence the importance of being able to calculate each contribution to a frequency.

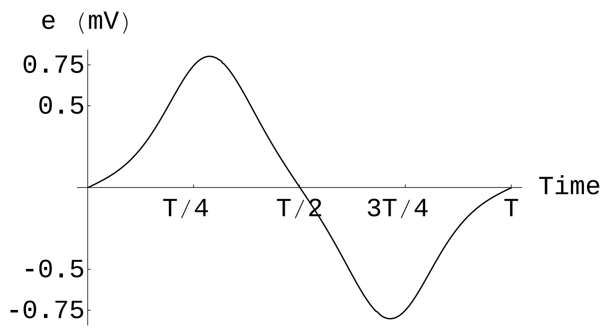

Fig. 6. Electromotive force for an excitation of the string $Y_I = 6mm$ along y, string initially shifted of $3mm$. String altitude:$d = 5mm$. Magnet position $L/4$: neck pickup.

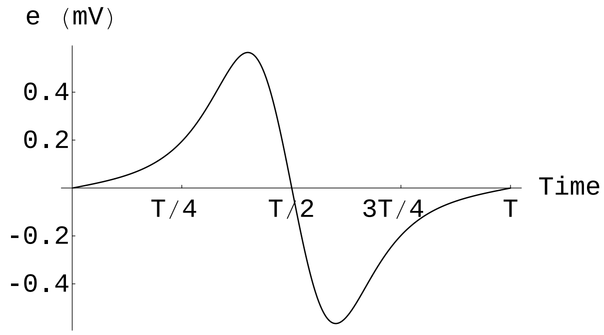

Fig. 7. Electromotive force for an excitation of the string $Z_I = 6mm$ along z. String altitude: $d = 5mm$. Magnet position $L/4$: neck pickup.

B. Movement in the xz plane

As said in the introduction, the real movement of the string is elliptical and we study both movements in the xy and in the xz planes. So, we want to study now the movement in a plane parallel to the xz plane. The cut flux is then calculated with the B_y component of the flux density. If the string is centred above the magnet and its position along the y axis is $y = 0$ nothing will happen, as the B_y component equals zero. So, we have to consider an initial position of the string shifted along the y axis, of $1mm$, for example, which corresponds to the quarter of the magnet's side. We consider an excitation along the z axis, $Z_I = 6mm$. This is largely exaggerated with regard with the real amplitude, but allows to emphazise the phenomenon for a better understanding. The cut surface is then the surface delimited in the xz plane by the straight line of the string at rest and the sine shape of the string when it moves. Fig.7 shows how the EMF gets distorted in this case and we can see that the distortion is not the same as the ones shown in the preceeding section. The real EMF will contain all the distortions, but the contribution of the ones in the xy plane have a heavier weight in the total signal.

This study only gives a hint of the real EMF. Indeed, during the ellipsoidal movement, the cut flux is not the same for each alternance of the string. And as the frequencies of the modes are slightly different, the string never goes through exactly the same area. The composition of the movements has to be made on the vectors, moduli and phases, so, is quite complicated.

Nevertheless, the interesting thing is that we are able to calculate the harmonics generated for more or less complicated hypotheses.

IV. Conclusion

This paper presents a method to study PM pickups for electric guitars. Through the study of the magnetic circuit, we show that the electromotive force in the pickup can be considered as proportional to the magnetic flux cut by the string when it moves. This is interesting, because the cut flux can be calculated analytically, and the analytical approach allows the calculation of magnetic fluxes and of their variations with a precision good enough to study quantitatively the harmonics of the electromotive force, so, of the sound. The paper thus gives a model of the pickup, which is simplified but that can be used to design a pickup on a quantitative basis. Indeed, a pickup is not a linear movement sensor, but is designed to generate harmonics, which are very important from an acoustical point of view, as the richness of a sound is clearly related to them. So, this study is a first step towards the dimensioning of pickups to have a specifically desired sound.

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